# Vibration in a Washing Machine Assembly 

## Introduction

Vibration and noise, due to non-uniform distribution of clothes, is a common problem in washing machines.

This model simulates a simplified multibody dynamics model of a horizontal-axis portable washing machine. An eigenfrequency analysis is performed to compute the natural frequencies and mode shapes of the entire assembly. Transient analysis is performed to find out the vibrations induced in the housing during the spinning cycle. The housing is modeled as a flexible shell.

## Model Definition

The geometry of the washing machine assembly is shown in Figure 1.


Figure 1: Model geometry (top, bottom, front, and left panels of the housing are hidden)

A key to the coloring of the parts constituting the washing machine is given below.
table I: IDENTIFICATION OF VARIOUS PARTS OF WASHING MACHINE ASSEMBLY

| Part | Color in figure |
| :--- | :--- |
| Clothes | Red |
| Drum | White |
| Tub | Cyan |
| Motor | Yellow |
| Pistons | Green |
| Cylinders | Magenta |
| Mountings | Blue |
| Base supports | Black |
| Housing | Gray |

The following assumptions are used:

- The housing is modeled using flexible shell elements.
- The other parts are modeled by rigid solids.
- The clothes do not move relative to the drum.


## CONNECTION DETAILS

The connections of the housing to the remaining parts are as follows:

- Connection with the base, at four locations on the bottom surface, using fixed joints.
- Connection with the mountings, at four locations on the bottom surface, using fixed joints.
- Connection with the tub, at front and back surfaces, using stabilizing springs. These springs are not shown in the geometry (Figure l) or in the model.

The details of all the connections in the assembly are shown below:


## Results and Discussion

The mode shapes of washing machine assembly can be seen in Figure 2 and Figure 3. In these figures, one of the modes showing the translation of the tub whereas the other one shows the rotation of the tub about the vertical axis. The corresponding housing deformation can also be seen. The magnitude of deformation of housing is very small compared to the motion of tub, so a different color table is used for better clarity.

Figure 4 and Figure 5 shows the tub displacement magnitude with the drum rotation or the position of unbalanced clothes for the full time duration. The color of the trajectory has the time information representing red as the initial time and blue as final time.


Figure 2: One of the eigenmodes of the washing machine assembly (with tub translating along vertical axis).

Eigenfrequency $=3.674+0.6775 i$ Surface: Total displacement (mm)


Figure 3: One of the eigenmodes of the washing machine assembly (with tub rotating about vertical axis).


Figure 4: In-plane ( $x-z$ ) displacement magnitude of the tub with the position of unbalanced clothes.

Tub displacement magnitude (out-of-plane)


Figure 5: Out-of-plane (y) displacement magnitude of the tub with the position of unbalanced clothes.

Tub rotation about all three axes with the rotation of drum is shown in Figure 6 below.


Figure 6: Tub rotation about all three axes.
Figure 7 shows the extension in the front and back stabilizing springs with the rotation of drum.

The relative displacement between piston and cylinder for the different struts is shown in Figure 8. The energy dissipation in the struts with the rotation of drum is plotted in Figure 9.

The deformation of the housing in the vertical direction with the drum rotation at the locations where mountings are attached is shown in Figure 10.


Figure 7: Extension of stabilizing springs.


Figure 8: Relative displacement between piston and cylinder of different struts.


Figure 9: Energy dissipation rate in different struts.


Figure 10: Vertical deformation of the housing at the mounting locations.

Figure 11 shows the deformation of the housing at a point on the right side wall.


Figure 11: Deformation of the housing in different directions at a point on the side wall.
Figure 12 shows the normal acceleration of the housing at a point in the middle of the right side wall. Normal acceleration is a measure of noise emitted by the side walls.

The frequency spectrum of this acceleration can be seen in Figure 13. The frequency spectrum shows that the frequency content of side wall vibration is mainly in the range $0-30 \mathrm{~Hz}$. This is interesting, since the excitation frequency, corresponding to the rotational speed of the drum or unbalanced mass is smaller, approximately 1.67 Hz .


Figure 12: Normal acceleration of the housing, at a point on the side wall.


Figure 13: Frequency spectrum of normal acceleration of the housing.

## Notes About the COMSOL Implementation

- Use Mass and Moment of Inertia subnode of the Rigid Domain node to enter the inertia properties given at a certain point.
- The connections set up in the model and the details of the system DOF and constraints can be seen in the Joints Summary and Rigid Body DOF Summary sections of the Multibody Dynamics node.
- Use the Attachment boundary condition in the Shell interface to establish the connection to the solid objects through joints and springs.
- The numbering used in the model for piston, cylinder, mounting, and base is such that $1,2,3$, and 4 corresponds to front-left, front-right, back-right, and back-left locations respectively.

```
Model Library path: Multibody_Dynamics_Module/Machinery_and_Robotics/
washing_machine_vibrations
```


## Modeling Instructions

From the File menu, choose New.

## NEW

I In the New window, click Model Wizard.

## MODEL WIZARD

I In the Model Wizard window, click 3D.
2 In the Select physics tree, select Structural Mechanics>Multibody Dynamics (mbd).
3 Click Add.
4 In the Select physics tree, select Structural Mechanics>Shell (shell).
5 Click Add.
6 Click Study.
7 In the Select study tree, select Preset Studies for Selected Physics Interfaces>Eigenfrequency.
8 Click Done.

## DEFINITIONS

Start by importing the model parameters and geometry.

## Parameters

I On the Model toolbar, click Parameters.
2 In the Settings window for Parameters, locate the Parameters section.
3 Click Load from File.
4 Browse to the model's Model Library folder and double-click the file washing_machine_vibration_parameters.txt.

## GEOMETRY I

I In the Model Builder window, under Component I (compl) click Geometry I.
2 In the Settings window for Geometry, locate the Units section.
3 From the Length unit list, choose mm.
Import I (impl)
I On the Model toolbar, click Import.
2 In the Settings window for Import, locate the Import section.

