Dipolar Microwave Plasma Source

Introduction

This model presents a 2D axisymmetric dipolar microwave plasma source sustained through resonant heating of the electrons. This is known as electron cyclotron resonance (ECR), which occurs when a suitable high magnetic flux density is present along with the microwaves.

This is an advanced model that showcases many of the features that make COMSOL unique, including:

- Infinite elements for the magnetostatic model.
- Functional-based mesh adaption to create a fine mesh on the ECR surface.
- PMLs for the electromagnetic waves to represent infinite space.
- Degrees of freedom for all 3 components of the high-frequency electric field despite the fact that the problem is geometrically axisymmetric.
- Full anisotropic tensors for the plasma conductivity and charged particle transport properties.
- Resonant power absorption in the ECR surface by the electrons.
- Equation-based modeling using integrated quantities to fix the total absorbed power.
- Solver sequencing to first compute the static magnetic field, then solve for all the plasma components.

Note: This model requires the Plasma Module, AC/DC Module, and RF Module.

Model Definition—Static Magnetic Field

For the static magnetic field, Ampère's law governs the azimuthal component of the magnetic vector potential:

$$\nabla \times \mu_{\rm r}^{-1} {\mu_0}^{-1} (\nabla \times A_{\varphi}) = J_{\varphi}$$

where the external current density, J_{ϕ} only has an azimuthal component and is defined in the coil as:

$$J_{\varphi} = \frac{NI}{A}$$

where N is the number of turns in the coil I is the total current and A is the cross sectional area. To represent the fact that the coil is in free space, infinite elements are used far away from the coil, as shown in Figure 1. A stationary study type is used to compute the static magnetic field. This field is then fed into a self consistent model for the plasma.

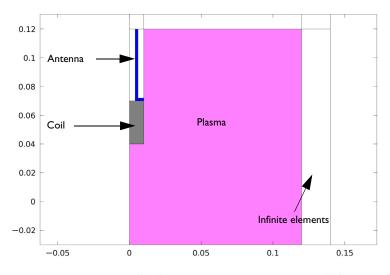


Figure 1: Basic concept for the plasma source. A stationary azimuthal current flows in the coil which generates a static magnetic field in the surroundings. Resonant heating of the electrons occurs on the contour of the critical magnetic flux density.

The plasma conductivity becomes a full tensor in the presence of a static magnetic field. At some critical magnetic field the electrons continually gain energy from both the electric and magnetic fields over one RF cycle. This leads to a resonance zone in the plasma where the incoming electromagnetic wave is completely absorbed over a very short distance. The critical magnetic field is only dependent on the angular frequency, the electron mass and charge:

$$B_{\rm cr} = \frac{\omega m_e}{q}$$

At 2.45 GHz the critical magnetic flux density is 875 gauss or 0.0875 T. Therefore you can use functional-based mesh adaption to ensure that the ECR surface is adequately meshed for the plasma model. The functional is somewhat arbitrary; it is chosen such that it is zero everywhere but becomes large at the resonant magnetic flux density. In this model, use the functional

$$f = \frac{1}{\|\mathbf{B}\| - 0.0875| + \delta}$$
(1)

where δ is a small number to prevent division by zero.

Model Definition—Microwave Plasma

In this example, you solve the following wave equation for the high-frequency component of the electric field in the frequency domain:

$$\nabla \times \mu_0^{-1} (\nabla \times \mathbf{E}) - k_0^2 \left(\varepsilon_r - \frac{j \boldsymbol{\sigma}}{\omega \varepsilon_0} \right) \cdot \mathbf{E} = 0$$

Here σ is the plasma conductivity, which is a full tensor and a function of the electron density, collision frequency, and the static magnetic flux density. Using the definitions

$$\alpha = \frac{q}{m_e(v_e + j\omega)} \tag{2}$$

where q is the electron charge, m_e is the electron mass, v_e is the electron-neutral collision frequency, and ω is the angular frequency. The inverse of the plasma conductivity is defined as

$$qn_e \mathbf{\sigma}^{-1} = \begin{bmatrix} 1 & -\alpha B_z & \alpha B_y \\ \alpha B_z & 1 & -\alpha B_x \\ -\alpha B_y & \alpha B_x & 1 \end{bmatrix}$$
(3)

where n_e is the electron number density. Using the inverse of the plasma conductivity is convenient because it can be written in a compact form. COMSOL automatically computes the tensor form of the plasma conductivity for you by inverting Equation 3. Because the plasma conductivity tensor is a full tensor, all three components of the electric field are computed despite the fact that the only excitation from the coaxial port occurs in the *rz*-plane. The nonlinearity in the plasma conductivity can be seen in Figure 2. The surface represents four of the components of the nondimensional plasma conductivity versus the *r*- and *z*-components of the magnetic flux density (indicated by the *x*-axis and *y*-axis, respectively) on a log scale. At the resonant flux density (0.0875 T) the plasma conductivity is more than 10^6 higher than the case where no static magnetic field is present.

In Ref. 1 the size of the resonance is smoothed over a distance which can be resolved by the mesh. It is argued that this has a physical basis corresponding to collision-less heating. In Ref. 2 the physical reasoning behind the broadening of the resonance zone is doppler shifting of the electrons into resonance. The same smoothing used in Ref. 1 is available in COMSOL by selecting the **Doppler broadening** check box in the Microwave Plasma interface properties. In this case, the collision frequency, v_e in Equation 2 is replaced by an effective collision frequency:

$$\tilde{\mathbf{v}}_e = \mathbf{v}_e + \frac{\omega}{\delta} \tag{4}$$

where δ is chosen to be 20. This is very simple from an implementation point of view but does lead to unphysical power absorption away from the resonance zone. The approach taken in Ref. 2 leads to the ECR surface being broadened only at the resonance zone.

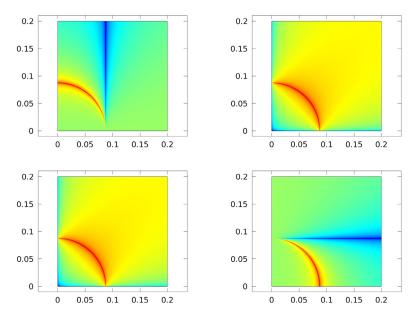


Figure 2: Plots of the four components of the plasma conductivity tensor.

