## Magnetic Lens

## Introduction

Scanning electron microscopes image samples by scanning with a high-energy beam of electrons. The subsequent electron interactions produce signals such as secondary and back-scattered electrons than contain information about the sample surface topography. Electromagnetic lenses are used to focus this electron beam down to a spot about 10 nm wide on the sample surface.

Note: This model requires the Particle Tracing Module.

## Model Definition

Particles (electrons) are released from near the bottom boundary of the simulation space and pass through a collimator. This collimator can typically be adjusted to remove stray electrons. A simple DC coil produces an axial magnetic field. This rotationally symmetric, inhomogeneous magnetic field results in non-axial electrons experiencing a radial force causing them to spiral about the axis. As they begin to spiral, they have a larger velocity component perpendicular to the mainly axial magnetic field, therefore the radius of their spiral/helical path decreases. Thus, a parallel beam of electrons entering the lens will converge to a point.

If the region in which the magnetic field acts upon the electrons is sufficiently small, this coil acts as a 'thin' convex lens and the thin lens expression holds.

## MODEL EQUATIONS

A simple model is set up to test the magnetic force within the Particle Tracing for ACDC interface. The equations solved are the equation of motion of a charged particle in a magnetic field (Lorentz force):

$$
\begin{equation*}
\frac{d}{d t}(m \mathbf{v})=q(\mathbf{v} \times \boldsymbol{B}) \tag{1}
\end{equation*}
$$

where $q$ is the particle charge (C), $\mathbf{v}$ is the particle velocity ( $\mathrm{m} / \mathrm{s}$ ) and $\mathbf{B}$ is the magnetic flux density (SI unit: T). The total work done on a particle by a magnetic field is zero.

## Results and Discussion

The magnetic flux density is plotted in Figure 1. The strength of the lens depends upon the coil configuration and current. The lens within electron microscopes are generally very strong, in some cases focusing the electron beam within the lens itself.


Figure 1: Plot of the magnetic flux density in the magnetic lens.
Figure 2 plots the electron trajectories as they travel through the coil. The electrons are focused at a point along the z -axis. The focal length is given by:

$$
\begin{equation*}
f=K \frac{V}{i} \tag{2}
\end{equation*}
$$

where $K$ is a constant based on the coil geometry and number of turns, $V$ is the accelerating voltage and $i$ is the coil current. The focal length increases with electron energy (i.e. V) because their high velocity means they spend less time experiencing a force due the magnetic field. However, as the current increases so does the magnetic
field strength, therefore the electrons spiral in tighter paths bringing the focal length closer.


Figure 2: Plot of the electron trajectories travelling through the magnetic lens.
The ability to change the focal length of a lens is useful as it allows the focusing onto a surface in addition to adjusting the magnification. The effect of the focusing can be seen in Figure 3 which shows a Poincaré map of the particle position at three different snapshots in time. The sharpness of the cross-over can be improved using multiple lenses.


Figure 3: Poincaré plot of the particle location in the xy-plane initially (red), at the focal point of the lens (blue) and at the last time step (black).

Reference

1. M.J.Pritchard, Manipulation of Ultracold Atoms Using Magnetic and Optical Fields, PhD thesis, Durham University, September 2006, http://massey.dur.ac.uk/ resources/mjpritchard/thesis_pritchard.pdf.

Model Library path: ACDC_Module/Particle_Tracing/magnetic_lens

## Modeling Instructions

## NEW

I In the New window, click the Model Wizard button.

## MODEL WIZARD

I In the Model Wizard window, click the 3D button.
2 In the Select physics tree, select AC/DC>Magnetic Fields (mf).
3 Click the Add button.
4 Click the Study button.
5 In the tree, select Preset Studies>Stationary.
6 Click the Done button.

## GLOBAL DEFINITIONS

## Parameters

I On the Home toolbar, click Parameters.
2 In the Parameters settings window, locate the Parameters section.
3 In the table, enter the following settings:

| Name | Expression |
| :--- | :--- |
| Vaccel | $500[\mathrm{~V}]$ |
| Ic | $0.32[\mathrm{~A}]$ |
| velec | $\operatorname{sqrt}\left(\left(2^{*}\right.\right.$ Vaccel*e_const $) /($ me_const $\left.)\right)$ |
| Nc | 1000 |

## GEOMETRY I

Build a simple coil geometry using cylinders.
I In the Model Builder window, under Component I click Geometry I.
2 In the Geometry settings window, locate the Units section.
3 From the Length unit list, choose mm.
Cylinder I
I On the Geometry toolbar, click Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 10.
4 In the Height edit field, type 2.5.