## 3 Click Browse.

4 Browse to the model's Model Library folder and double-click the file washing_machine_vibration.mphbin.

## 5 Click Import.

## Form Union (fin)

I In the Model Builder window, under Component I (comp I)>Geometry I click Form Union (fin).

2 In the Settings window for Form Union/Assembly, locate the Form Union/Assembly section.

3 From the Action list, choose Form an assembly.
4 Clear the Create pairs check box.
5 On the Model toolbar, click Build All.

## DEFINITIONS

Hiding the front panels of the geometry will make it more convenient to set up the model.

View I
I In the Model Builder window, expand the Component I (compl)>Definitions node, then click View I.

2 In the Settings window for View, locate the View section.
3 Select the Wireframe rendering check box.
4 On the 3D view toolbar, click Hide Geometry Objects.
5 In the Settings window for Hide Geometry Objects, locate the Selection section.
6 From the Geometric entity level list, choose Boundary.
7 On the object fin, select Boundaries $1,2,5,6,10,12$, and 14 only.
Define some selections to be used later.

## Explicit I

I On the Definitions toolbar, click Explicit.
2 In the Settings window for Explicit, type Housing in the Label text field.
3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.

4 Select Boundary 36 only.
5 Select the Group by continuous tangent check box.

## Explicit 2

I On the Definitions toolbar, click Explicit.
2 In the Settings window for Explicit, locate the Input Entities section.
3 From the Geometric entity level list, choose Edge.
4 Select Edge 228 only.
5 In the Label text field, type Strut axis 1.

## Strut axis 1.2

I Right-click Component I (compl)>Definitions>Explicit 2 and choose Duplicate.
2 In the Settings window for Explicit, type Strut axis 2 in the Label text field.
3 Locate the Input Entities section. Click Clear Selection.
4 Select Edge 444 only.

## Strut axis 2.2

I Right-click Component I (compl)>Definitions>Strut axis 1.2 and choose Duplicate.
2 In the Settings window for Explicit, type Strut axis 3 in the Label text field.
3 Locate the Input Entities section. Click Clear Selection.

4 Select Edge 484 only.
Strut axis 3.2
I Right-click Component I (compl)>Definitions>Strut axis 2.2 and choose Duplicate.
2 In the Settings window for Explicit, type Strut axis 4 in the Label text field.
3 Locate the Input Entities section. Click Clear Selection.
4 Select Edge 268 only.
Define various components of the washing machine assembly.

## MULTIBODY DYNAMICS (MBD)

## Rigid Domain I

I On the Physics toolbar, click Domains and choose Rigid Domain.
2 In the Settings window for Rigid Domain, type Clothes in the Label text field.
3 Select Domain 13 only.
Set the density of the selected domain to zero. Use Mass and moment of inertia subnode instead to specify the mass of the domain.

4 Locate the Rigid Domain section. From the $\rho$ list, choose User defined.

## Mass and Moment of Inertia I

I On the Physics toolbar, click Attributes and choose Mass and Moment of Inertia.
2 In the Settings window for Mass and Moment of Inertia, locate the Mass and Moment of Inertia section.

3 In the $m$ text field, type Mc.
Use the Applied Force subnode to account for the gravitational force.
Applied Force I
I In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd) right-click Clothes and choose Applied Force.

2 In the Settings window for Applied Force, locate the Applied Force section.
3 Specify the $\mathbf{F}$ vector as

| 0 | $x$ |
| :--- | :--- |
| 0 | $y$ |
| - Mc*g_const | $z$ |

## Rigid Domain 2

I On the Physics toolbar, click Domains and choose Rigid Domain.
2 In the Settings window for Rigid Domain, type Drum in the Label text field.
3 Select Domain 8 only.
Select the Force initial values in Consistent initialization, to enforce the parts, which are connected to the drum such as clothes, to rotate with the same angular speed.

4 Locate the Initial Values section. From the list, choose Locally defined.
5 From the Consistent initialization list, choose Force initial values.
6 Select the Translation along first axis check box.
7 Select the Translation along second axis check box.
8 Select the Translation along third axis check box.
9 Select the Total rotation check box.

## Initial Values I

I In the Model Builder window, expand the Component I (compl)>Multibody Dynamics $\mathbf{( m b d})>$ Drum node, then click Initial Values I.

2 In the Settings window for Initial Values, locate the Center of Rotation section.
3 From the list, choose Centroid of selected entities.
4 From the Entity level list, choose Point.
5 Locate the Initial Values: Rotational section. Specify the $\omega$ vector as

| 0 | $x$ |
| :--- | :--- |
| omega | $y$ |
| 0 | $z$ |

## Center of Rotation: Point I

I In the Model Builder window, expand the Initial Values I node, then click Center of Rotation: Point I.

2 Select Points 141 and 151 only.
Rigid Domain 3
I On the Physics toolbar, click Domains and choose Rigid Domain.
2 In the Settings window for Rigid Domain, type Tub in the Label text field.
3 Select Domains 5, 15, and 16 only.

## Rigid Domain 4

I On the Physics toolbar, click Domains and choose Rigid Domain.
2 In the Settings window for Rigid Domain, type Motor in the Label text field.
3 Select Domain 14 only.
4 Locate the Rigid Domain section. From the $\rho$ list, choose User defined.

## Mass and Moment of Inertia I

I On the Physics toolbar, click Attributes and choose Mass and Moment of Inertia.
2 In the Settings window for Mass and Moment of Inertia, locate the Mass and Moment of Inertia section.

3 In the $m$ text field, type Mm.
4 In the I text field, type Im.

## Rigid Domain 5

I On the Physics toolbar, click Domains and choose Rigid Domain.
2 In the Settings window for Rigid Domain, type Piston 1 in the Label text field.
3 Select Domain 11 only.
Similarly create more components by duplicating Piston I and resetting the inputs using the information given in the table below.