Compute the electron number density and electron energy density by solving a pair of drift-diffusion equations:

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot \left[-n_e(\mathbf{\mu}_e \bullet \mathbf{E}) - \mathbf{D}_e \bullet \nabla n_e \right] = R_e$$
$$\frac{\partial}{\partial t}(n_{\varepsilon}) + \nabla \cdot \left[-n_{\varepsilon}(\mathbf{\mu}_{\varepsilon} \bullet \mathbf{E}) - \mathbf{D}_{\varepsilon} \bullet \nabla n_{\varepsilon} \right] + \mathbf{E} \cdot \mathbf{\Gamma}_e = R_{\varepsilon}$$

The electron source R_e and the energy loss due to inelastic collisions R_{ϵ} are defined later. The electron diffusivity, energy mobility and energy diffusivity are calculated from the electron mobility using

$$\mathbf{D}_e = \mathbf{\mu}_e T_e, \mathbf{\mu}_{\varepsilon} = \left(\frac{5}{3}\right) \mathbf{\mu}_e, \mathbf{D}_{\varepsilon} = \mathbf{\mu}_{\varepsilon} T_e$$

The electron transport properties are, like the plasma conductivity, full tensors. The electron mobility in the direction of the magnetic field lines is up to 8 orders of magnitude higher than the cross-field electron mobility. As such, electrons are only transported along magnetic field lines. The inverse of the electron mobility can be written in compact form as

$$\boldsymbol{\mu}_{e}^{-1} = \begin{bmatrix} \frac{1}{\mu_{dc}} & -B_{z} & B_{y} \\ B_{z} & \frac{1}{\mu_{dc}} & -B_{x} \\ -B_{y} & B_{x} & \frac{1}{\mu_{dc}} \end{bmatrix}$$
(5)

where μ_{dc} is the electron mobility in the absence of a magnetic field. COMSOL automatically inverts the matrix in Equation 5 for you. The source coefficients in the above equations are determined by the plasma chemistry and are written using rate coefficients. Suppose that there are M reactions which contribute to the growth or decay of electron density and P inelastic electron-neutral collisions. In general P >> M. In the case of rate coefficients, the electron source term is given by

$$R_e = \sum_{j=1}^{M} x_j k_j N_n n_e$$

where x_j is the mole fraction of the target species for reaction j, k_j is the rate coefficient for reaction j (m³/s), and N_n is the total neutral number density (1/m³). The electron energy loss is obtained by summing the collisional energy loss over all reactions:

$$R_{\varepsilon} = \sum_{j=1}^{P} x_j k_j N_n n_e \Delta \varepsilon_j$$

where $\Delta \varepsilon_j$ is the energy loss from reaction j (V). The electron source and inelastic energy loss are automatically computed by the multiphysics interface. The rate coefficients may be computed from cross section data by the following integral:

$$k_k = \gamma \int_0^\infty \varepsilon \sigma_k(\varepsilon) f(\varepsilon) d\varepsilon$$

where $\gamma = (2q/m_e)^{1/2} (C^{1/2}/kg^{1/2})$, m_e is the electron mass (kg), ε is energy (V), σ_k is the collision cross section (m²), and *f* is the electron energy distribution function. In this model the distribution function is chosen to be Maxwellian:

$$f(\varepsilon) = \phi^{-3/2} \beta_1 \exp(-(\varepsilon \beta_2 / \phi))$$

where ϕ is the mean electron energy:

$$\phi = \frac{n_{\varepsilon}}{n_{e}}$$

and

$$\beta_1 = \Gamma(5/2)^{3/2} \Gamma(3/2)^{-5/2}, \beta_2 = \Gamma(5/2) \Gamma(3/2)^{-1}$$

where Γ is the gamma function. The heating term, $\mathbf{E} \cdot \Gamma_e$ has two components, one due to electron motion in the ambipolar field in the *rz*-plane and one due to heating of the electrons by the microwaves. Heating due to the microwaves is handled in the same way as described in Ref. 1. The power transferred from the electromagnetic field to the electrons is normalized so that 10 W of total power is absorbed by the electrons. This is accomplished by multiplying the heating term by a factor, α , which is defined as:

$$\alpha = \frac{10 W}{\iiint Q_{\text{ind}} dV} \tag{6}$$

The result of this is that exactly 10 W of total power is transferred from the electromagnetic field to the electrons. If you did not apply this renormalization of the

absorbed power then there would be nothing to stop the plasma from simply self-extinguishing or absorbing an inordinate amount of power. This approach is perfectly valid due to the fact that the microwave equations are linear. The only drawback from this method is that the S-parameters given on the coaxial port will not be valid because the fields are decoupled from the plasma model. Furthermore, it is not possible to self-consistently compute the ratio of the absorbed and reflected power through the excitation port, a quantity that may be of interest.

For nonelectron species, the following equation is solved for the mass fraction of species *k*:

$$\rho \frac{\partial}{\partial t} (w_k) + \rho (\mathbf{u} \cdot \nabla) w_k = \nabla \cdot \mathbf{j}_k + R_k$$
(7)

As with the electrons, the ion transport properties are functions of the static magnetic flux density. The magnetic force is included as it can generate a significant ion velocity in the azimuthal direction close to the antenna. The ion mobility is also a function of the ambipolar electric field and is specified as a look up table. The ion diffusion velocity, \mathbf{v}_k , is related to the diffusive flux via

$$\mathbf{j}_k = \rho \omega \mathbf{v}_k$$

where

$$\mathbf{v}_{k} = D_{m} \nabla \ln(w) + D_{m} \nabla \ln(M) + Z \mu(\mathbf{E} + \mathbf{v}_{k} \times \mathbf{B})$$
(8)

Equation 7 can be re-arranged to give an expression for the diffusion velocity as a function of the other variables:

$$\mathbf{v}_{k} = \mathbf{A}^{-1}[D_{m}\nabla \ln(w) + D_{m}\nabla \ln(M) + Z\mu\mathbf{E}]$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & -Z\mu B_z & Z\mu B_y \\ Z\mu B_z & 1 & -Z\mu B_x \\ -Z\mu B_y & Z\mu B_x & 1 \end{bmatrix}$$

COMSOL automatically inverts Equation 8 when defining the diffusion velocity for each of the ionic species. The electrostatic field is computed using the following equation:

$$-\nabla \cdot \varepsilon_0 \varepsilon_r \nabla V = \rho$$

The space charge density ρ is automatically computed based on the plasma chemistry specified in the model using the formula

$$\rho = q \left(\sum_{k=1}^{N} Z_k n_k - n_e \right)$$

For detailed information about electrostatics see Theory for the Electrostatics Interface in the *Plasma Module User's Guide*.

PLASMA CHEMISTRY

The model considers argon plasma chemistry with the following set of collisions including elastic, excitation, direct ionization and stepwise ionization. Penning ionization and metastable quenching are also included in the model.

