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Universal Dielectric Response of Atmospheric Ice using COMSOL

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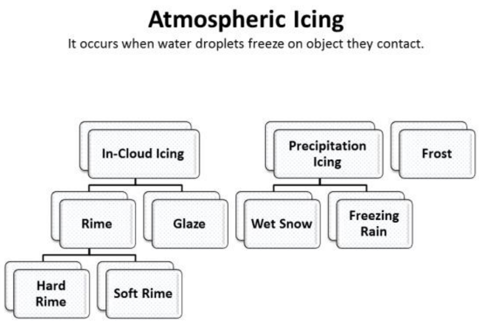
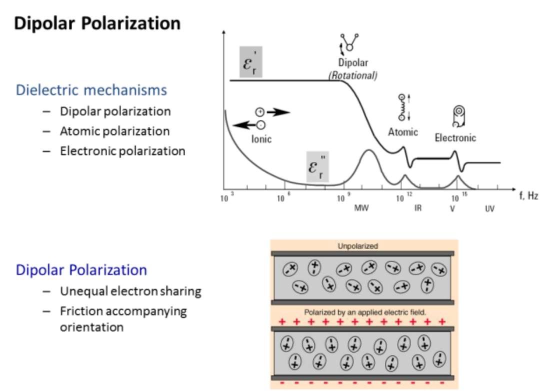
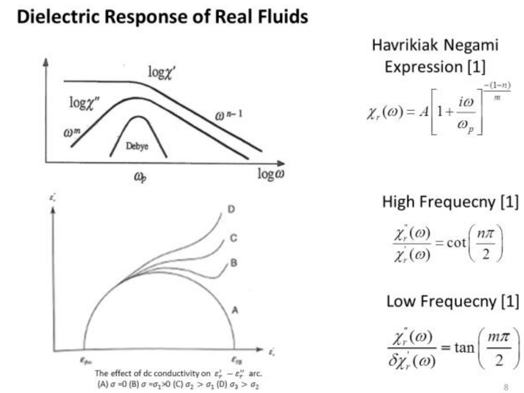
My Today's Talk Will Comprise