TABLE 2: RIGID DOMAINS

| Name | Selection |
| :--- | :--- |
| Piston 2 | 17 |
| Piston 3 | 18 |
| Piston 4 | 12 |
| Cylinder I | 9 |
| Cylinder 2 | 19 |
| Cylinder 3 | 20 |
| Cylinder 4 | 10 |
| Mounting 1 | 6 |
| Mounting 2 | 21 |
| Mounting 3 | 22 |
| Mounting 4 | 7 |
| Base | $1-4$ |

Right-click Component I (compI)>Multibody Dynamics (mbd)>Base and choose Fixed Constraint.

## MATERIALS

## Material I (matl)

I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.

2 In the Settings window for Material, locate the Material Contents section.
3 In the table, enter the following settings:

| Property | Name | Value | Unit | Property group |
| :--- | :--- | :--- | :--- | :--- |
| Density | rho | rho0 | $\mathrm{kg} / \mathrm{m}^{3}$ | Basic |

4 On the Model toolbar, click More Windows and choose Add Material.

## ADD MATERIAL

I Go to the Add Material window.
2 In the tree, select Built-In>Aluminum.
3 Click Add to Component in the window toolbar.

## MATERIALS

## Aluminum (mat2)

I In the Model Builder window, under Component I (comp I)>Materials click Aluminum (mat2).

2 In the Settings window for Material, locate the Geometric Entity Selection section.
3 From the Geometric entity level list, choose Boundary.
4 From the Selection list, choose Housing.
5 On the Model toolbar, click Add Material to close the Add Material window.

## SHELL (SHELL)

On the Physics toolbar, click Multibody Dynamics (mbd) and choose Shell (shell).
I In the Model Builder window, under Component I (compl) click Shell (shell).
2 In the Settings window for Shell, type Shell [Housing] in the Label text field.
3 Locate the Boundary Selection section. From the Selection list, choose Housing.

4 Locate the Thickness section. In the $d$ text field, type $0.001[\mathrm{~m}]$.
Use linear elements for shell to reduce the computation time.
5 In the Model Builder window's toolbar, click the Show button and select Discretization in the menu.

6 Click to expand the Discretization section. From the Displacement field list, choose Linear.

As the amplitude of vibration is going to be small in the shell, select the Force linear strains option in Linear Elastic Material I node. This will reduce the analysis time.

## SHELL [HOUSING] (SHELL)

On the Physics toolbar, click Shell (shell) and choose Shell [Housing] (shell).

## Linear Elastic Material I

I In the Model Builder window, under Component I (compl)>Shell [Housing] (shell) click Linear Elastic Material I.

2 In the Settings window for Linear Elastic Material, locate the Geometric Nonlinearity section.

3 Select the Force linear strains check box.
Define the attachments in the Shell interface to couple it with the Multibody Dynamics interface.

## Attachment I

I On the Physics toolbar, click Edges and choose Attachment.
2 In the Settings window for Attachment, type Front spring in the Label text field.
3 Select Edges 45, 46, 49, and 50 only.
Similarly, create more attachments by duplicating Front Spring and resetting the inputs using the information given in the table below:

TABLE 3: SHELL ATTACHMENTS

| Name | Selection (edge) |
| :--- | :--- |
| Back Spring | $47,48,51$, and 52 |
| Mounting I | $37,38,41$, and 42 |
| Mounting 2 | $53,54,57$, and 58 |
| Mounting 3 | $55,56,59$, and 60 |
| Mounting 4 | $39,40,43$, and 44 |
| Base I | $29,30,33$, and 34 |

TABLE 3: SHELL ATTACHMENTS

| Name | Selection (edge) |
| :--- | :--- |
| Base 2 | $61,62,65$, and 66 |
| Base 3 | $63,64,67,68$ |
| Base 4 | $31,32,35,36$ |

Use the Spring-Damper node to model the stabilizing springs.

## MULTIBODY DYNAMICS (MBD)

## Spring-Damper 1

I On the Physics toolbar, click Global and choose Spring-Damper.
2 In the Settings window for Spring-Damper, type Housing-tub (front) in the Label text field.

3 Locate the Attachment Selection section. From the Source list, choose Front spring (shell).

4 Locate the Spring-Damper section. In the $k$ text field, type kt.

## Destination Point I

I In the Model Builder window, expand the Component I (compl)>Multibody Dynamics (mbd) $>$ Housing-tub (front) node, then click Destination Point I.
2 Select Points 246 and 248 only.

## Housing-tub (front) I

I In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd) right-click Housing-tub (front) and choose Duplicate.
2 In the Settings window for Spring-Damper, type Housing-tub (back) in the Label text field.

3 Locate the Attachment Selection section. From the Source list, choose Back spring (shell).

## Destination Point I

I In the Model Builder window, expand the Component I (compl)>Multibody Dynamics (mbd) $>$ Housing-tub (back) node, then click Destination Point I.
2 In the Settings window for Destination Point, locate the Point Selection section.
3 Click Clear Selection.
4 Select Points 258 and 260 only.
Define the connection between the drum and the tub using a Hinge joint.

## Hinge Joint I

I On the Physics toolbar, click Global and choose Hinge Joint.
2 In the Settings window for Hinge Joint, type Tub-drum in the Label text field.
3 Locate the Attachment Selection section. From the Source list, choose Tub.
4 From the Destination list, choose Drum.
5 Locate the Center of Joint section. From the Entity level list, choose Point.
6 Locate the Axis of Joint section. Specify the $\mathbf{e}_{0}$ vector as

| 0 | $x$ |
| :--- | :--- |
| 1 | $y$ |
| 0 | $z$ |

7 Locate the Joint Forces and Moments section. From the list, choose Do not compute.

## Center of Joint: Point I

I In the Model Builder window, expand the Component I (compl)>Multibody Dynamics (mbd)>Tub-drum node, then click Center of Joint: Point I.

2 Select Points 141 and 151 only.

## Prescribed Motion I

I In the Model Builder window, under Component I (comp I)>Multibody Dynamics (mbd) right-click Tub-drum and choose Prescribed Motion.