REACTION	FORMULA	ТҮРЕ	$\Delta\epsilon(eV)$
I	e+Ar=>e+Ar	Elastic	0
2	e+Ar=>e+Ars	Excitation	11.5
3	e+Ars=>e+Ar	Superelastic	-11.5
4	e+Ar=>2e+Ar+	Ionization	15.8
5	e+Ars=>2e+Ar+	Ionization	4.24
6	Ars+Ars=>e+Ar+Ar+	Penning ionization	-
7	Ars+Ar=>Ar+Ar	Metastable quenching	-

TABLE I: TABLE OF COLLISIONS AND REACTIONS MODELED

On surfaces, the following two reactions are considered:

TABLE 2: TABLE OF SURFACE REACTIONS

REACTION	FORMULA	STICKING COEFFICIENT
1	Ar+=>Ar	1
2	Ars=>Ar	I

BOUNDARY CONDITIONS

The above partial differential equations must be supplemented with a suitable set of boundary conditions. The coaxial port boundary condition is used to drive the electromagnetic waves. The port power is inconsequential due to the normalization scheme used on the absorbed power. For the electrons, neglect reflection as well as secondary and thermal emission to get the following boundary condition on the electron flux:

$$-\mathbf{n}\cdot\Gamma_{e} = \left(\frac{1}{2}\mathbf{v}_{e,\,\mathrm{th}}n_{e}\right)$$

and the electron energy flux:

$$-\mathbf{n}\cdot\Gamma_{\varepsilon} = \left(\frac{5}{6}\mathbf{v}_{e,\,\mathrm{th}}n_{\varepsilon}\right)$$

Losses at the wall for the heavy species is due to surface reactions and migration due to the ambipolar field:

$$-\mathbf{n} \cdot \mathbf{j}_k = M_w R_k + M_w c_k Z \mu_k [(\mathbf{A}^{-1} \cdot \mathbf{E}) \cdot \mathbf{n}] [(Z_k \mu_k (\mathbf{A}^{-1} \cdot \mathbf{E}) \cdot \mathbf{n}) > 0]$$

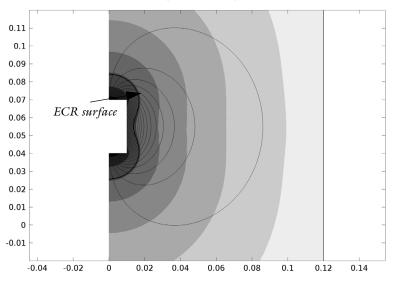
The reactor walls are grounded.

Notes About the COMSOL Implementation

You solve this problem in two stages. First, compute the static magnetic field using adaptive mesh refinement. Then, in a separate study step, you solve for the electron density, electron energy density, mass fraction of argon ions, and mass fraction of electronically excited argon atoms, as well as the plasma potential and the 3 components of the high-frequency electric field. The magnetic flux density computed in the first study step is used to define the tensor plasma conductivity as well as electron and ion transport properties.

Results and Discussion—Static Magnetic Field Model

Figure 3 and Figure 4 present the results from the first study step. As expected, the azimuthal current in the coil generates a static magnetic field that has a "3"-shaped contour at a flux density of 0.0875 T. The magnetic field lines form a circular pattern around the coil, which is important to bear in mind when discuss the transport of the charged particles later.



Contour: log10(mf.normB+eps) Contour: Magnetic vector potential, phi component (Wb/m) Contour: Magnetic flux density norm (T)

Figure 3: Plot of the static magnetic flux density on a log scale (filled contour), magnetic field lines (thin lines) and the ECR surface at 0.0875 T (thick black line).

In Figure 4 the mesh, which has adapted based on the functional given in Equation 1 is shown. The mesh has clearly been significantly refined around the contour of the resonant magnetic flux density. This is required to accurately resolve the region where all the power deposition to the electrons occurs. Functional-based mesh adaption is a feature that makes the finite element approach more attractive than the finite difference approach for ECR modeling.

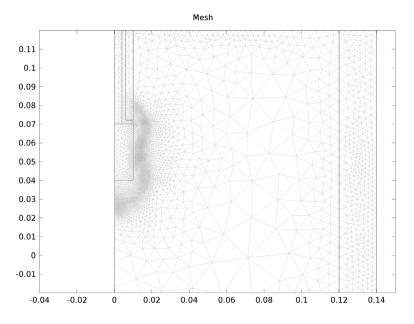
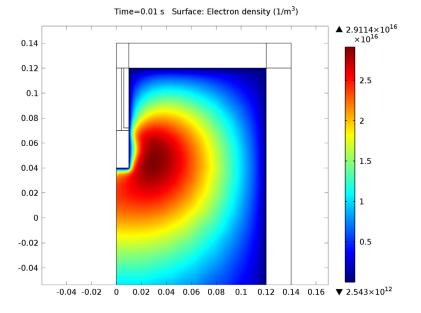


Figure 4: Mesh generated after one refinement using functional based mesh adaption. The mesh is very fine on the ECR surface and relatively coarse away from the resonance zone.

Results and Discussion—Microwave Plasma Model

The electron density at the quasi steady state solution is plotted in Figure 5. The peak electron density is around $5 \cdot 10^{16} \text{ m}^{-3}$ and peaks radially outwards from the center of the coil. The magnitude of the electron number density and its profile agree well with



the results in Ref. 1.

Figure 5: Plot of the electron density at the quasi steady state condition. The peak electron density is still below the critical electron density at the chosen operating frequency.

Despite the sharply peaked heating the electron temperature, plotted in Figure 6 does not show such peaks. Recall from Figure 3 that the magnetic field lines show the circular pattern away from the coil. This leads to strong energy transport along the field lines and very little transport across the magnetic field lines. Indeed the circular pattern along which the electron temperature is constant is consistent with the magnetic field lines. The peak electron temperature is around 3.8 eV and around 1.78 eV below the coil which is again, consistent with the results in Ref. 2.

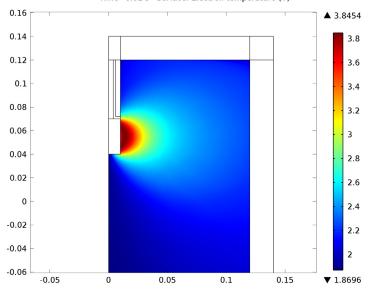


Figure 6: Plot of the electron temperature which peaks at around 3.8 eV

Despite the fact that power is only deposited to the plasma on the ECR surface, the electron temperature, plotted in Figure 6 is not sharply peaked at the critical magnetic flux density. Recall from Figure 3 that the magnetic field lines show the circular pattern away from the coil. The high degree of anisotropy in the electron transport properties results in strong energy transport along the magnetic field lines and little transport across the magnetic field lines. Indeed the circular pattern along which the electron temperature is constant is consistent with the magnetic field lines. The peak electron temperature is around 3.8 eV and around 1.78 eV below the coil which is again, consistent with the results in Ref. 2.

Time=0.01 s Surface: Electron temperature (V)

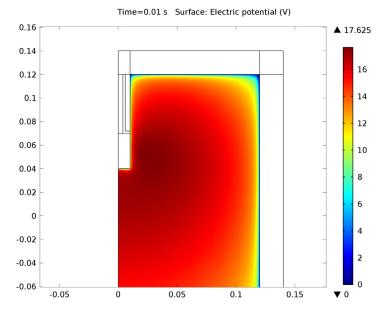


Figure 7: Plot of the plasma potential.

The electron density profile shows no signs of the resonance zone which is clearly seen in Figure 8. The power deposition is very high, peaking at 35 W/cm^3 . All of the power deposition into the plasma from the electromagnetic field occurs in this resonance zone.

