Fatigue Damage Evaluation on Mechanical Components Under Multiaxial Loadings

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Abstract: This paper is concerned with the fatigue behaviour of complex, three-dimensional, stress concentrations under multiaxial loadings. Starting from the stress field obtained from a linear elastic analysis and taking advantage of the so-called implicit gradient approximation, an effective stress index connected with the material strength is calculated. The effective stress is calculated by solving a second-order differential equation over all the component (the implicit gradient approach) independently of its geometric shape. Besides this work summarizes a first investigation into the possibility of applying the implicit gradient approach to real components under both uniaxial and multiaxial loading conditions by introducing an appropriate multiaxial criterion into the implicit gradient framework. Generally speaking many multiaxial criteria could be used to this purpose, namely critical plane approaches, stress-invariant based approaches and integral approaches. However, since the aim of the present work is to obtain a numerically efficient other than the effective method, in this first attempt attention has been focused on stress-invariant based approaches that is suitable for high-cycle fatigue evaluation. The damage evaluation is obtained by analyzing the loading path on the 5-dimensinal deviatoric Euclidean space. Explicit analytical solutions of the proposed criterion are available in case of biaxial sinusoidal loads. The method in conjunction with implicit gradient approach has been applied to experimental results generated by testing notched specimens of low-carbon steel containing severe 3D stress raisers subjected to uniaxial and multiaxial in-phase and out-of-

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1. Introduction

Reliability design of real components with three-dimensional (3D), stress concentrations under fatigue loads is a subject of great practical interest to industrial engineers.

Three-dimensional solid modelling tools are largely used by design engineers for virtual prototyping of new products or structures. In order to capitalize on the increased capability of these numerical aids, the applied research have to meet the needs of designers so that calculation tools must be developed to be integrated into solid modelling and FEM analysis of three-dimensional components.

In this context, the structural problems are often complicated by the presence of multiaxial stress state due to the complex geometrical shapes itself or complex loading conditions. In order to take into account the notch stress effect, notch stress approaches can be applied. In this way fatigue strength could be related to the stress–strain condition very close to the crack initiation point. Several methods have been proposed over the last few decades to estimate the fatigue life of notched components: Peterson [1] and Neuber [2] method; the Theory of Critical Distance [3]; the strain-life method [4]; the Notch Stress Intensity Factors approach [5]. Unfortunately, for real structures subjected to complex multiaxial loading conditions, this methods cannot easily be applied directly.

Recently, Tovo et al. have proposed a “non-local” model (*Implicit Gradient approach*) initially to predict static failure [6] and subsequently to evaluate the stress gradient effect on the fatigue strength of steel welded joints [7]. These methods are called ‘‘non-local” because the strength at a given ‘‘local” point is related to the stress–strain condition of the surrounding ‘‘non-local” material. Starting from a stress field obtained from a linear elastic analysis, this method provides a finite reliable solution of an effective stress all over the investigated solid, also at the sharp notches. Hence the implicit gradient is an interesting way of addressing the problem of fatigue strength evaluation of components affected by high stress raising effects.

The aim of this paper is developing a numerical tool in conjunction with three-dimensional modelling tools to be used by industrial engineers to predict multiaxial fatigue assessment of mechanical components, affected by high stress concentrations and complex loading conditions. A first validation of the method will be proposed by analysing experimental data that we generated by testing 3D-notched cylindrical samples made of a commercial cold-rolled low-carbon steel, subjected to combined tension and torsion loading, both in-phase and out-of-phase.

2. Implicit gradient approach

Early developments on non-local models were proposed by Eringen, Edelen [8] and Kroner in 1960 and were based on the assumption that the principle of local action could be violated. In particular it was proposed that the stress $\bar{\bar{\sigma}}$ depends not only on the strain $\bar{\bar{\varepsilon}}$ at the point $\bar{x}$, but also a weighted average over a reference volume with weight proportional to the distance from $\bar{x}$. So in a body with volume V and surface S, the non local stress tensor $\tilde{\bar{\bar{\sigma}}}\left(\bar{x}\right)$ at the point $\bar{x}=(x_1,x_2,x_3)$ in V, can be obtained from the weighted average of local stress tensor $\bar{\bar{\sigma}}\left(\bar{x}\right)$ through the following expression:

$$\tilde{\bar{\bar{\sigma}}}\left(\bar{x}\right)=\frac{1}{V_r\left(\bar{x}\right)}\int_V \alpha\left(\bar{x},\bar{y}\right)\cdot\bar{\bar{\sigma}}\left(\bar{y}\right)d\bar{y} \tag{1}$$

In equation (1) $\alpha(\bar{x},\bar{y})$ indicates a scaling of the weight function that depends on the Euclidean distance $\|\bar{x}-\bar{y}\|$ between point $\bar{x}$ and every point $\bar{y}=(y_1,y_2,y_3)$ of V.