2 In the Settings window for Prescribed Motion, locate the Prescribed Rotational Motion section.

3 From the Prescribed motion through list, choose Angular velocity.
4 In the $\omega_{\mathrm{p}}$ text field, type omega.
Define Fixed joints between motor-tub and drum-clothes.

## Fixed Joint I

I On the Physics toolbar, click Global and choose Fixed Joint.
2 In the Settings window for Fixed Joint, type Motor - tub in the Label text field.
3 Locate the Attachment Selection section. From the Source list, choose Motor.
4 From the Destination list, choose Tub.
5 Locate the Joint Forces and Moments section. From the list, choose Do not compute.

## Center of Joint: Boundary I

I In the Model Builder window, expand the Component I (comp I)>Multibody Dynamics (mbd)>Motor-tub node, then click Center of Joint: Boundary I.
2 Select Boundaries 168 and 171 only.

## Motor-tub I

I In the Model Builder window, under Component I (comp I)>Multibody Dynamics (mbd) right-click Motor-tub and choose Duplicate.

2 In the Settings window for Fixed Joint, type Drum-clothes in the Label text field.
3 Locate the Attachment Selection section. From the Source list, choose Drum.
4 From the Destination list, choose Clothes.

## Center of Joint: Boundary I

I In the Model Builder window, expand the Component I (compI)>Multibody Dynamics (mbd)>Drum-clothes node, then click Center of Joint: Boundary I.

2 In the Settings window for Center of Joint: Boundary, locate the Boundary Selection section.

3 Click Clear Selection.
4 Select Boundary 157 only.
Model the struts using Prismatic joints.

## Prismatic Joint I

I On the Physics toolbar, click Global and choose Prismatic Joint.
2 In the Settings window for Prismatic Joint, type Cylinder 1 -piston 1 in the Label text field.

3 Locate the Attachment Selection section. From the Source list, choose Cylinder I.
4 From the Destination list, choose Piston I.
5 Locate the Axis of Joint section. From the list, choose Select a parallel edge.
6 Locate the Joint Forces and Moments section. From the list, choose Do not compute.

## Center of Joint: Boundary I

I In the Model Builder window, expand the Component I (compI)>Multibody Dynamics (mbd)>Cylinder I-piston I node, then click Center of Joint: Boundary I.

2 Select Boundary 144 only.

## Joint Axis I

I In the Model Builder window, under Component I (compI)>Multibody Dynamics (mbd) $>$ Cylinder I-piston I click Joint Axis I.
2 In the Settings window for Joint Axis, locate the Edge Selection section.
3 From the Selection list, choose Strut axis I.

## Spring and Damper I

I On the Physics toolbar, click Attributes and choose Spring and Damper.
2 In the Settings window for Spring and Damper, locate the Spring and Damper: Translational section.

3 In the $k_{\mathrm{u}}$ text field, type ks .
4 In the $c_{\mathrm{u}}$ text field, type cs.
Create the other three prismatic joints by duplicating Cylinder I-piston I and resetting the inputs using the information given in the table below:

TABLE 4: PRISMATIC JOINTS

| Name | Source | Destination | Center of Joint <br> selection <br> (boundary) | joint axis <br> selection <br> (edge) |
| :--- | :--- | :--- | :--- | :--- |
| Cylinder 2-piston 2 | Cylinder 2 | Piston 2 | 195 | Strut axis 2 |
| Cylinder 3-piston 3 | Cylinder 3 | Piston 3 | 201 | Strut axis 3 |
| Cylinder 4-piston 4 | Cylinder 4 | Piston 4 | 150 | Strut axis 4 |

## Fixed Joint 3

I On the Physics toolbar, click Global and choose Fixed Joint.
2 In the Settings window for Fixed Joint, type Tub-piston 1 in the Label text field.
3 Locate the Attachment Selection section. From the Source list, choose Tub.
4 From the Destination list, choose Piston I.
5 Locate the Axis of Joint section. From the list, choose Select a parallel edge.
6 Locate the Joint Elasticity section. From the list, choose Elastic joint.
7 Locate the Joint Forces and Moments section. From the list, choose Do not compute.

## Center of Joint: Boundary I

I In the Model Builder window, expand the Component I (compl)>Multibody Dynamics (mbd)>Tub-piston I node, then click Center of Joint: Boundary I.

2 Select Boundary 149 only.

## Joint Axis I

I In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd)>Tub-piston I click Joint Axis I.
2 In the Settings window for Joint Axis, locate the Edge Selection section.
3 From the Selection list, choose Strut axis I.

## Joint Elasticity I

I In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd)>Tub-piston I click Joint Elasticity I.
2 In the Settings window for Joint Elasticity, locate the Elastic Degrees of Freedom section.

3 Clear the First axis check box.
4 Clear the Second axis check box.
5 Clear the Third axis check box.
6 Locate the Spring section. In the $\mathbf{k}_{\theta}$ text field, type kbr .
7 Locate the Viscous Damping section. In the $\mathbf{c}_{\theta}$ text field, type cbr.
Similarly create other seven bushings by duplicating Tub I-piston I and resetting the inputs using the information given in the table below:

TABLE 5: FIXED JOINTS (WITH ELASTICITY)

| Name | Source | Destination | Center of <br> Joint <br> selection <br> (boundary) | Joint axis <br> selection <br> (Edge) |
| :--- | :--- | :--- | :--- | :--- |
| Tub-piston 2 | Tub | Piston 2 | 190 | Strut axis 2 |
| Tub-piston 3 | Tub | Piston 3 | 196 | Strut axis 3 |
| Tub-piston 4 | Tub | Piston 4 | 155 | Strut axis 4 |
| Cylinder 1-mounting 1 | Cylinder 1 | Mounting 1 | 74 | Strut axis 1 |
| Cylinder 2-mounting 2 | Cylinder 2 | Mounting 2 | 247 | Strut axis 2 |
| Cylinder 3-mounting 3 | Cylinder 3 | Mounting 3 | 257 | Strut axis 3 |
| Cylinder 4-mounting 4 | Cylinder 4 | Mounting 4 | 84 | Strut axis 4 |

## Cylinder 2-piston 2

I In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd) click Cylinder 2-piston 2.
2 In the Settings window for Prismatic Joint, locate the Axis of Joint section.
3 Select the Reverse direction check box.