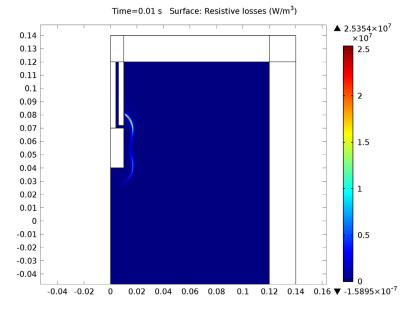


Figure 8: Plot of the power deposition into the plasma. Nearly all the power deposition occurs on the ECR surface.

The ionization source, plotted in Figure 9 is more highly localized around the coil. This corresponds to the region where the electron density and electron temperature are highest. Because the ionization rate scales linearly with the electron density and exponentially with the electron temperature this is to be expected.

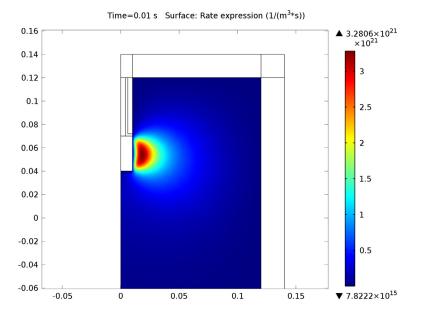


Figure 9: Plot of the rate expression for electrons generated via ionization.

The plasma potential, plotted in Figure 7, peaks at around 16 V. The plasma potential is uniform throughout the plasma, even though the electron temperature shows large variations. The physical basis for the flat plasma potential is explained in Ref. 1.

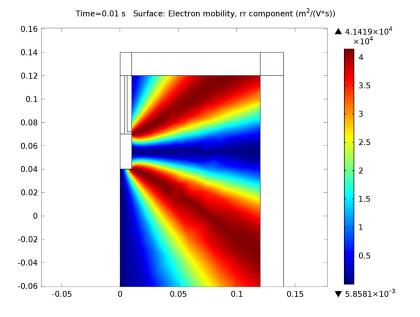


Figure 10: Plot of the electron mobility tensor's rr-component. The mobility varies by 8 orders of magnitude over the space of only a couple of centimeters.

The degree of anisotropy in the electron transport properties can be seen in Figure 10 and Figure 11. In Figure 10 the electron mobility varies by 8 orders of magnitude, it is $4 \cdot 10^4 \text{ m}^2/(\text{Vs})$ towards the coil edges and $10^{-4} \text{ m}^2/(\text{Vs})$ radially outwards from the coil center. In Figure 11 the opposite is true, the electron mobility is very high in the *z* direction at the center of the coil, and very small towards the coil edges. This leads to migration of electrons along the magnetic field lines when they are produced in the ionization region, Figure 9.

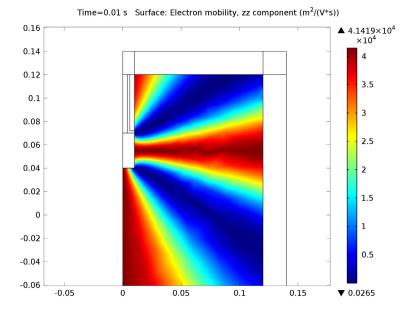


Figure 11: Plot of the electron mobility tensor's zz-component.

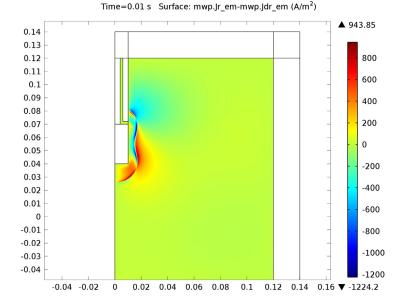


Figure 12: Unnormalized radial component of the microwave conduction current.

The conduction current due to the microwaves is plotted in Figure 12 - Figure 14. The largest component of the conduction current is actually in the azimuthal direction despite the coaxial port only propagating in the TM mode. Despite this, the heating (cooling) due to the dot product of the azimuthal components of the current and electric fields is small, due to the much lower value of the azimuthal component of the electric field.

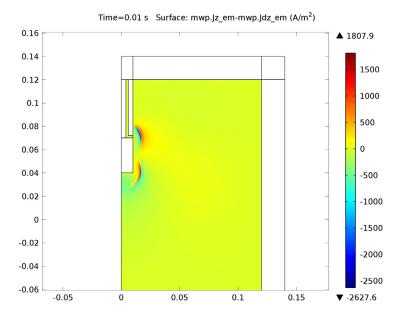


Figure 13: Unnormalized axial component of the microwave conduction current.

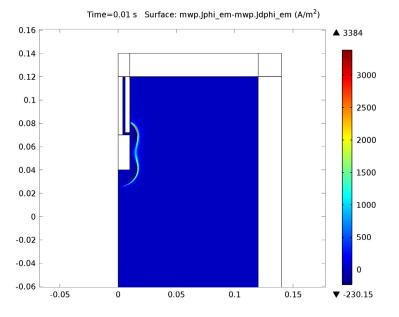


Figure 14: Unnormalized azimuthal component of the microwave conduction current.

Finally, the trace of the plasma conductivity is plotted in Figure 15. The resonance zone is evident and the locally high electrical conductivity leads to the propagating electromagnetic waves to be absorbed.

It is worth mentioning that the electron density in this example model is below the critical plasma density everywhere $(7.4 \cdot 10^{16} \text{ m}^{-3} \text{ at } 2.45 \text{ GHz})$. If either the pressure or the power is increased, the power absorption can shift from the ECR surface to the contour where the plasma density is equal to the critical plasma density. On this contour the phase velocity approaches infinity whereas the group velocity approaches zero. The numerical instabilities caused by this are also smoothed out by adding an effective collision frequency to the actual collision frequency using Equation 4.

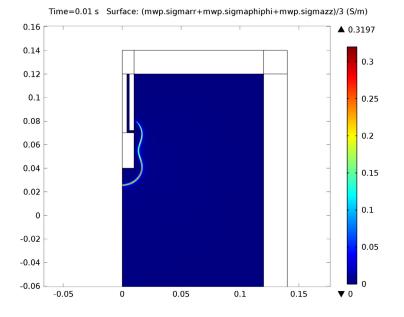


Figure 15: Plot of the trace of the plasma conductivity tensor.

Reference

1. G.J.M. Hagelaar, K. Makasheva, L. Garrigues, and J.-P. Boeuf, "Modelling of a dipolar microwave plasma sustained by electron cyclotron resonance," *J. Phys. D: Appl. Phys.*, vol. 42, p. 194019 (12pp), 2009.

2. R.L. Kinder and M.J. Kushner, "Consequences of mode structure on plasma properties in electron cyclotron resonance sources," *J. Vac. Sci. Technol. A*, vol. 175, Sep/Oct 1999.

Model Library path: Plasma_Module/Wave-Heated_Discharges/ dipolar_ecr_source

Modeling Instructions

From the File menu, choose New.

NEW

I In the New window, click the Model Wizard button.

MODEL WIZARD

- I In the Model Wizard window, click the 2D Axisymmetric button.
- 2 In the Select physics tree, select AC/DC>Magnetic Fields (mf).
- **3** Click the **Add** button.
- 4 Click the **Study** button.
- 5 In the tree, select Preset Studies>Stationary.
- 6 Click the Done button.
- 7 In the Model Builder window's toolbar, click the Show button and select Advanced Physics Options in the menu.