A variant of non-local integral definitions, defined as “implicit gradient model” was initially proposed by Peerlings et al. [9]. In fact, starting from the definition in terms of full non-local model (1), could be developed a gradient expansion of the non local scalar [10]:

$$\tilde{\zeta}\left(\bar{x}\right)\cong\zeta\left(\bar{x}\right)+c^2\nabla^2\tilde{\zeta}\left(\bar{x}\right) \tag{2}$$

where c is a characteristic length related to the weight function $\alpha(\bar{x},\bar{y})$ defined on the whole volume V. For engineering applications, c is assumed to be related only to relevant material properties. In equation (2) the Laplacian operator is applied to the non-local equivalent stress, so that $\tilde{\zeta}\left(\bar{x}\right)$ can be obtained by solving an implicit type differential equation. In this type of analysis, usually, as boundary conditions are taken into consideration just the amount of Newmann, expressing the orthogonality of the gradient of the solution sought by the outgoing normal to the edge of the domain of integration:

$$\nabla\tilde{\zeta}\cdot\bar{n}=0 \tag{3}$$

where $\bar{n}$ is the normal to the surface of the body of volume V [11]-[9]. It results that eq. (2) is much easier to be solved numerically than eq.(1). For this reason, it can be applied to a three-dimensional domain, where the actual critical point cannot be previously assumed as it has to be localized by means of the numerical investigation. PDE Modes of COMSOL Multiphysics® has been used to solve eq. (2). Second order tetrahedral elements have been employed to mesh the three-dimensional models, as Neumann-type boundary condition has been applied at all side surfaces of the models.

3. Fatigue life prediction

The fatigue behaviour is related to stress and,or strain variations. In this work, we will consider only non-proportional constant amplitute loading, so that fatigue life can be simply expressed by an equivalent stress amplitude. By means of eq. (2), an equivalent deviatoric stress amplitude $\tilde{\sigma}_{d,a}$, linked to fatigue life, can be defined assuming that $\tilde{\sigma}_{d,a}=\tilde{\zeta}$ and assuming as local scalar the equivalent deviatoric stress amplitude resulting from an isotropic linear elastic solution $\sigma_{d,a}=\zeta$. Eqs. (2) and (3) become:

$$\tilde{\sigma}_{d,a}\cong\sigma_{d,a}+c^2\nabla^2\tilde{\sigma}_{d,a} \tag{4}$$

$$\nabla\left(\tilde{\sigma}_{d,a}\right)\cdot\mathbf{n}=0 \tag{5}$$

However, to correctly define the equivalent deviatoric component, it must be considered that under non-proportional loading the principal stress directions are not constant. For this reason, it could be necessary to apply a multiaxial criterion together with the stress gradient approach under mixed-mode loadings, in order to correctly evaluate fatigue strength.

3.1. Definition of the equivalent amplitude of the deviatoric component

From a theoretical point of view many multiaxial criteria could be used into the implicit gradient approach, namely critical plane approaches, stress-invariant based approaches and integral approaches. In fact, such criteria make use of scalar quantities that can be introduced as equivalent stress in eq. (4).

In this work, multiaxial fatigue damage calculation is then performed by means a stress-invariant based criterion (*PbP* approach) proposed by Cristofori et al. [12]-[13]. It makes use of deviatoric component $\bar{\bar{\sigma}}_d(t)$ and hydrostatic component $\sigma_H(t)$ to evaluate the damage due to generic fatigue loading.