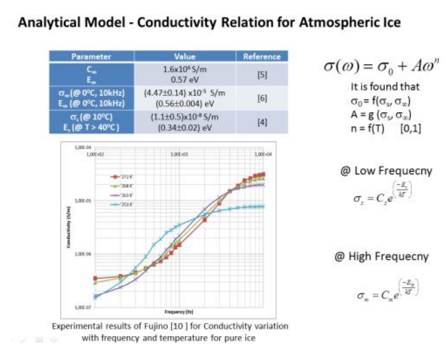
Atmospheric Ice



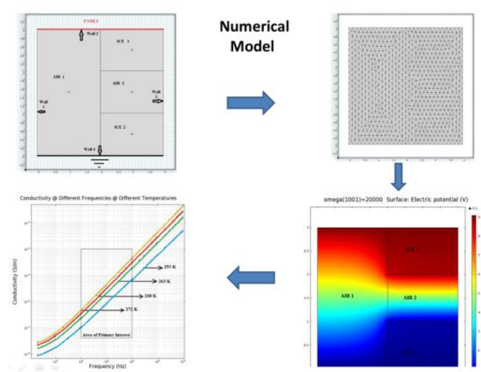
Dipolar Polarization



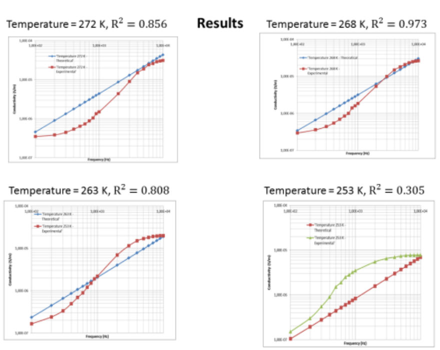
Real Dielectric Response



UDR For Atmospheric Ice



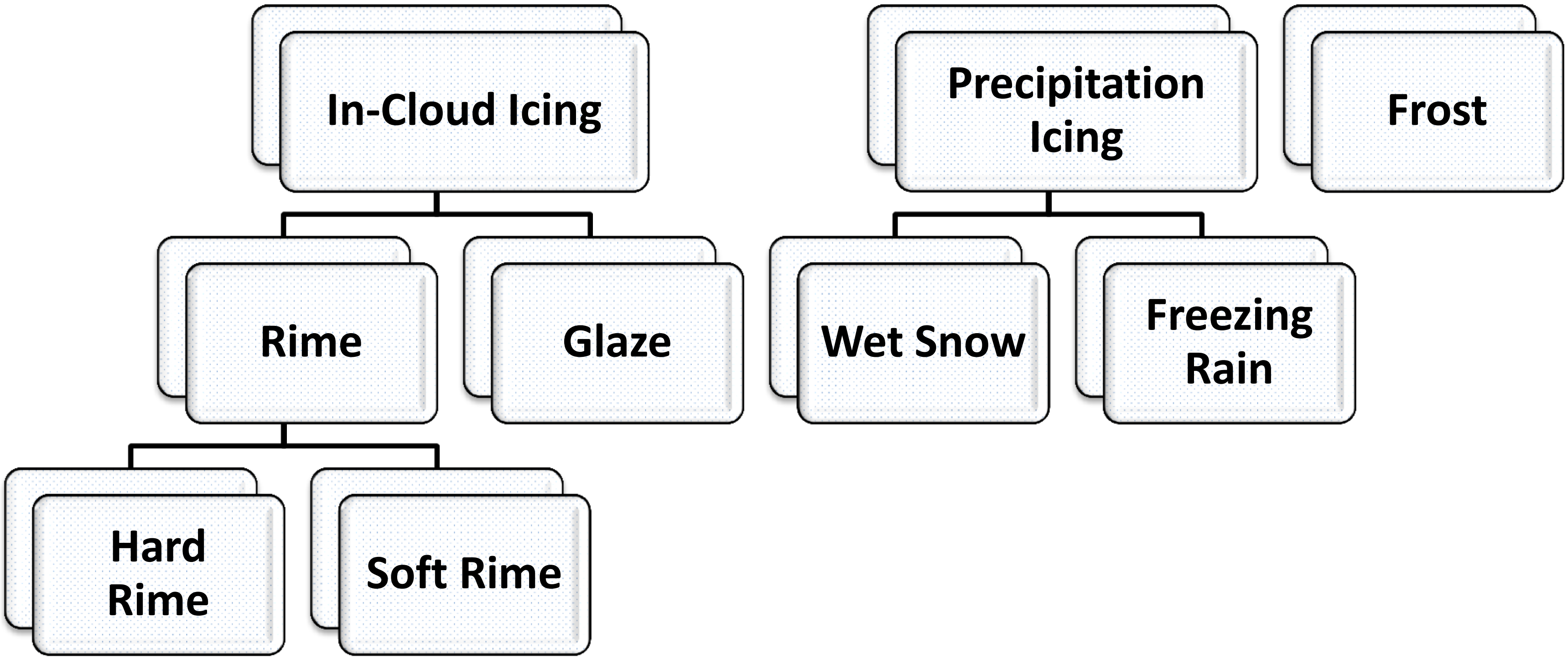
Numerical Setup



Results

Atmospheric Icing

It occurs when water droplets freeze on object they contact.



Water Molecule (H_2O)