## Cylinder 2

I On the Geometry toolbar, click Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.

3 In the Radius edit field, type 6.
4 In the Height edit field, type 2.5.
5 Click the Build Selected button.

## Cylinder 3

I In the Model Builder window, under Component I>Geometry I right-click Cylinder I and choose Duplicate.

2 In the Cylinder settings window, locate the Position section.
3 In the $\mathbf{z}$ edit field, type -7.5.
Cylinder 4
I On the Geometry toolbar, click Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 2.
4 In the Height edit field, type 2.5.
5 Locate the Position section. In the $\mathbf{z}$ edit field, type -7.5.

## Cylinder 5

I Right-click Cylinder I and choose Duplicate.
2 In the Cylinder settings window, locate the Position section.
3 In the $\mathbf{z}$ edit field, type -2.5.

## Cylinder 6

I On the Geometry toolbar, click Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 3.
4 In the Height edit field, type 2.5.
5 Locate the Position section. In the $\mathbf{z}$ edit field, type -2.5.

## Cylinder 7

I Right-click Cylinder I and choose Duplicate.
2 In the Cylinder settings window, locate the Position section.
3 In the $\mathbf{z}$ edit field, type 2.5.

## Cylinder 8

I On the Geometry toolbar, click Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.

3 In the Radius edit field, type 3.
4 In the Height edit field, type 2.5.
5 Locate the Position section. In the $\mathbf{z}$ edit field, type 2.5.
Cylinder 9
I On the Geometry toolbar, click Cylinder.
2 In the Cylinder settings window, locate the Size and Shape section.
3 In the Radius edit field, type 20.
4 In the Height edit field, type 50.
5 Locate the Position section. In the $\mathbf{z}$ edit field, type -15.
Difference I
I On the Geometry toolbar, click Difference.
2 Select the objects cyl5, cyl3, cyII, and cyl7 only.
3 In the Difference settings window, locate the Difference section.
4 Select the Objects to subtract toggle button.
5 Select the objects cyl2, cyl8, cyl6, and cyl4 only.
6 Click the Build Selected button.
7 Click the Go to Default 3D View button on the Graphics toolbar.

## Work Plane I

I On the Geometry toolbar, click Work Plane.
2 Click the Wireframe Rendering button on the Graphics toolbar.
3 In the Work Plane settings window, locate the Plane Definition section.
4 From the Plane type list, choose Face parallel.
5 Select the Planar face toggle button.
6 On the object difI, select Boundary 3 only.

## Plane Geometry

Click the Zoom Extents button on the Graphics toolbar.

## Circle I

I In the Model Builder window, under Component I>Geometry I>Work Plane I right-click Plane Geometry and choose Circle.

2 In the Circle settings window, locate the Size and Shape section.
3 In the Radius edit field, type 2.

4 Click the Build Selected button.
Last, create a circular edge to be used in the Multi-Turn Coil feature as a reference edge.

## Work Plane 2

I On the Geometry toolbar, click Work Plane.
2 In the Work Plane settings window, locate the Plane Definition section.
3 From the Plane type list, choose Face parallel.
4 Select the Planar face toggle button.
5 On the object difI, select Boundary 13 only.


## Circle I

I In the Model Builder window, under Component I>Geometry I>Work Plane 2 right-click Plane Geometry and choose Circle.
2 In the Circle settings window, locate the Object Type section.
3 From the Type list, choose Curve.
4 Locate the Size and Shape section. In the Radius edit field, type 8.
5 On the Home toolbar, click Build All.

## MATERIALS

Add materials for the air domain and metal collimator and coil.

## Material I

I In the Model Builder window, under Component I right-click Materials and choose New Material.