GLOBAL DEFINITIONS

Parameters

- I On the Home toolbar, click Parameters.
- 2 In the Parameters settings window, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description	
r0	0.12	0.12000	Plasma source radius	
z0	0.24	0.24000	Plasma source height	

GEOMETRY I

Rectangle 1

- I In the Model Builder window, under Component I right-click Geometry I and choose Rectangle.
- 2 In the Rectangle settings window, locate the Size section.
- 3 In the Width edit field, type r0.
- 4 In the **Height** edit field, type z0.
- **5** Locate the **Position** section. In the z edit field, type -z0/2.

Rectangle 2

- I In the Model Builder window, right-click Geometry I and choose Rectangle.
- 2 In the Rectangle settings window, locate the Size section.

- **3** In the **Width** edit field, type 0.01.
- 4 In the **Height** edit field, type 0.03.
- **5** Locate the **Position** section. In the **z** edit field, type **0.04**.

Rectangle 3

- I Right-click Geometry I and choose Rectangle.
- 2 In the Rectangle settings window, locate the Size section.
- **3** In the **Width** edit field, type 0.004.
- 4 In the **Height** edit field, type 0.048.
- 5 Locate the **Position** section. In the **r** edit field, type 0.006.
- 6 In the z edit field, type 0.072.

Rectangle 4

- I Right-click Geometry I and choose Rectangle.
- 2 In the **Rectangle** settings window, locate the **Size** section.
- **3** In the **Width** edit field, type 0.004.
- 4 In the **Height** edit field, type 0.05.
- **5** Locate the **Position** section. In the **z** edit field, type **0.07**.

Rectangle 5

- I Right-click Geometry I and choose Rectangle.
- 2 In the **Rectangle** settings window, locate the **Size** section.
- 3 In the Width edit field, type r0-0.01.
- 4 In the **Height** edit field, type 0.02.
- 5 Locate the **Position** section. In the **r** edit field, type 0.01.
- 6 In the z edit field, type 0.12.

Rectangle 6

- I Right-click Geometry I and choose Rectangle.
- 2 In the **Rectangle** settings window, locate the **Size** section.
- **3** In the **Width** edit field, type **0.02**.
- 4 In the **Height** edit field, type 0.02.
- **5** Locate the **Position** section. In the **r** edit field, type **r0**.
- 6 In the z edit field, type 0.12.

Rectangle 7

- I Right-click Geometry I and choose Rectangle.
- 2 In the **Rectangle** settings window, locate the **Size** section.
- **3** In the **Width** edit field, type **0.02**.
- 4 In the **Height** edit field, type z0.
- **5** Locate the **Position** section. In the **r** edit field, type **r0**.
- 6 In the z edit field, type -z0/2.

Rectangle 8

- I Right-click Geometry I and choose Rectangle.
- 2 In the Rectangle settings window, locate the Size section.
- **3** In the **Width** edit field, type 0.02.
- 4 In the **Height** edit field, type 0.02.
- **5** Locate the **Position** section. In the **r** edit field, type **r0**.
- 6 In the z edit field, type -0.02-z0/2.

Rectangle 9

- I Right-click Geometry I and choose Rectangle.
- 2 In the Rectangle settings window, locate the Size section.
- **3** In the **Width** edit field, type r0.
- 4 In the **Height** edit field, type 0.02.
- 5 Locate the Position section. In the z edit field, type -0.02 z0/2.

Rectangle 10

- I Right-click Geometry I and choose Rectangle.
- 2 In the Rectangle settings window, locate the Size section.
- 3 In the Width edit field, type 0.01.
- 4 In the **Height** edit field, type 0.02.
- **5** Locate the **Position** section. In the **z** edit field, type **0.12**.

Bézier Polygon I

- I Right-click Geometry I and choose Bézier Polygon.
- 2 In the Bézier Polygon settings window, locate the Polygon Segments section.
- 3 Find the Added segments subsection. Click the Add Linear button.
- 4 Find the **Control points** subsection. In row I, set r to 0.01.

- **5** In row **I**, set **z** to **0.072**.
- 6 In row 2, set r to 0.01.
- 7 In row 2, set z to 0.07.

DEFINITIONS

- I In the Model Builder window, expand the Component I>Definitions node.
- 2 Right-click Definitions and choose View.

Axis

- I In the Model Builder window, expand the View 2 node, then click Axis.
- 2 In the Axis settings window, locate the Axis section.
- **3** In the **x minimum** edit field, type -0.05.
- 4 In the **x maximum** edit field, type 0.16.
- 5 In the y minimum edit field, type -0.02.
- 6 In the y maximum edit field, type 0.12.
- 7 Click the **Apply** button.

View 2

- I In the Model Builder window, under Component I>Definitions click View 2.
- 2 In the View settings window, locate the View section.
- 3 Select the Lock axis check box.

Explicit 1

- I On the **Definitions** toolbar, click **Explicit**.
- 2 In the Explicit settings window, locate the Input Entities section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select Boundaries 4, 6, 18–20, 22, and 26 only.
- 5 Right-click Component I>Definitions>Explicit I and choose Rename.
- 6 Go to the Rename Explicit dialog box and type Walls in the New name edit field.
- 7 Click OK.

Infinite Element Domain 1

- I On the Definitions toolbar, click Infinite Element Domain.
- 2 In the Infinite Element Domain settings window, locate the Geometry section.
- 3 From the Type list, choose Cylindrical.
- 4 Select Domains 1, 5, and 8–11 only.

MATERIALS

Material I

- I In the Model Builder window, under Component I right-click Materials and choose New Material.
- 2 Select Domains 1, 2, 4, 5, and 7–11 only.
- 3 In the Material settings window, locate the Material Contents section.
- **4** In the table, enter the following settings:

Property	Name	Value	Unit	Property group
Electrical conductivity	sigma	0	S/m	Basic
Relative permittivity	epsilonr	1	I	Basic
Relative permeability	mur	1	I.	Basic

Material 2

- I In the Model Builder window, right-click Materials and choose New Material.
- 2 Select Domain 3 only.
- 3 In the Material settings window, locate the Material Contents section.
- **4** In the table, enter the following settings:

Property	Name	Value	Unit	Property group
Electrical conductivity	sigma	6e7	S/m	Basic
Relative permittivity	epsilonr	1	I.	Basic
Relative permeability	mur	1	1	Basic

Material 3

- I Right-click Materials and choose New Material.
- **2** Select Domain 6 only.
- 3 In the Material settings window, locate the Material Contents section.
- **4** In the table, enter the following settings:

Property	Name	Value	Unit	Property group
Electrical conductivity	sigma	0	S/m	Basic
Relative permittivity	epsilonr	2	I	Basic
Relative permeability	mur	1	I	Basic

DEFINITIONS

Integration 1

- I On the Definitions toolbar, click Component Couplings and choose Integration.
- **2** Select Domain 2 only.

