Coupling Stochastic Boundary Perturbations with Fiber Drawing Heat Transfer

A. Emery[1]

[1]University of Washington, Seattle, WA, USA

Abstract

The production of polymer fibers is done by drawing the raw material (preform) in a vertical cylindrical furnace whose heated walls radiantly heat the preform, Figure 1. When drawing glass fibers the wall temperatures are very high and the dominant heat transfer to the fiber is by radiation with little effect from the convective flow of the gas in the furnace. In contrast, for polymer fibers the convection contribution is large and oscillations or stochastic variations in the gas flow can cause random diameter variations, Figure 2.

Clearly the random diameter variations coupled with stochastic fluid variations affect the convective heat transfer and the fiber temperature. As a first step in understanding the effects, we have simulated the heating of a axisymmetric fiber with diameters varying with the axial length, z. The heat transfer coefficient and the radius are represented by polynomial chaos through the representation as:

r(z)= R0(z)+ a(z) and h(z,t)= Sum over i of [hi (z,t) Hi(b(z))] (1a)

where R0(z) is the deterministic value of the radius, Hi(b) are hermite polynomials and a(z) and b(z) represent the stochastic realization. The fiber temperature is represented correspondingly in the form:

T(z,r,t)= Sum over i of [Ti (z,r,t) Hi(a(z)) Hi(b(z))] (1b)

The problem is how to apply these forms to a stochastically varying diameter. Typically, Eq. 1 have been applied only to fixed regions, i.e., R(z) is deterministic, although with random initial conditions. In their current method of application, they cannot handle randomly defined boundaries. To accommodate the random boundaries, we must convert the problem into a fixed region. We follow the method used in the analysis of a stochastic phase front in which the boundary condition, which is defined in terms of the latent heat, is recast into a stochastic specific heat and is thus amenable to representation in terms of Hermite polynomials. This will be done as follows: Let the original coordinates be z, r, defined in the region [0, L]x[0,R0(z)] where the radius of the fiber, i.e., the boundary of the fiber-fluid flow, is defined in terms of its deterministic value, R0(z) and a stochastic perturbation a(z), by R(z)=R0(z) +a(z). The flow region is mapped to u,v, [0,L]x[0,1] where y=z, and r= v(R0(z) +a(z)). The transformation then effectively converts the original partial differential equation with deterministic properties into one

with stochastic conductivity through the Jacobian of transformation, but in a fixed deterministic region.

The solution is based upon the ability of COMSOL Multiphysics® to separately solve the conduction equation with user defined boundary conditions. Each of the Ti terms introduces another coupled partial differential equation.

Solutions are given in terms of the correlation lengths of the perturbations of the heat transfer, b(z), and of the radius, a^{ϵ} (z). The results are compared to a previous presentation at the 2009 conference [1] where temperatures were made on plane slabs with stochastic geometry in contrast to the method proposed here. These results were obtained with a(z) based upon a Kahrunen-Loeve explansion using COMSOL driven from MATLAB®. The geometry was changed for each resolution of a(z) and the results post processed using MATLAB. Figure 3 compares the effect of the correlation length, L, compared to the length, H, of the slab.

The new results will be based on a Monte Carlo simulation for a(z) and b(z) being based on white noise using the random function in COMSOL 4.3 driven by sweeping parameters that are functions of time. The results are then post processed in COMSOL to obtain mean values and standard deviations of the heat flux. Although the fiber diameter varies with z, in order to compare with the previous results, R0(x) is set equal to a constant. If possible computations will be made using LiveLink™ for MATLAB® to evaluate these two ways of treating stochastic problems.

Reference

COMSOL Conference, 2009, , “The Effect of Spatially Correlated Roughness on the Conduction of Heat through a Slab”

Figures used in the abstract

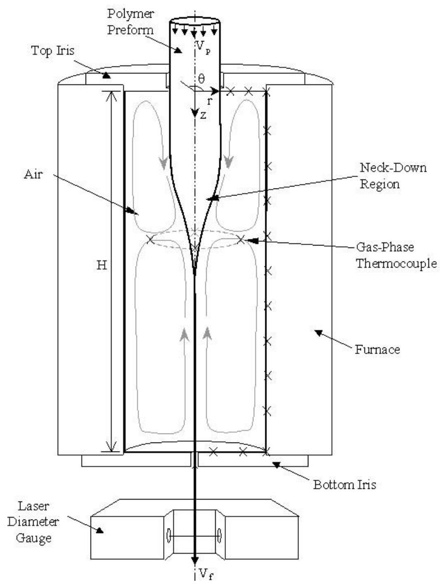

Figure 1: Schematic of the POF draw furnace.

Figure 1: Schematic of a Polmer Fiber being drawn.

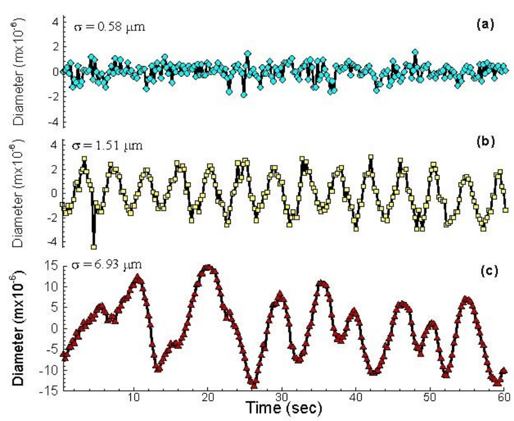

Figure 2: Diameter Histories: a) laminar, b) oscillatory,
c) chaotic [Note the difference in scales]

Figure 2: Showing the variation of Diameter with axial position.

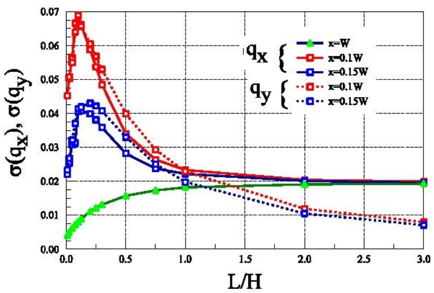

Figure 3 Effect of correlation length on the standard deviation of the heat fluxes.

Figure 3: Effect of Correlation on the Heat Flux.