## Cylinder 3-piston 3

I In the Model Builder window, under Component I (comp I)>Multibody Dynamics (mbd) click Cylinder 3-piston 3.
2 In the Settings window for Prismatic Joint, locate the Axis of Joint section.
3 Select the Reverse direction check box.

## Tub-piston 2

I In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd) click Tub-piston 2.
2 In the Settings window for Fixed Joint, locate the Axis of Joint section.
3 Select the Reverse direction check box.

## Tub-piston 3

I In the Model Builder window, under Component I (comp I)>Multibody Dynamics (mbd) click Tub-piston 3.

2 In the Settings window for Fixed Joint, locate the Axis of Joint section.
3 Select the Reverse direction check box.

## Cylinder 2-mounting 2

I In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd) click Cylinder 2-mounting 2.

2 In the Settings window for Fixed Joint, locate the Axis of Joint section.
3 Select the Reverse direction check box.
Cylinder 3-mounting 3
I In the Model Builder window, under Component I (comp I)>Multibody Dynamics (mbd) click Cylinder 3-mounting 3.

2 In the Settings window for Fixed Joint, locate the Axis of Joint section.
3 Select the Reverse direction check box.
Define the connection between mountings-housing and housing-base.

## Cylinder 4-mounting 4.1

I In the Model Builder window, under Component I (compI)>Multibody Dynamics (mbd) right-click Cylinder 4-mounting 4 and choose Duplicate.

2 In the Settings window for Fixed Joint, type Mounting 1-housing in the Label text field.

3 Locate the Attachment Selection section. From the Source list, choose Mounting I.

4 From the Destination list, choose Mounting I (shell).
5 Locate the Axis of Joint section. From the list, choose Specify direction.
6 Locate the Joint Elasticity section. From the list, choose Rigid joint.
7 Locate the Center of Joint section. From the list, choose Centroid of destination.
Similarly create other seven connections by duplicating Mounting I-housing and resetting the inputs using the information given in the table below:

TABLE 6: FIXED JOINTS (RIGID)

| Name | Source | Destination |
| :--- | :--- | :--- |
| Mounting 2-housing | Mounting 2 | Mounting 2(shell) |
| Mounting 3-housing | Mounting 3 | Mounting 3(shell) |
| Mounting 4-housing | Mounting 4 | Mounting 4(shell) |
| Housing-base I | Base I (shell) | Base |
| Housing-base 2 | Base 2 (shell) | Base |
| Housing-base 3 | Base 3 (shell) | Base |
| Housing-base 4 | Base 4 (shell) | Base |

## MESH I

I In the Model Builder window, under Component I (compI) click Mesh I.
2 In the Settings window for Mesh, locate the Mesh Settings section.
3 From the Element size list, choose Fine.
4 Click the Build All button.

## STUDY I

## Step 1: Eigenfrequency

I In the Model Builder window, under Study I click Step I: Eigenfrequency.
2 In the Settings window for Eigenfrequency, locate the Study Settings section.
3 In the Search for eigenfrequencies around text field, type 2.
4 In the Model Builder window, click Study I.
5 In the Settings window for Study, locate the Study Settings section.
6 Clear the Generate default plots check box.
7 On the Model toolbar, click Compute.
Use the instructions below to reproduce the mode shape as shown in Figure 2.

## RESULTS

## 3D Plot Group I

I On the Model toolbar, click Add Plot Group and choose 3D Plot Group.
2 In the Settings window for 3D Plot Group, type Mode Shape in the Label text field.
3 Locate the Data section. From the Data set list, choose None.

## Mode Shape

I Right-click Results>3D Plot Group I and choose Surface.
2 In the Model Builder window, under Results>Mode Shape right-click Surface I and choose Deformation.

3 Right-click Mode Shape and choose Surface.
4 In the Settings window for Surface, locate the Expression section.
5 In the Expression text field, type shell. disp.
6 Click to expand the Title section. From the Title type list, choose None.
7 Locate the Coloring and Style section. From the Color table list, choose GrayScale.
8 Select the Reverse color table check box.
9 Right-click Results>Mode Shape>Surface 2 and choose Deformation.
10 In the Settings window for Deformation, locate the Expression section.
II In the $\mathbf{x}$ component text field, type $u 2$.
12 In the $\mathbf{y}$ component text field, type v 2 .
13 In the $\mathbf{z}$ component text field, type w2.
14 In the Model Builder window, click Mode Shape.
I5 In the Settings window for 3D Plot Group, locate the Data section.
16 From the Data set list, choose Study I/Solution I.
$\mathbf{1 7}$ From the Eigenfrequency list, choose $\mathbf{2 . 6 5 2 I + 0 . 6 5 9 4 3 i}$.
I8 On the 3D plot group toolbar, click Plot.
19 Click the Zoom Extents button on the Graphics toolbar.
Change the eigenfrequency to obtain the eigenmode as shown in Figure 3.
20 From the Eigenfrequency list, choose $\mathbf{3 . 6 7 2 3 + 0 . 6 7 7 0 9 i}$.
21 On the 3D plot group toolbar, click Plot.
$\mathbf{2}$ Click the Zoom Extents button on the Graphics toolbar.

Before performing a transient analysis, define additional variables to use them in post-processing.

## DEFINITIONS

## Variables Ia

I In the Model Builder window, under Component I (comp I) right-click Definitions and choose Variables.