Oxygen

Atomic Number : 8

Electronegativity: 4.44

Hydrogen

Atomic Number : 1

Electronegativity : 2.2

Electronegative Critical Values

Least Electronegative : 0.7

Highly Electronegative : 4

Non Polar Covalent Bond : 0

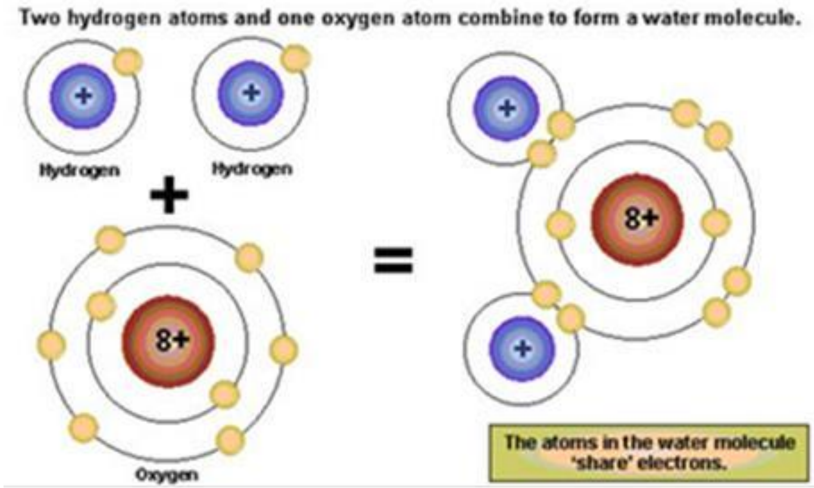
Linear Polar Covalent Bond : 0-0.7

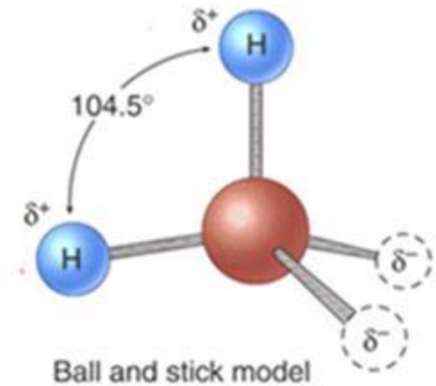
Nonlinear Polar Covalent Bond : 0.7-1.7

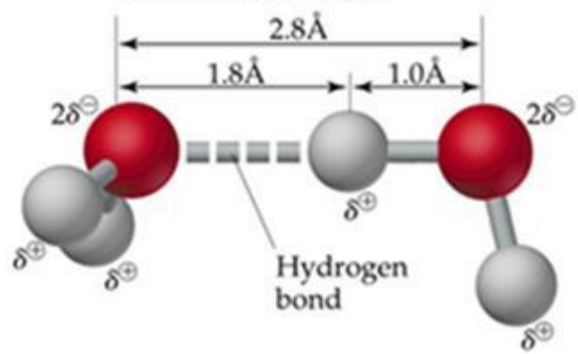
Ionic Bond : > 1.7

Oxygen and Hydrogen

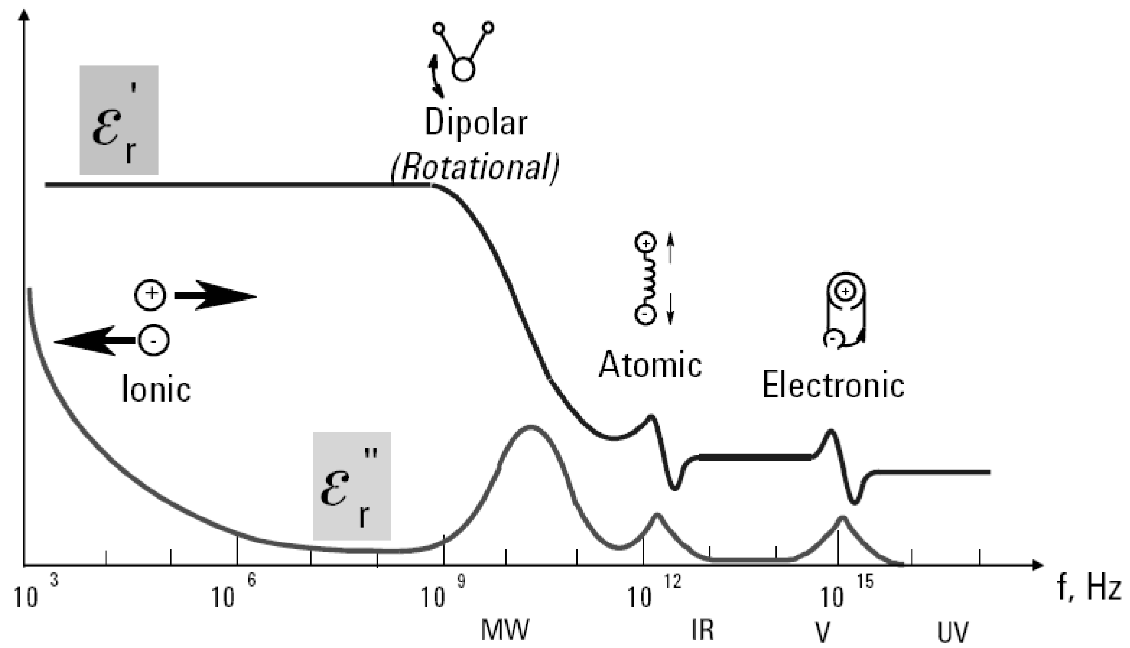
$0.7 < (4.44-2.2)=1.22 < 1.7$







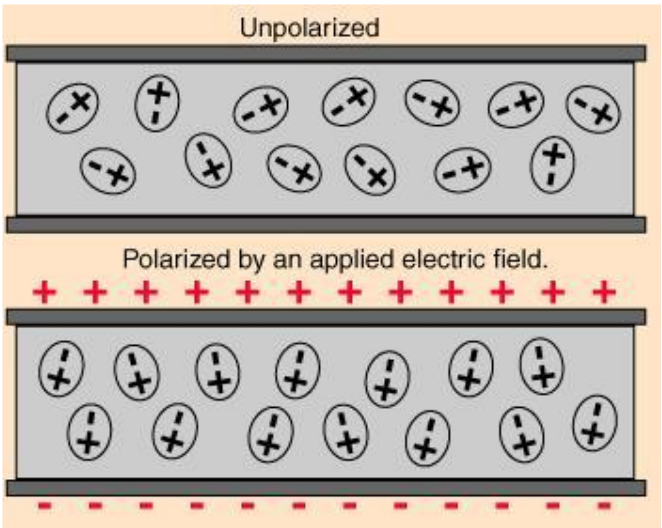
Dipolar Polarization



Dielectric mechanisms

- Electronic polarization
- Ionic polarization
- Interfecial polarization
- Dipolar polarization

$$\varepsilon = \varepsilon_0 \left(\chi_{electronic} + \chi_{ionic} + \chi_{dipolar} + \chi_{interfecial} \right)$$



Dipolar Polarization

- Unequal electron sharing
- Friction accompanying orientation

i. Dielectric Susceptibility 'χ'

$$\chi_r(\omega) = \chi_r^{'}(\omega) - \chi_r^{''}(\omega) = \varepsilon_r(\omega) - \varepsilon_{r\infty}$$

ii. Models to determine Dielectric Susceptibility

Kramer's Kroing Model