MAGNETIC FIELDS

Multi-Turn Coil I

- I On the Physics toolbar, click Domains and choose Multi-Turn Coil.
- **2** Select Domain 3 only.
- 3 In the Multi-Turn Coil settings window, locate the Multi-Turn Coil section.
- **4** In the *N* edit field, type **5000**.
- **5** In the I_{coil} edit field, type 14.
- 6 Click the Zoom Extents button on the Graphics toolbar.

MESH I

Edge 1

I In the Model Builder window, under Component I right-click Mesh I and choose Edge.

2 Select Boundary 19 only.

Size 1

- I Right-click Component I>Mesh I>Edge I and choose Size.
- 2 In the Size settings window, locate the Element Size section.
- 3 From the Predefined list, choose Extremely fine.
- 4 Click the **Custom** button.
- **5** Locate the **Element Size Parameters** section. Select the **Maximum element size** check box.
- 6 In the associated edit field, type 0.0005.

Edge 2

- I In the Model Builder window, right-click Mesh I and choose Edge.
- 2 Select Boundaries 6, 18, 20, and 22 only.

Size 1

- I Right-click Component I>Mesh I>Edge 2 and choose Size.
- 2 In the Size settings window, locate the Element Size section.

- 3 From the Predefined list, choose Extremely fine.
- 4 Click the **Custom** button.
- **5** Locate the **Element Size Parameters** section. Select the **Maximum element size** check box.
- 6 In the associated edit field, type 0.0015.

Free Triangular 1

- I In the Model Builder window, right-click Mesh I and choose Free Triangular.
- 2 In the Free Triangular settings window, locate the Domain Selection section.
- **3** From the Geometric entity level list, choose Domain.
- **4** Select Domain 2 only.

Size 1

- I Right-click Component I>Mesh I>Free Triangular I and choose Size.
- 2 In the Size settings window, locate the Element Size section.
- 3 From the Predefined list, choose Extra fine.

Free Triangular 2

- I In the Model Builder window, right-click Mesh I and choose Free Triangular.
- 2 In the Free Triangular settings window, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domain 6 only.

Size 1

- I Right-click Component I>Mesh I>Free Triangular 2 and choose Size.
- 2 In the Size settings window, locate the Element Size section.
- 3 From the Predefined list, choose Extremely fine.
- 4 Click the **Custom** button.
- **5** Locate the **Element Size Parameters** section. Select the **Maximum element size** check box.
- 6 In the associated edit field, type 0.001.

Free Triangular 3

- I In the Model Builder window, right-click Mesh I and choose Free Triangular.
- 2 Right-click Free Triangular 3 and choose Build All.

STUDY I

Step 1: Stationary

- I In the Model Builder window, expand the Study I node, then click Step I: Stationary.
- 2 In the Stationary settings window, click to expand the Study extensions section.
- 3 Locate the Study Extensions section. Select the Adaptive mesh refinement check box.

Solver I

- I On the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Study I>Solver Configurations>Solver I>Stationary Solver I node, then click Adaptive Mesh Refinement.
- 3 In the Adaptive Mesh Refinement settings window, locate the General section.
- 4 In the Maximum number of refinements edit field, type 1.
- 5 Locate the Error Estimation section. From the Error estimate list, choose Functional.
- 6 In the Functional edit field, type intop1(1/(abs(mf.normB-0.0875)+1e-4)).
- 7 Locate the Mesh Refinement section. From the Refinement method list, choose Mesh initialization.
- 8 In the Model Builder window, click Study I.
- 9 In the Study settings window, locate the Study Settings section.

IO Clear the **Generate default plots** check box.

II On the Home toolbar, click Compute.

RESULTS

Reproduce the magnetic flux density plot in Figure 3 with the following steps.

Data Sets

- I In the Model Builder window, expand the Results>Data Sets node.
- 2 Right-click Solution I and choose Add Selection.
- 3 In the Selection settings window, locate the Geometric Entity Selection section.
- 4 From the Geometric entity level list, choose Domain.
- **5** Select Domains 2, 4, 6, and 7 only.

2D Plot Group I

- I On the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the Model Builder window, under Results right-click 2D Plot Group I and choose Contour.

- 3 In the Contour settings window, locate the Expression section.
- **4** In the **Expression** edit field, type log10(mf.normB+eps).
- 5 Locate the Levels section. In the Total levels edit field, type 8.
- 6 Locate the Coloring and Style section. From the Contour type list, choose Filled.
- 7 From the Color table list, choose GrayScale.
- 8 Clear the Color legend check box.
- **9** Select the **Reverse color table** check box.
- IO In the Model Builder window, right-click 2D Plot Group I and choose Contour.
- II In the Contour settings window, locate the Expression section.
- **12** In the **Expression** edit field, type Aphi.
- **I3** Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 14 From the Color list, choose Black.
- **I5** Clear the **Color legend** check box.
- **I6** Right-click **Results>2D Plot Group I>Contour 2** and choose **Duplicate**.
- 17 In the Contour settings window, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Magnetic Fields>Magnetic>Magnetic flux density norm (mf.normB).
- **18** Locate the Levels section. From the Entry method list, choose Levels.
- **I9** Click the **Range** button.
- **20** Go to the **Range** dialog box.
- 21 From the Entry method list, choose Number of values.
- **2** In the **Start** edit field, type 0.086.
- **23** In the **Stop** edit field, type 0.089.
- 24 In the Number of values edit field, type 20.
- **25** Click the **Replace** button.
- **26** On the **2D plot group** toolbar, click **Plot**.

Next, visualize the refined mesh.

2D Plot Group 2

- I On the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the 2D Plot Group settings window, locate the Data section.
- 3 From the Data set list, choose Solution 2.
- 4 Right-click Results>2D Plot Group 2 and choose Mesh.

- 5 In the Mesh settings window, locate the Color section.
- 6 From the Element color list, choose None.
- 7 From the Wireframe color list, choose Custom.

Click the **Color** button. In the **Custom color** dialog box, click **Define Custom Colors** and then set the values of Red, Green, and Blue to 192, 192, and 192, respectively. Click **OK**.

Compare the result with the plot in Figure 4.

DEFINITIONS

View 1

Now add the plasma physics to the model.

COMPONENT I

On the Home toolbar, click Add Physics.

ADD PHYSICS

- I Go to the Add Physics window.
- 2 In the Add physics tree, select Plasma>Microwave Plasma (mwp).
- 3 In the Add physics window, click Add to Component.

ROOT

On the **Home** toolbar, click **Add Study**.

ADD STUDY

- I Go to the Add Study window.
- 2 Find the Studies subsection. In the tree, select Preset Studies>Frequency-Transient.
- 3 In the Add study window, click Add Study.

MICROWAVE PLASMA

- I In the Model Builder window, under Component I click Microwave Plasma.
- 2 In the Microwave Plasma settings window, locate the Transport Settings section.
- **3** Select the Full expression for diffusivity check box.
- 4 Select the Compute tensor ion transport properties check box.
- **5** Locate the **Plasma Properties** section. Select the **Compute tensor electron transport properties** check box.