According to the notation used for Crossland invariant criterion [14], the deviatoric tensor $\bar{\bar{\sigma}}_d(t)$ can be concisely represented as a 5-element vector defined as follows:

$$\bar{s}(t)=\begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \\ s_5(t) \end{bmatrix}=\begin{bmatrix} \frac{\sqrt{3}}{6}\left(2\sigma_x(t)-\sigma_y(t)-\sigma_z(t)\right) \\ \frac{1}{2}\left(\sigma_y(t)-\sigma_z(t)\right) \\ \tau_{xy}(t) \\ \tau_{xz}(t) \\ \tau_{yz}(t) \end{bmatrix} \tag{6}$$

As an external load is applied the tip of the vector $\bar{s}(t)$ describes a curve Γ, the deviatoric stress component loading path, as show in Fig. 1. There are several various ways of defining the range of the deviatoric vector so identified [15]-[16]. *PbP* approach takes as a starting point the hypothesis that the damage related to a generic loading path Γ can be estimated by considering the single contributions $\Gamma_{p,i}$ calculated by projecting the loading path itself along the axes of a frame of reference chosen as a base of the Euclidean space.

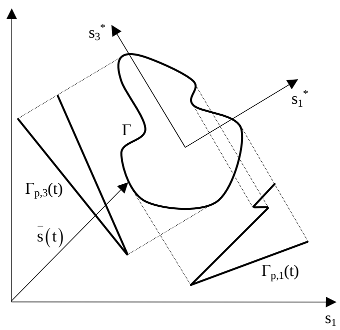


Figure 1. Loading path.

In order to propose a procedure suitable for addressing the above problem, the principal axes of path Γ can be attempted to be used to define a unique frame of reference suitable for accounting for the presence of non-zero out-of-phase angles. For this purpose, to correctly calculate the directions of the above principal axes, path Γ can be treated as a continuum, so that, its centroid can be determined as follows[17]:

$$s_{m,i}=\frac{1}{T}\int_T s_i(t)\,dt \quad i=1,.....,5 \tag{7}$$

where T is the period. Moreover, using a definition similar to the one adopted to define continua's moment of inertia, the rectangular moments of inertia of path Γ can be represented with respect to its centroid by using the following symmetric square matrix of order five:

$$C_{ij}=\int_T \left(s_i(t)-s_{m,i}\right)\cdot\left(s_j(t)-s_{m,j}\right)dt \quad i,j=1,.....,5 \tag{8}$$

This matrix has five eigenvalues and five orthogonal eigenvectors. The eigenvalues are the principal moments of inertia, whereas the eigenvectors are the principal directions of the tensor path calculated with respect to the centroid itself. Finally, the loading path Γ can be projected along each frame axis (suffix i) so that, the equivalent deviatoric stress amplitude, $\sigma_{d,a}$, can be calculated, in according to the *PbP* approach, as follow:

$$\sigma_{d,a}=\sqrt{\sum_i\left(\sigma_{d,a}\right)_i^2} \tag{9}$$

where $(\sigma_{d,a})_i$ is the amplitude of the projection along ith axis.

3.2. Multiaxial high-cycle fatigue criteria

Fatigue limit calculation is perform by the biparametric method, proposed by Lazzarin and Susmel [18], formulated in terms of stress tenson invariants. According to this approach, fatigue bihaviour is related to the multiaxial stress ratio, ρ_{FL}, between the non local values of the hydrostatic component, $\tilde{\sigma}_{H,max}$ and the deviatoric stress component, $\tilde{\sigma}_{d,a}$:

$$\rho_{FL} = \sqrt{3} \cdot \frac{\tilde{\sigma}_{H,max}}{\tilde{\sigma}_{d,a}} \tag{10}$$

For a given loading condition, the estimated fatigue limit value, $\sigma_{d,A}\big|_{\rho_{FL}}$, can be calculated by assuming a linear variation of $\sigma_{d,A}\big|_{\rho_{FL}}$ to respect to ρ_{FL}. Generally, the uniaxial fatigue limit, $\sigma_{d,A}\big|_{\rho_{FL}=1}$ and the torsional fatigue limit $\sigma_{d,A}\big|_{\rho_{FL}=0}$ are used to calibrate the criterion:

$$\sigma_{d,A}\big|_{\rho_{FL}} = \sigma_{d,A}\big|_{\rho_{FL}=0} + \rho_{FL} \cdot \left(\sigma_{d,A}\big|_{\rho_{FL}=0} - \sigma_{d,A}\big|_{\rho_{FL}=1} \right) \tag{11}$$