2 In the Material settings window, locate the Material Contents section.
3 In the table, enter the following settings:

| Property | Name | Value |
| :--- | :--- | :--- |
| Electrical conductivity | sigma | 6 e 7 |
| Relative permittivity | epsilonr | 1 |
| Relative permeability | mur | 1 |

## Material 2

I In the Model Builder window, right-click Materials and choose New Material.
2 Select Domain 1 only.
3 In the Material settings window, locate the Material Contents section.
4 In the table, enter the following settings:

| Property | Name | Value |
| :--- | :--- | :--- |
| Electrical conductivity | sigma | 0 |
| Relative permittivity | epsilonr | 1 |
| Relative permeability | mur | 1 |

MAGNETIC FIELDS

## Multi-Turn Coil I

I On the Physics toolbar, click Domains and choose Multi-Turn Coil.
2 Select Domain 4 only.
3 In the Multi-Turn Coil settings window, locate the Coil Type section.
4 From the list, choose Circular.
5 Locate the Multi-Turn Coil section. In the $N$ edit field, type Nc.
6 In the $I_{\text {coil }}$ edit field, type Ic.
Specify the reference edges to be used in the calculation of the current path for the circular coil. To obtain the best results, the selected edges should have a radius close
to the average coil radius. In this case, select the edges intentionally created in previous steps.

Reference Edge I
I Right-click Component I>Magnetic Fields>Multi-Turn Coil I and choose Edges $>$ Reference Edge.
2 In the Reference Edge settings window, locate the Edge Selection section.
3 Click Clear Selection.
4 Select Edges 22, 23, 57, and 82 only.

MESH I
Scale I
I In the Model Builder window, under Component I right-click Mesh I and choose Scale.
2 In the Scale settings window, locate the Geometric Entity Selection section.
3 From the Geometric entity level list, choose Domain.
4 Select Domains 2-5 only.
5 Locate the Scale section. In the Element size scale edit field, type 0.5.

## Free Triangular I

I On the Mesh toolbar, click Boundary and choose Free Triangular.
2 Select Boundary 30 only.

## Size I

I Right-click Component I>Mesh I>Free Triangular I and choose Size.
2 In the Size settings window, locate the Element Size section.
3 From the Predefined list, choose Extremely fine.
4 Click to expand the Element size parameters section. Locate the Element Size section. Click the Custom button.

5 Locate the Element Size Parameters section. Select the Maximum element size check box.

6 In the associated edit field, type 0.4.

## Free Tetrahedral I

I In the Model Builder window, right-click Mesh I and choose Free Tetrahedral.
2 In the Settings window, click Build AII.

## STUDY I

On the Home toolbar, click Compute.

## RESULTS

## Magnetic Flux Density Norm (mf)

I In the Model Builder window, expand the Results>Magnetic Flux Density Norm (mf) node and click Multislice I.

2 In the Multislice settings window, locate the Multiplane Data section.
3 Find the z-planes subsection. In the Planes edit field, type 0.
4 On the 3D plot group toolbar, click Plot.

## COMPONENT I

On the Home toolbar, click Add Physics.

## ADD PHYSICS

I Go to the Add Physics window.
2 In the Add physics tree, select AC/DC>Charged Particle Tracing (cpt).
3 Find the Physics in study subsection. In the table, enter the following settings:

| Studies | Solve |
| :--- | :--- |
| Study I | $\times$ |

4 In the Add physics window, click Add to Component.
5 On the Home toolbar, click Add Physics.

ROOT
On the Home toolbar, click Add Study.

## ADD STUDY

I Go to the Add Study window.
2 Find the Studies subsection. In the tree, select Preset Studies>Time Dependent.
3 Find the Physics in study subsection. In the table, enter the following settings:

| Physics | Solve |
| :--- | :--- |
| Magnetic Fields (mf) | $\times$ |
| 4 In the Add study window, click Add Study. |  |

5 On the Home toolbar, click Add Study.

## GEOMETRY I

In the Model Builder window, collapse the Component I>Geometry I node.

## CHARGED PARTICLE TRACING

I In the Model Builder window, click Component I>Charged Particle Tracing.
2 In the Charged Particle Tracing settings window, locate the Domain Selection section.
3 Click Clear Selection.
4 Select Domain 1 only.

## Magnetic Force I

You need to provide the forces acting on the particles; in this case, the magnetic (Lorentz) force.