2 In the Settings window for Variables, locate the Variables section.
3 In the table, enter the following settings:

| Name | Expression | Unit | Description |
| :--- | :--- | :--- | :--- |
| uin_tub | sqrt(mbd.rd3.u^2+ <br> mbd.rd3.w^2) | m | Tub displacement <br> magnitude (in-plane) |
| uout_tub | abs(mbd.rd3.v) | m | Tub displacement <br> magnitude (out-of-plane) |
| th_drum | mbd.hgj1.th | rad | Drum rotation |
| n_cycle | th_drum/360[deg] | rad | Number of cycles |

ROOT
On the Model toolbar, click More Windows and choose Add Study.

## ADD STUDY

I Go to the Add Study window.
2 Find the Studies subsection. In the Select study tree, select Preset Studies>Time Dependent.

3 Click Add Study in the window toolbar.
4 On the Model toolbar, click Add Study to close the Add Study window.

## STUDY 2

## Step I: Time Dependent

I In the Model Builder window, under Study $\mathbf{2}$ click Step I: Time Dependent.
2 In the Settings window for Time Dependent, locate the Study Settings section.
3 In the Times text field, type range ( $0,0.01,2$ ).
4 Select the Relative tolerance check box.
5 In the associated text field, type 0.001 .
6 In the Model Builder window, click Study 2.

7 In the Settings window for Study, locate the Study Settings section.
8 Clear the Generate default plots check box.
9 On the Study toolbar, click Show Default Solver.
Modify the default solver settings in such a way that both physics are solved together.

## Solution 2

I In the Model Builder window, expand the Solution 2 node, then click Time-Dependent Solver I.

2 In the Settings window for Time-Dependent Solver, click to expand the Time stepping section.

3 Locate the Time Stepping section. From the Steps taken by solver list, choose Intermediate.

4 Right-click Study $2>$ Solver Configurations $>$ Solution $\mathbf{2}>$ Time-Dependent Solver $\mathbf{I}$ and choose Fully Coupled.
5 In the Settings window for Fully Coupled, click to expand the Method and termination section.

6 Locate the Method and Termination section. From the Jacobian update list, choose On every iteration.

7 In the Maximum number of iterations text field, type 15.
8 On the Study toolbar, click Compute.
Duplicate the mode shape plot and change the data set to plot the displacement obtained in transient analysis.

## RESULTS

## Mode Shape I

I In the Model Builder window, under Results right-click Mode Shape and choose Duplicate.

2 In the Settings window for 3D Plot Group, type Displacement in the Label text field.

3 Locate the Data section. From the Data set list, choose Study 2/Solution 2.
4 From the Time (s) list, choose $\mathbf{I} .0000$.
5 Locate the Plot Settings section. From the Frame list, choose Spatial (x, y, z).

## Displacement

I In the Model Builder window, expand the Results>Displacement>Surface I node, then click Deformation I.

2 In the Settings window for Deformation, locate the Scale section.
3 Select the Scale factor check box.
4 In the associated text field, type 1.
5 In the Model Builder window, expand the Results>Displacement>Surface 2 node, then click Deformation I.

6 In the Settings window for Deformation, locate the Scale section.
7 Select the Scale factor check box.
8 In the associated text field, type 1.
9 On the 3D plot group toolbar, click Plot.
10 Click the Zoom Extents button on the Graphics toolbar.
To get the polar plots of the tub displacement as shown in Figure 4 and Figure 5, follow the instructions below.

## Polar Plot Group 3

I On the Model toolbar, click Add Plot Group and choose Polar Plot Group.
2 In the Settings window for Polar Plot Group, type Tub displacement magnitude (in-plane) in the Label text field.

3 Locate the Data section. From the Data set list, choose Study 2/Solution 2.
4 Click to expand the Title section. From the Title type list, choose Manual.
5 In the Title text area, type Tub displacement magnitude (in-plane).

## Tub displacement magnitude (in-plane)

I On the Polar plot group toolbar, click Global.
2 In the Settings window for Global, click Replace Expression in the upper-right corner of the $\mathbf{r}$-axis data section. From the menu, choose Model>Component I> Definitions $>$ Variables $>$ uin_tub - Tub displacement magnitude (in-plane).

3 Locate the $\theta$ Angle Data section. From the Parameter list, choose Expression.
4 Click Replace Expression in the upper-right corner of the $\theta$ angle data section. From the menu, choose Model>Component I >Definitions>Variables> th_drum - Drum rotation.
5 Click to expand the Coloring and style section. Locate the Coloring and Style section. Find the Line style subsection. In the Width text field, type 2.

6 Click to expand the Legends section. Clear the Show legends check box.
7 Right-click Results>Tub displacement magnitude (in-plane)>Global I and choose Color Expression.

8 In the Settings window for Color Expression, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Model>Solver>t - Time.

9 Locate the Coloring and Style section. Select the Reverse color table check box.
10 On the Polar plot group toolbar, click Plot.
II Click the Zoom Extents button on the Graphics toolbar.
Tub displacement magnitude (in-plane) I
I In the Model Builder window, right-click Tub displacement magnitude (in-plane) and choose Duplicate.

2 In the Settings window for Polar Plot Group, type Tub displacement magnitude (out-of-plane) in the Label text field.

3 Locate the Title section. In the Title text area, type Tub displacement magnitude (out-of-plane).

## Tub displacement magnitude (out-of-plane)

I In the Model Builder window, expand the Results> Tub displacement magnitude (out-of-plane) node, then click Global I.

2 In the Settings window for Global, click Replace Expression in the upper-right corner of the $\mathbf{r}$-axis data section. From the menu, choose Model>Component I> Definitions $>$ Variables $>$ uout_tub - Tub displacement magnitude (out-of-plane).

3 On the Polar plot group toolbar, click Plot.
4 Click the Zoom Extents button on the Graphics toolbar.
Use the instructions below to generate the plot of tub rotation and stabilizing springs extension as shown in Figure 6 and Figure 7 respectively.

## ID Plot Group 5

I On the Model toolbar, click Add Plot Group and choose ID Plot Group.
2 In the Settings window for 1D Plot Group, type Tub rotation in the Label text field.