Finally, the general form for this criterion can be written by comparing the non local value of the deviatoric stress component and the estimated fatigue limit value:

$$\tilde{\sigma}_{d,a} \le \sigma_{d,A}\big|_{\rho_{FL}} \tag{12}$$

4. Experimental details and results

In order to check the validity of the fatigue assessment procedure proposed previously, 70 tests were carried out by testing cylindrical 3D-notched specimens having cross diameter equal to 8 mm and made of a commercial cold-rolled low-carbon steel, En3B. This material had an ultimate tensile strength, σ_{UTS}, equal to 676 MPa, a yield stress, σ_Y , equal to 653 MPa and a Young's modulus equal to 208500 MPa. The geometry of the tested fatigue samples is show in Fig. 2.

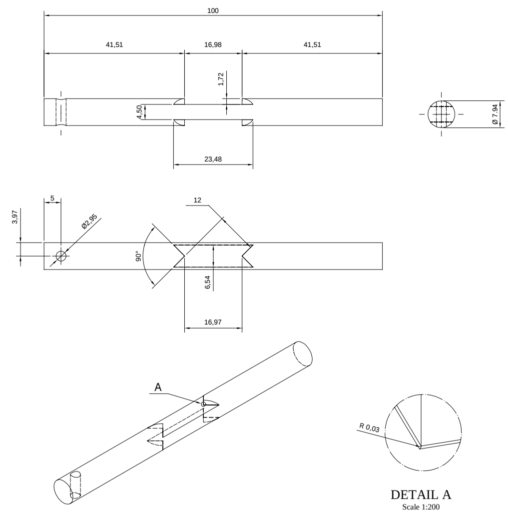


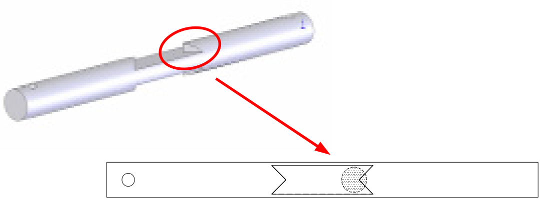
Figure 2. Geometry of the tested specimen.

The specimens are characterized by severe stress raisers with root radius equal to 0.03mm. Such specimens were tested under pure tension, pure torsion and mixed tension–torsion loading in-phase and 90° out-of-phase, by imposing a uniaxial and torsional load ratio, R, equal to −1. Under biaxial load, two different ratios, δ, between the amplitudes of the tensile, $\sigma_{x,a}$, and torsional, $\tau_{xy,a}$, stress refers to the gross section were considered, that is, $\delta = 1$ and $\delta = \sqrt{3}$. All tests were carried out in the Department of Mechanical Engineering, Trinity College of Dublin on a Instron® 8874 servo-hydraulic axial-torsional testing system with an axial cell ±10 KN and a torsion cell of ±100 Nm. All tests have been performed under load control, with a frequency ranging from 8 and 16 Hz, as a function of load value. During all fatigue tests, the specimen stiffness has been monitored and the fatigue failure was defined by 10% stiffness drop, which resulted in cracks, emanated from the tip of the notch, having length of about 1 mm. In agreement with observed fatigue behaviour, high-cycle endurance limits were extrapolated at $N_A = 2 \cdot 10^6$ cycles. A summary of the results obtained from the experimental investigations of all series is given in Table 1.

Table 1. Summary of the experimental fatigue results.

Series	Loading conditions σ_a [Mpa]	τ_a [Mpa]	φ	R	No. data	Slope k	$\sigma_{A,50\%}$ at $2\cdot10^6$ cycles [MPa]
Uniaxial	1	0	0°	-1	15	3.76	75.7
Torsional	0	1	0°	-1	16	6.87	66.1
Biaxial in phase	1	1	0°	-1	13	4.98	68.7
Biaxial out of phase	1	1	90°	-1	11	4.61	66.7
Biaxial in phase	1.73	1	0°	-1	8	5.67	52.7
Biaxial out of phase	1.73	1	90°	-1	7	5.36	52.9

In Fig. 3, the pictures showing the generated crack paths on the surfaces of different specimens for various loadings conditions.