- 6 Select the Compute tensor plasma conductivity check box.
- **7** In the δ edit field, type 20.
- **8** Select Domains 1, 2, 6, and 8–11 only.

Wave Equation, Electric 1

- I On the Physics toolbar, click Domains and choose Wave Equation, Electric.
- **2** Select Domain 6 only.

Perfectly Matched Layers 1

- I On the Physics toolbar, click Domains and choose Perfectly Matched Layers.
- **2** Select Domains 1 and 8–11 only.
- **3** In the **Perfectly Matched Layers** settings window, locate the **Geometric Settings** section.
- 4 From the Type list, choose Cylindrical.

Cross Section Import 1

- I On the Physics toolbar, click Domains and choose Cross Section Import.
- 2 In the Cross Section Import settings window, locate the Cross Section Import section.
- **3** Click the **Browse** button.
- **4** Browse to the model's Model Library folder and double-click the file Ar_xsecs.txt.

Reaction 1

- I On the Physics toolbar, click Domains and choose Reaction.
- 2 In the **Reaction** settings window, locate the **Reaction Formula** section.
- **3** In the **Formula** edit field, type Ars+Ar=>Ar+Ar.
- **4** Locate the **Kinetics Expressions** section. In the k^{f} edit field, type 1807.

Reaction 2

- I On the Physics toolbar, click Domains and choose Reaction.
- 2 In the **Reaction** settings window, locate the **Reaction Formula** section.
- 3 In the Formula edit field, type Ars+Ars=>e+Ar+Ar+.
- **4** Locate the **Kinetics Expressions** section. In the k^{f} edit field, type **3.734E8**.

Species: Ar

- I In the Model Builder window, under Component I>Microwave Plasma click Species: Ar.
- 2 In the Species settings window, locate the Species Formula section.

- **3** Select the **From mass constraint** check box.
- 4 Locate the General Parameters section. From the Preset species data list, choose Ar.

Species: Ars

- I In the Model Builder window, under Component I>Microwave Plasma click Species: Ars.
- 2 In the Species settings window, locate the General Parameters section.
- 3 From the Preset species data list, choose Ar.
- **4** In the x_0 edit field, type 1E-4.

Species: Ar+

- I In the Model Builder window, under Component I>Microwave Plasma click Species: Ar+.
- 2 In the Species settings window, locate the Species Formula section.
- **3** Select the **Initial value from electroneutrality constraint** check box.
- 4 Locate the General Parameters section. From the Preset species data list, choose Ar.
- 5 Click to expand the Mobility and diffusivity expressions section. Locate the Mobility and Diffusivity Expressions section. From the Specification list, choose Specify mobility, compute diffusivity.
- 6 Click to expand the Mobility specification section. Locate the Mobility Specification section. From the Specify using list, choose Lookup table.
- 7 Click Load from File.
- 8 Browse to the model's Model Library folder and double-click the file ion_mobility_data.txt.

Plasma Model I

- I In the Model Builder window, under Component I>Microwave Plasma click Plasma Model I.
- 2 In the Plasma Model settings window, locate the Model Inputs section.
- **3** Specify the **B** vector as

mf.Br	r
0	phi
mf.Bz	z

- **4** In the T edit field, type 300.
- **5** In the p_A edit field, type 1.

6 Locate the DC Electron Mobility section. In the μ_{dc} edit field, type 1E25/mwp.Nn.

Surface Reaction 1

- I On the Physics toolbar, click Boundaries and choose Surface Reaction.
- 2 In the Surface Reaction settings window, locate the Reaction Formula section.
- 3 In the Formula edit field, type Ar+=>Ar.
- 4 Locate the Boundary Selection section. From the Selection list, choose Walls.

Surface Reaction 2

- I On the Physics toolbar, click Boundaries and choose Surface Reaction.
- 2 In the Surface Reaction settings window, locate the Reaction Formula section.
- 3 In the Formula edit field, type Ars=>Ar.
- 4 Locate the Boundary Selection section. From the Selection list, choose Walls.

Wall I

- I On the Physics toolbar, click Boundaries and choose Wall.
- 2 In the Wall settings window, locate the Boundary Selection section.
- 3 From the Selection list, choose Walls.

Ground I

- I On the Physics toolbar, click Boundaries and choose Ground.
- 2 In the Ground settings window, locate the Boundary Selection section.
- 3 From the Selection list, choose Walls.

Port I

- I On the Physics toolbar, click Boundaries and choose Port.
- 2 Select Boundary 14 only.
- 3 In the Port settings window, locate the Port Properties section.
- 4 From the Type of port list, choose Coaxial.
- 5 From the Wave excitation at this port list, choose On.
- **6** In the $P_{\rm in}$ edit field, type 600.

Initial Values 1

- I In the Model Builder window, under Component I>Microwave Plasma click Initial Values I.
- 2 In the Initial Values settings window, locate the Initial Values section.
- **3** In the $n_{e,0}$ edit field, type 1E14.

Plasma Model I

- I In the Model Builder window's toolbar, click the Show button and select Equation View in the menu.
- 2 In the Model Builder window, expand the Plasma Model I node, then click Equation View.
- 3 In the Equation View settings window, locate the Weak Expressions section.
- **4** In the table, change the following row:

Weak expression	Integration frame	Selection
2*mwp.Qrh*test(En)*pi*r/e_const	Material	Domain 2

to

Weak expression	Integration frame	Selection
2*mwp.Qind*test(En)*pi*r*alpha	Material	Domain 2

The multiplication by alpha, a power scaling factor that you will define next in accordance with Equation 6.

DEFINITIONS

Variables I a

- I In the Model Builder window, under Component I right-click Definitions and choose Variables.
- 2 In the Variables settings window, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
Psp	10[W]	W	Total power absorbed by the plasma setpoint
Pabs	<pre>intop1(2*pi*r*e_con st*mwp.Qind[W/ (m^3*C)])</pre>	m·kg²/(s?·A)	Total power absorbed by the plasma
alpha	Psp/nojac(Pabs+0.1)	m·s?·A/kg	Power scaling factor

COMPONENT I

On the Mesh toolbar, click Add Mesh.

MESH 3

Reference 1

- I In the Model Builder window, under Component I>Meshes right-click Mesh 3 and choose More Operations>Reference.
- 2 In the **Reference** settings window, locate the **Reference** section.
- 3 From the Mesh list, choose Mesh 2.

Refine 1

- I In the Model Builder window, right-click Mesh 3 and choose More Operations>Refine.
- 2 In the Refine settings window, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 Select Domain 6 only.
- 5 Locate the Refine Options section. In the Number of refinements edit field, type 2.

Boundary Layers 1

- I Right-click Mesh 3 and choose Boundary Layers.
- 2 In the Boundary Layers settings window, locate the Domain Selection section.
- **3** From the Geometric entity level list, choose Domain.
- **4** Select Domain 2 only.