$$\chi_r^{'}(\omega) = \frac{2}{\pi}\int_0^{+\infty} \frac{\omega\chi_r^{''}(\omega)}{\omega^2 - \omega_r^2} d\omega$$

$$\chi_r^{''}(\omega) = -\frac{2\omega_r}{\pi}\int_0^{+\infty} \frac{\chi_r^{'}(\omega)}{\omega^2 - \omega_r^2} d\omega$$

Debye Model

$$\varepsilon_r^{'} = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} + \varepsilon_{r\infty}}{1 + \omega^2\tau_0^2}$$

$$\varepsilon_r^{''} = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})\omega\tau_0}{1 + \omega^2\tau_0^2} + \frac{\sigma}{\omega\varepsilon_0}$$

where

'ε_r' is the relative permittivity

'ω_r' is the reference frequency

'ω' is the frequency from 0 to ∞.

'ε_{rs}' is relative permittivity at D.C

'$\varepsilon_{r\infty}$' is relative permittivity at high frequency

'τ_0' is the relaxation time

Dielectric Responses of Ideal Fluid (Debye Fluids), Ref. [8]

$$\frac{\chi_r^{''}(\omega)}{\chi_r^{'}(\omega)} = \omega\tau$$

$$\varepsilon_r^{'} = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} + \varepsilon_{r\infty}}{1+\omega^2\tau_0^2}$$

$$\varepsilon_r^{''} = \frac{\left(\varepsilon_{rs} - \varepsilon_{r\infty}\right)\omega\tau_0}{1+\omega^2\tau_0^2}$$