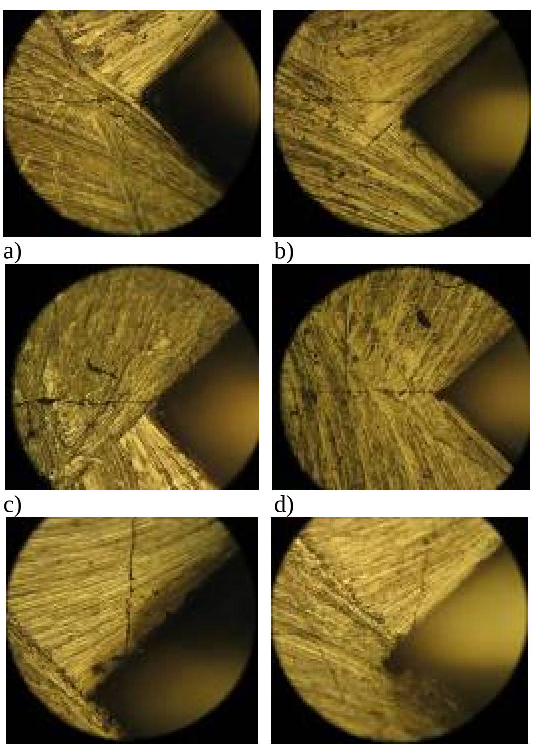


Figure 3. Examples of the observed crack path.

5. Procedure and calculation tools

To clarify this procedure it is important highlight here the fundamental steps that we need to obtain a multiaxial fatigue damage evaluation by means of aforementioned non local approach in conjunction with *PbP* criterion:

1) Linear elastic stress analysis has to be carried out for any external loading applied.

2) Maximum variance reference frame are calculated for any nodal point.

3) Local values of the equivalent deviatoric stress amplitude, $\sigma_{d,a}$, and hydrostatic component, $\sigma_{H,max}$, are evaluated for any nodal point.

4) Non local values of the $\tilde{\sigma}_{d,a}$ and $\tilde{\sigma}_{H,max}$ are calculated by means of the implicit gradient approach:

$$\tilde{\sigma}_{d,a} \cong \sigma_{d,a} + c^2\nabla^2\tilde{\sigma}_{d,a} \tag{13}$$

$$\nabla\left(\tilde{\sigma}_{d,a}\right)\cdot\mathbf{n} = 0 \tag{14}$$

$$\tilde{\sigma}_{H,max} \cong \sigma_{H,max} + c^2\nabla^2\tilde{\sigma}_{H,max} \tag{15}$$

$$\nabla\left(\tilde{\sigma}_{H,max}\right)\cdot\mathbf{n} = 0 \tag{16}$$

5) Multiaxial stress ratio, ρ_{FL}, between the non local values of the hydrostatic and deviatoric stress components is evaluated:

$$\rho_{FL} = \sqrt{3}\cdot\frac{\tilde{\sigma}_{H,max}}{\tilde{\sigma}_{d,a}} \tag{17}$$

6) Fatigue limit value, $\sigma_{d,A}\big|_{\rho_{FL}}$, for a given loading condition is calculated by means eq. (11).

7) Fatigue life estimation is finally performed at each nodal point:

$$\tilde{\sigma}_{d,a} \le \sigma_{d,A}\big|_{\rho_{FL}} \tag{18}$$

To conclude, the flow-chart reported in Fig. 4 summarizes the procedure above explained and highlights the software used. In particular, have been used the Structural Mechanics Models of COMSOL Multiphysics® to perform linear elastic stress analysis and Matlab® to calculate the projection system frame, equivalent deviatoric stress amplitude, $\sigma_{d,a}$, and hydrostatic component, $\sigma_{H,max}$, for any nodal point. Finally PDE Modes of COMSOL Multiphysics® has been used to solve eq. (13)-(15) and to perform the fatigue life estimation. Moreover, to shift working environment , it was necessary to establish suitable import and extraction's routine of the stress nodal quantities.