3 Locate the Data section. From the Data set list, choose Study 2/Solution 2.
4 Click to expand the Title section. From the Title type list, choose None.
5 Locate the Plot Settings section. Select the $\mathbf{x}$-axis label check box.

6 In the associated text field, type Number of cycles.
7 Select the $\mathbf{y}$-axis label check box.
8 In the associated text field, type Tub rotation (deg).

## Tub rotation

I On the ID plot group toolbar, click Global.
2 In the Settings window for Global, click Replace Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I> Multibody Dynamics>Rigid domains>Tub>Rigid body rotation (Spatial)> mbd.rd3.thx - Rigid body rotation, $x$ component.
3 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I>Multibody Dynamics>Rigid domains>Tub> Rigid body rotation (Spatial)>mbd.rd3.thy - Rigid body rotation, y component.

4 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I>Multibody Dynamics>Rigid domains>Tub> Rigid body rotation (Spatial)>mbd.rd3.thz - Rigid body rotation, z component.

Change the units to degrees from radians.
5 Locate the $\boldsymbol{y}$-Axis Data section. In the table, enter the following settings:

| Expression | Unit | Description |
| :--- | :--- | :--- |
| mbd.rd3.thx | deg | Rigid body rotation, x component |
| mbd.rd3.thy | deg | Rigid body rotation, y component |
| mbd.rd3.thz | deg | Rigid body rotation, z component |

6 Click Replace Expression in the upper-right corner of the $\mathbf{x}$-axis data section. From the menu, choose Model>Component I >Definitions>Variables> n_cycle - Number of cycles.

7 Click to expand the Coloring and style section. Locate the Coloring and Style section. Find the Line style subsection. In the Width text field, type 2.

8 Find the Line markers subsection. From the Marker list, choose Cycle.
9 In the Number text field, type 20.
10 Click to expand the Legends section. From the Legends list, choose Manual.

II In the table, enter the following settings:

## Legends

x component
y component
z component
I2 On the ID plot group toolbar, click Plot.
I3 Click the Zoom Extents button on the Graphics toolbar.

## Tub rotation I

I In the Model Builder window, right-click Tub rotation and choose Duplicate.
2 In the Settings window for 1D Plot Group, type Stabilizing spring extension in the Label text field.

3 Locate the Plot Settings section. Clear the $\mathbf{y}$-axis label check box.
4 Click to expand the Legend section. From the Position list, choose Lower left.

## Stabilizing spring extension

I In the Model Builder window, expand the Results>Stabilizing spring extension node, then click Global I.

2 In the Settings window for Global, click Replace Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I>
Multibody Dynamics>Spring-Dampers>Housing-tub (front)> mbd.spdI.dl-Spring extension.
3 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component l>Multibody Dynamics>Spring-Dampers> Housing-tub (back)>mbd.spd2.dl - Spring extension.

4 Locate the Legends section. In the table, enter the following settings:

| Legends |
| :--- |
| Front spring |
| Back spring |

5 On the ID plot group toolbar, click Plot.
6 Click the Zoom Extents button on the Graphics toolbar.
To plot the strut displacement and energy dissipation rate in struts as shown in Figure 8 and Figure 9 respectively, follow the instructions below.

## Stabilizing spring extension I

I In the Model Builder window, right-click Stabilizing spring extension and choose Duplicate.

2 In the Settings window for 1D Plot Group, type Strut displacement in the Label text field.

3 Locate the Legend section. From the Position list, choose Upper right.

## Strut displacement

I In the Model Builder window, expand the Results>Strut displacement node, then click Global I.

2 In the Settings window for Global, click Replace Expression in the upper-right corner of the $\mathbf{y}$-axis data section. From the menu, choose Model>Component I> Multibody Dynamics>Prismatic joints>Cylinder I-piston I> mbd.prjI.u - Relative displacement.

3 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component l>Multibody Dynamics>Prismatic joints> Cylinder 2-piston 2>mbd.prj2.u - Relative displacement.

4 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I>Multibody Dynamics>Prismatic joints> Cylinder 3-piston $\mathbf{3 > m b d}$.prj3.u - Relative displacement.

5 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component l>Multibody Dynamics>Prismatic joints> Cylinder 4-piston 4>mbd.prj4.u - Relative displacement.

6 Locate the Legends section. In the table, enter the following settings:

## Legends

Strut 1
Strut 2
Strut 3
Strut 4
7 On the ID plot group toolbar, click Plot.
8 Click the Zoom Extents button on the Graphics toolbar.

## Strut displacement I

I In the Model Builder window, right-click Strut displacement and choose Duplicate.
2 In the Settings window for 1D Plot Group, type Energy dissipation rate in the Label text field.

## Energy dissipation rate

I In the Model Builder window, expand the Results>Energy dissipation rate node, then click Global I.

2 In the Settings window for Global, locate the $\mathbf{y}$-Axis Data section.
3 In the table, enter the following settings:

| Expression | Unit | Description |
| :--- | :--- | :--- |
| mbd.prj1.Qdamper | W | Energy dissipation rate in damper |
| mbd.prj2.Qdamper | W | Energy dissipation rate in damper |
| mbd.prj3.Qdamper | W | Energy dissipation rate in damper |
| mbd.prj4.Qdamper | W | Energy dissipation rate in damper |

4 On the ID plot group toolbar, click Plot.
5 Click the Zoom Extents button on the Graphics toolbar.
Use the instructions below to plot the shell deformation components at mountings and side wall as shown in Figure 10 and Figure 11 respectively.

## Energy dissipation rate I

I In the Model Builder window, right-click Energy dissipation rate and choose Duplicate.
2 In the Settings window for 1D Plot Group, type Shell deformation (mountings) in the Label text field.
3 Click to expand the Title section. From the Title type list, choose Manual.
4 In the Title text area, type Shell deformation (mountings).

## Shell deformation (mountings)

I In the Model Builder window, expand the Results>Shell deformation (mountings) node, then click Global I.