I On the Physics toolbar, click Domains and choose Magnetic Force.
2 Select Domain 1 only.
3 In the Magnetic Force settings window, locate the Magnetic Force section.
4 From the $\mathbf{B}$ list, choose Magnetic flux density (mf).
Inlet I
I On the Physics toolbar, click Boundaries and choose Inlet.
2 Select Boundary 30 only.
3 In the Inlet settings window, locate the Initial Position section.
4 From the Initial position list, choose Projected plane grid.
5 In the $N$ edit field, type 10000.
6 Locate the Initial Velocity section. Specify the $\mathbf{v}_{0}$ vector as

| 0 | $x$ |
| :--- | :--- |
| 0 | $y$ |
| velec | $z$ |

STUDY 2

## Step I: Time Dependent

I In the Model Builder window, expand the Study 2 node, then click Step I: Time Dependent.

2 In the Time Dependent settings window, click to expand the Values of dependent variables section.

3 Locate the Values of Dependent Variables section. Select the Values of variables not solved for check box.

4 From the Method list, choose Solution.
5 From the Study list, choose Study I, Stationary.
6 Locate the Study Settings section. Click the Range button.
7 Go to the Range dialog box.
8 From the Entry method list, choose Number of values.
9 In the Stop edit field, type 5e-9.
10 In the Number of values edit field, type 25.
II Click the Replace button.
I2 On the Home toolbar, click Compute.

## RESULTS

## Particle Trajectories (cpt)

I In the Model Builder window, under Results>Particle Trajectories (cpt) click Particle Trajectories I.

2 In the Particle Trajectories settings window, locate the Coloring and Style section.
3 Find the Line style subsection. From the Type list, choose Line.
4 Find the Point style subsection. From the Type list, choose None.
5 In the Model Builder window, under Results>Particle Trajectories (cpt)>Particle Trajectories I click Color Expression I.

6 In the Color Expression settings window, locate the Expression section.
7 In the Expression edit field, type sqrt(cpt.Ftx^2+cpt.Fty^2+cpt.Ftz^2).

8 On the 3D plot group toolbar, click Plot.


Now construct a Poincaré map to visualize the radial distribution of particles initially, at the focal point and at the exit of the modeling domain.

## Data Sets

I On the Results toolbar, click Cut Plane.
2 In the Cut Plane settings window, locate the Plane Data section.
3 From the Plane list, choose xy-planes.
4 In the z-coordinate edit field, type - 6 .
5 Locate the Data section. From the Data set list, choose Particle I.
6 Right-click Results>Data Sets>Cut Plane I and choose Duplicate.
7 In the Cut Plane settings window, locate the Plane Data section.
8 In the z-coordinate edit field, type 7.
9 Right-click Results>Data Sets>Cut Plane 2 and choose Duplicate.
10 In the Cut Plane settings window, locate the Plane Data section.
II In the z-coordinate edit field, type 34.

## 2D Plot Group 3

I On the Home toolbar, click Add Plot Group and choose 2D Plot Group.

2 On the 2D plot group toolbar, click More Plots and choose Poincaré Map.
3 In the Poincaré Map settings window, locate the Data section.
4 From the Cut plane list, choose Cut Plane 3.
5 Locate the Coloring and Style section. From the Color list, choose Black.
6 Click the Zoom Extents button on the Graphics toolbar.
7 Select the Radius scale factor check box.
8 In the associated edit field, type 4E-2.
9 Right-click Results>2D Plot Group 3>Poincaré Map I and choose Duplicate.
10 In the Poincaré Map settings window, locate the Data section.
II From the Cut plane list, choose Cut Plane I.
I2 Locate the Coloring and Style section. In the Radius scale factor edit field, type 2E-2.
13 From the Color list, choose Red.
14 Right-click Results>2D Plot Group 3>Poincaré Map 2 and choose Duplicate.
15 In the Poincaré Map settings window, locate the Data section.
16 From the Cut plane list, choose Cut Plane 2.
17 Locate the Coloring and Style section. From the Color list, choose Blue.
18 In the Radius scale factor edit field, type 5E-3.
19 On the 2D plot group toolbar, click Plot.

20 Click the Zoom Extents button on the Graphics toolbar.
Poincaré map Poincaré map Poincaré map