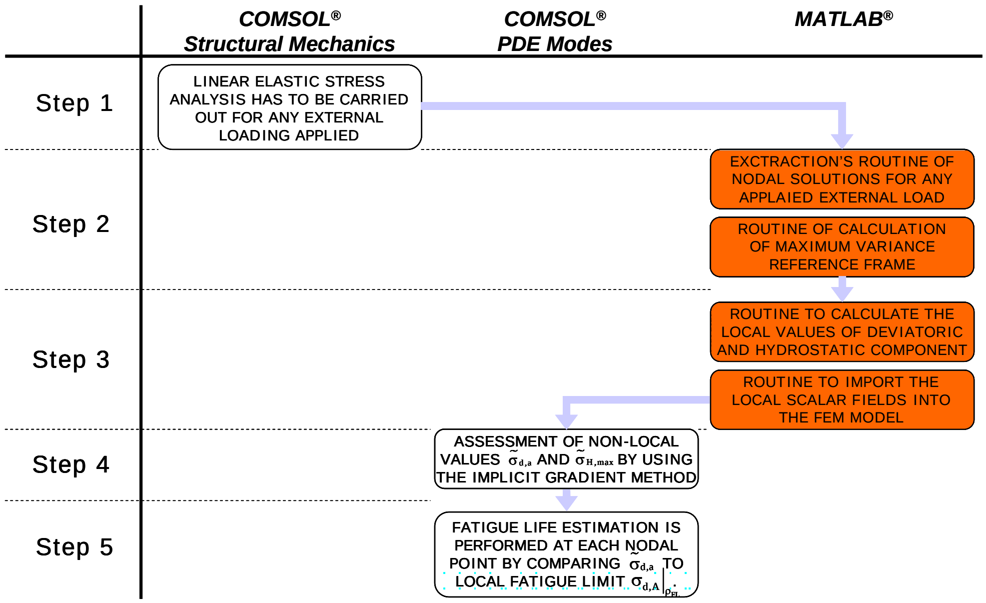


Figure 4. Procedure framework.

8. Numerical results

The following summarises the application of the above procedure. The error factor is defined as:

$$E(\%) = \frac{\sigma_{A,exp} - \sigma_{A,fem}}{\sigma_{A,fem}} \times 100 \qquad (19)$$

Hence, an error factor of greater than zero indicates a conservative prediction.

Post-processing and the numerical value of the damage index are depicted in Fig. 5. Sub-modeling technique was used to employ refined mesh near the singularity points in order to obtain the convergence of the non local stress field. The results in terms of tensile and torsional stress amplitude refered to the gross section, are summarised in Table 2.

The locations of the the hot-spot predicted by FEA (maximum damege index) agree with the experimental crack initiation points.

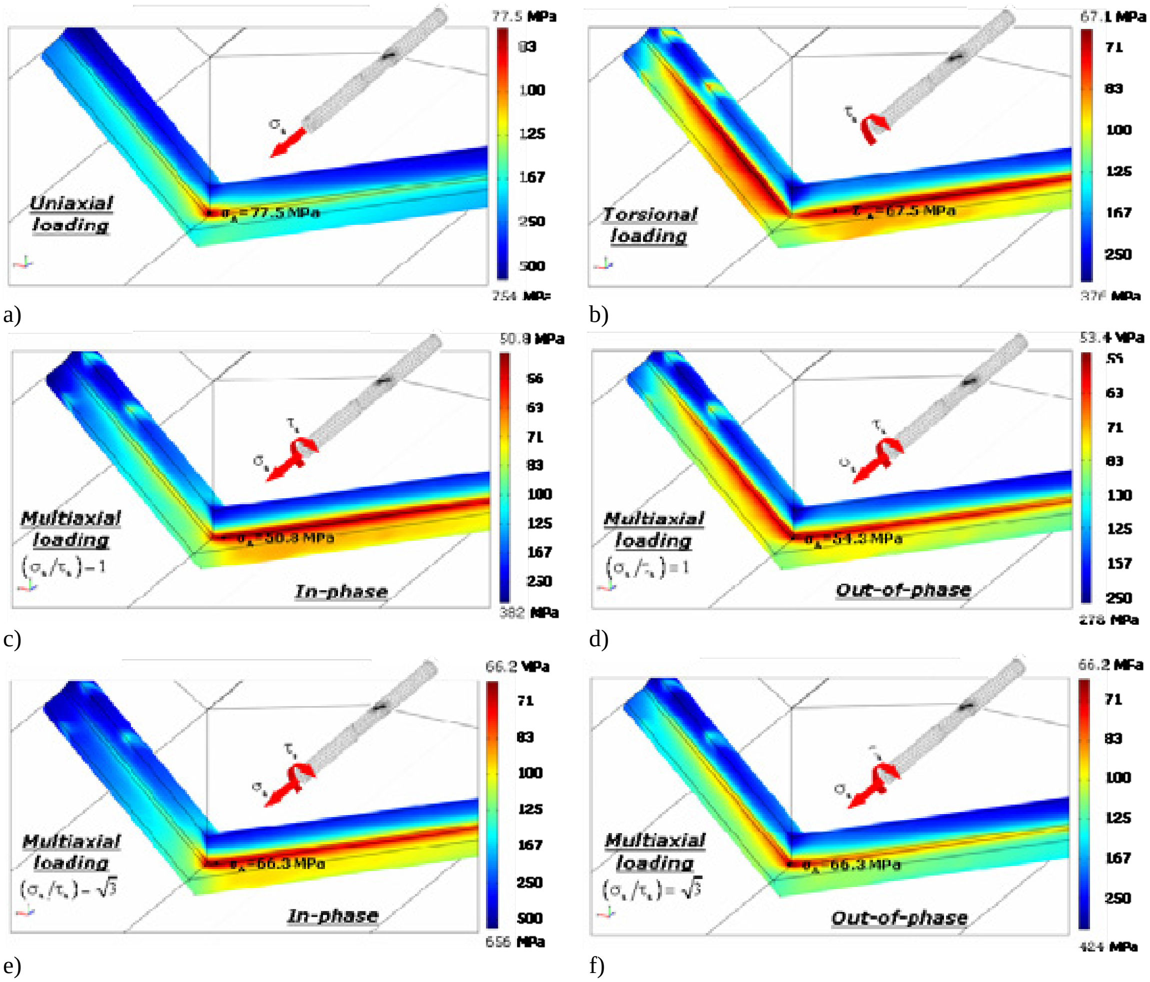
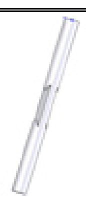