2 In the Settings window for Global, click Replace Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I> Shell [Housing]>Attachments>Mounting I>Rigid body displacement> shell.att3.w - Rigid body displacement, $\mathbf{z}$ component.
3 Click Add Expression in the upper-right corner of the $\mathbf{y}$-axis data section. From the menu, choose Model>Component I>Shell [Housing]>Attachments>Mounting 2> Rigid body displacement>shell.att4.w - Rigid body displacement, z component.
4 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I>Shell [Housing]>Attachments>Mounting 3> Rigid body displacement>shell.att5.w - Rigid body displacement, z component.

5 Click Add Expression in the upper-right corner of the $\boldsymbol{y}$-axis data section. From the menu, choose Model>Component I>Shell [Housing]>Attachments>Mounting 4> Rigid body displacement>shell.att6.w - Rigid body displacement, z component.
6 Locate the Legends section. In the table, enter the following settings:

## Legends

Mounting 1
Mounting 2
Mounting 3
Mounting 4
7 On the ID plot group toolbar, click Plot.
8 Click the Zoom Extents button on the Graphics toolbar.

## Data Sets

I On the Results toolbar, click Cut Point 3D.
2 In the Settings window for Cut Point 3D, locate the Data section.
3 From the Data set list, choose Study 2/Solution 2.
4 Locate the Point Data section. In the $\mathbf{X}$ text field, type 600.
5 In the $\mathbf{Y}$ text field, type 250.
6 In the $\mathbf{Z}$ text field, type 450.
7 Select the Snap to closest boundary check box.
8 Click the Plot button.
ID Plot Group 10
I On the Results toolbar, click ID Plot Group.
2 In the Settings window for 1D Plot Group, type Shell deformation (side wall) in the Label text field.

3 Locate the Data section. From the Data set list, choose Cut Point 3D I.
4 Locate the Title section. From the Title type list, choose Manual.
5 In the Title text area, type Shell deformation (side wall).
6 Locate the Plot Settings section. Select the $\mathbf{x}$-axis label check box.
7 In the associated text field, type Number of cycles.
8 Select the $\mathbf{y}$-axis label check box.
9 In the associated text field, type Displacement (mm).

10 Locate the Legend section. From the Position list, choose Lower left.

## Shell deformation (side wall)

I On the ID plot group toolbar, click Point Graph.
2 In the Settings window for Point Graph, locate the $\boldsymbol{y}$-Axis Data section.
3 In the Expression text field, type u2.
4 Click Replace Expression in the upper-right corner of the $\mathbf{x}$-axis data section. From the menu, choose Model>Component I>Definitions>Variables> n_cycle - Number of cycles.

5 Click to expand the Coloring and style section. Locate the Coloring and Style section. Find the Line style subsection. In the Width text field, type 2.

6 Find the Line markers subsection. From the Marker list, choose Cycle.
7 In the Number text field, type 20.
8 Click to expand the Legends section. Select the Show legends check box.
9 From the Legends list, choose Manual.
10 In the table, enter the following settings:

## Legends

x component
II Right-click Results>Shell deformation (side wall)>Point Graph I and choose Duplicate.
12 In the Settings window for Point Graph, locate the $\mathbf{y}$-Axis Data section.
13 In the Expression text field, type v2.
14 Locate the Legends section. In the table, enter the following settings:

## Legends

y component
I5 Right-click Results>Shell deformation (side wall)>Point Graph 2 and choose Duplicate.
16 In the Settings window for Point Graph, locate the $\mathbf{y}$-Axis Data section.
17 In the Expression text field, type w2.
I8 Locate the Legends section. In the table, enter the following settings:

## Legends

z component
19 On the ID plot group toolbar, click Plot.

20 Click the Zoom Extents button on the Graphics toolbar.
To plot the shell normal acceleration with time and it's frequency spectrum as shown in Figure 12 and Figure 13 respectively, follow the steps below.

ID Plot Group II
I On the Model toolbar, click Add Plot Group and choose ID Plot Group.
2 In the Settings window for 1D Plot Group, type Shell normal acceleration (side wall): time in the Label text field.

3 Locate the Data section. From the Data set list, choose Cut Point 3D I.
4 Click to expand the Title section. From the Title type list, choose Manual.
5 In the Title text area, type Shell normal acceleration (side wall).
6 Locate the Plot Settings section. Select the $\mathbf{x}$-axis label check box.
7 In the associated text field, type Number of cycles.
Shell normal acceleration (side wall): time
I On the ID plot group toolbar, click Point Graph.
2 In the Settings window for Point Graph, locate the $\mathbf{y}$-Axis Data section.
3 In the Expression text field, type shell.u_ttX.
4 In the Unit field, type $\mathrm{mm} / \mathrm{s}^{\wedge} 2$.
5 Click Replace Expression in the upper-right corner of the $\mathbf{x}$-axis data section. From the menu, choose Model>Component I>Definitions>Variables> n_cycle - Number of cycles.

6 Click to expand the Coloring and style section. Locate the Coloring and Style section. Find the Line style subsection. In the Width text field, type 2.
7 On the ID plot group toolbar, click Plot.
8 Click the Zoom Extents button on the Graphics toolbar.

## Shell normal acceleration (side wall): time I

I In the Model Builder window, right-click Shell normal acceleration (side wall): time and choose Duplicate.

2 In the Settings window for 1D Plot Group, type Shell normal acceleration (side wall): frequency in the Label text field.

3 Locate the Title section. In the Title text area, type Shell normal acceleration (side wall).

4 Locate the Plot Settings section. Clear the $\mathbf{x}$-axis label check box.

## Shell normal acceleration (side wall): frequency

I In the Model Builder window, expand the Results>Shell normal acceleration (side wall): frequency node, then click Point Graph I.
2 In the Settings window for Point Graph, locate the x-Axis Data section.
3 From the Parameter list, choose Frequency spectrum.
4 On the ID plot group toolbar, click Plot.
5 Click the Zoom Extents button on the Graphics toolbar.