Boundary Layer Properties

- I In the Model Builder window, under Component I>Meshes>Mesh 3>Boundary Layers I click Boundary Layer Properties.
- **2** In the **Boundary Layer Properties** settings window, locate the **Boundary Selection** section.
- 3 From the Selection list, choose Walls.
- **4** Locate the **Boundary Layer Properties** section. In the **Number of boundary layers** edit field, type **4**.
- 5 In the Boundary layer stretching factor edit field, type 1.4.
- 6 Click the Build All button.

STUDY 2

Step 1: Frequency-Transient

I In the Model Builder window, expand the Study 2 node, then click Step I: Frequency-Transient.

- 2 In the Frequency-Transient settings window, locate the Study Settings section.
- 3 In the Times edit field, type 0.
- 4 Click the **Range** button.
- 5 Go to the Range dialog box.
- 6 From the Entry method list, choose Number of values.
- 7 In the Start edit field, type -8.
- 8 In the Stop edit field, type -2.
- 9 In the Number of values edit field, type 11.
- **IO** From the **Function to apply to all values** list, choose **expl0**.
- II Click the Add button.
- 12 In the Frequency-Transient settings window, locate the Study Settings section.
- **I3** In the **Frequency** edit field, type **2.45E9**.
- **14** Locate the **Physics and Variables Selection** section. In the table, enter the following settings:

Physics	Solve for	Discretization
Magnetic Fields	×	physics

IS Click to expand the **Mesh selection** section. Locate the **Mesh Selection** section. In the table, enter the following settings:

Geometry Mesh Geometry I mesh3

Solver 3

- I On the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Study 2>Solver Configurations node.
- **3** In the **Model Builder** window, expand the **Solver 3** node, then click **Dependent Variables I**.
- 4 In the Dependent Variables settings window, locate the General section.
- **5** From the **Defined by study step** list, choose **User defined**.
- 6 Locate the Values of Variables Not Solved For section. From the Method list, choose Solution.
- 7 From the Solution list, choose Adaptive Mesh Refinement 2.

- 8 In the Model Builder window, under Study 2>Solver Configurations>Solver 3 right-click Time-Dependent Solver 1 and choose Segregated.
- 9 In the Model Builder window, expand the Study 2>Solver Configurations>Solver
 3>Time-Dependent Solver 1>Segregated 1 node, then click Segregated Step.
- 10 In the Segregated Step settings window, locate the General section.
- II In the Variables list, choose Electric field (compl.E) and compl.Sparaml.
- 12 Under Variables, click Delete.
- **13** Click to expand the **Method and termination** section. Locate the **Method and Termination** section. In the **Number of iterations** edit field, type 2.
- I4 In the Model Builder window, under Study 2>Solver Configurations>Solver
 3>Time-Dependent Solver I right-click Segregated I and choose Segregated Step.
- IS In the Segregated Step settings window, locate the Method and Termination section.
- **I6** From the **Jacobian update** list, choose **Once per time step**.
- 17 Locate the General section. Under Variables, click Add.
- **I8** Go to the **Add** dialog box.
- 19 In the Variables list, choose Electric field (compl.E) and compl.Sparaml.
- **20** Click the **OK** button.
- 21 On the Home toolbar, click Compute.

RESULTS

DEFINITIONS

RESULTS

Three new default plots show the electron density, electron temperature, and plasma potential respectively; compare with those in Figure 5, Figure 6 and Figure 7.

Follow the instructions below to reproduce Figure 8 through Figure 15.

2D Plot Group 6

- I On the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the 2D Plot Group settings window, locate the Data section.
- 3 From the Data set list, choose Solution 3.
- 4 Right-click Results>2D Plot Group 6 and choose Surface.

- 5 In the Surface settings window, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Microwave Plasma (Electromagnetic Waves)>Heating and losses>Resistive losses (mwp.Qrh).
- 6 On the 2D plot group toolbar, click Plot.

Compare with Figure 8.

2D Plot Group 7

- I In the Model Builder window, right-click 2D Plot Group 6 and choose Duplicate.
- 2 In the Model Builder window, expand the 2D Plot Group 7 node, then click Surface 1.
- 3 In the Surface settings window, locate the Expression section.
- 4 In the Expression edit field, type mwp.Re.
- 5 On the 2D plot group toolbar, click Plot.

Compare with Figure 9.

2D Plot Group 8

- I In the Model Builder window, right-click 2D Plot Group 7 and choose Duplicate.
- 2 In the Model Builder window, expand the 2D Plot Group 8 node, then click Surface 1.
- 3 In the Surface settings window, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Microwave Plasma (Drift Diffusion)>Electron transport properties>Electron mobility>Electron mobility, rr component (mwp.muerr).
- 4 On the 2D plot group toolbar, click Plot.

Compare with Figure 10.

2D Plot Group 9

- I In the Model Builder window, right-click 2D Plot Group 8 and choose Duplicate.
- 2 In the Model Builder window, expand the 2D Plot Group 9 node, then click Surface 1.
- 3 In the Surface settings window, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Microwave Plasma (Drift Diffusion)>Electron transport properties>Electron mobility>Electron mobility, zz component (mwp.muezz).
- 4 On the 2D plot group toolbar, click Plot.

Compare with Figure 11.

2D Plot Group 10

I On the Home toolbar, click Add Plot Group and choose 2D Plot Group.

- 2 In the 2D Plot Group settings window, locate the Data section.
- 3 From the Data set list, choose Solution 3.
- 4 Right-click Results>2D Plot Group 10 and choose Surface.
- 5 In the Surface settings window, locate the Expression section.
- 6 In the Expression edit field, type mwp.Jr_em-mwp.Jdr_em.
- 7 On the 2D plot group toolbar, click Plot.

Compare with Figure 12.

2D Plot Group 11

- I In the Model Builder window, right-click 2D Plot Group 10 and choose Duplicate.
- 2 In the Model Builder window, expand the 2D Plot Group 11 node, then click Surface 1.
- 3 In the Surface settings window, locate the Expression section.
- 4 In the **Expression** edit field, type mwp.Jz_em-mwp.Jdz_em.
- 5 On the 2D plot group toolbar, click Plot.

Compare with Figure 13.

2D Plot Group 12

- I In the Model Builder window, right-click 2D Plot Group II and choose Duplicate.
- 2 In the Model Builder window, expand the 2D Plot Group 12 node, then click Surface 1.
- 3 In the Surface settings window, locate the Expression section.
- 4 In the Expression edit field, type mwp.Jphi_em-mwp.Jdphi_em.
- 5 On the 2D plot group toolbar, click Plot.

Compare with Figure 14.

2D Plot Group 13

- I In the Model Builder window, under Results right-click 2D Plot Group 8 and choose Duplicate.
- 2 In the Model Builder window, expand the 2D Plot Group 13 node, then click Surface 1.
- 3 In the Surface settings window, locate the Expression section.
- 4 In the Expression edit field, type (mwp.sigmarr+mwp.sigmaphiphi+mwp.sigmazz)/3.
- 5 On the 2D plot group toolbar, click Plot.

Compare with Figure 15.

Solved with COMSOL Multiphysics 4.4