Figure 5. Damage index.

Table 2. Synthesis of the numerical results.

GEOMETRY	LOAD CONDITIONS				EXPERIMENTAL RESULTS		FATIGUE STRENGTH PREDICTION		ERROR INDEX	CRITICAL POINT LOCATION		
	σa [Mpa]	τa [Mpa]	φ	R	σ_A [Mpa]	τ_A [Mpa]	σ_A [Mpa]	τ_A [Mpa]	E(%)	x [mm]	y [mm]	z [mm]
	1	0	0°	-1	75.7	–	77.5	–	**-2.3%**	0.00	0.00	0.00
	0	1	0°	-1	–	66.1	–	67.5	**-2.1%**	0.84	0.00	0.85
	1	1	0°	-1	52.7	–	50.8	–	**3.7%**	-0.25	0.00	0.25
	1	1	90°	-1	52.9	–	54.3	–	**-2.6%**	-0.00	0.02	0.04
	1.73	1	0°	-1	68.7	–	66.3	–	**3.6%**	-0.03	0.01	0.06
	1.73	1	90°	-1	66.7	–	66.3	–	**0.6%**	-0.00	0.02	0.04

7. Conclusions

In this work was considered both the effect on the fatigue strength due to the presence of complex three-dimensional (3D) notches (*gradient effect*) and the multiaxial effect caused by external loadings as well as by multiaxial stress fields due to severe stress raisers (*multiaxial effect*). Moreover we developed a numerical tool in conjunction with three-dimensional modelling tools to be used by industrial engineers. Finally, in order to validate the proposed approach, theoretical fatigue estimations and experimental data was compared. The major conclusions can be summarised as follow:

- The devised approach is seen to be highly accurate in estimating high-cycle fatigue damage in mechanical components without the need for assuming a priori the position of the critical point.
- This approach is capable of efficiently taking into account the presence of both multiaxial loading and non zero out-of-phase angles.
- The implicit gradient method applied in conjunction with *PbP* approach proved to be a powerful engineering tool capable of efficiently designing complex, i.e. three-dimensional (3D), stress concentrations against multiaxial fatigue.
- The fatigue life estimation technique proposed in the present work is suitable for being used in situations of practical interest by directly post-processing simple linear-elastic FE models.
- Even if the results obtained so far are very satisfactory, more work needs to be done for a complete validation of this method.

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