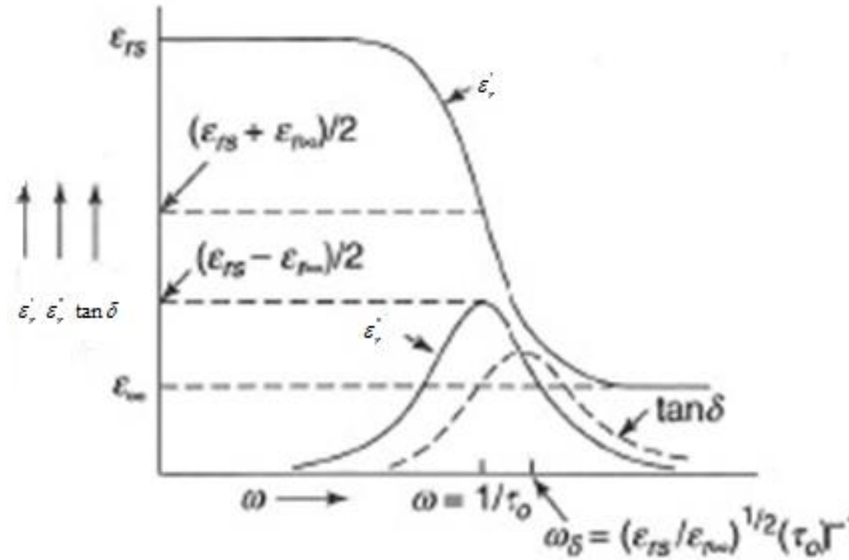
$$\omega_p = \frac{1}{\tau_0} = v_0 e^{\left(-\frac{W}{kT}\right)}$$

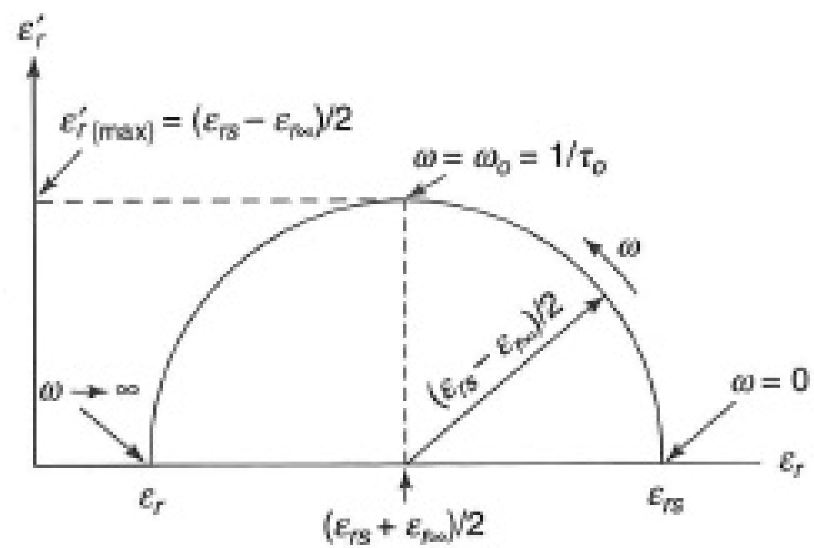
where

'v_0' is the attempt to jump frequency

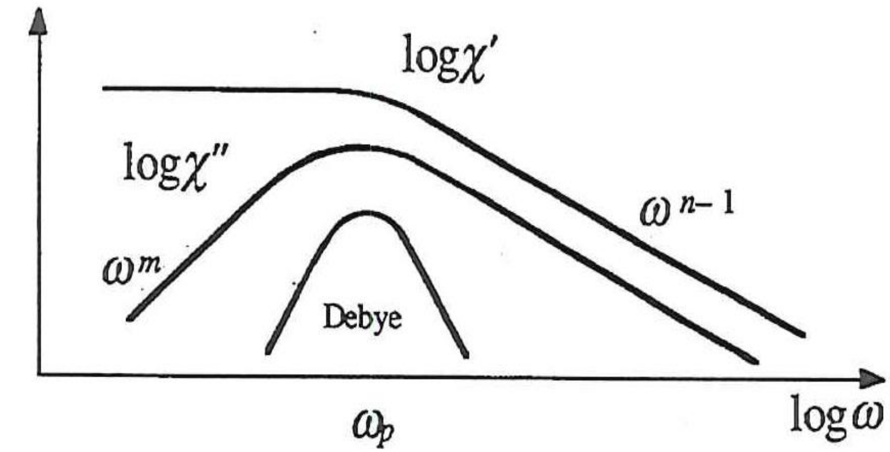
'W' is the activation energy

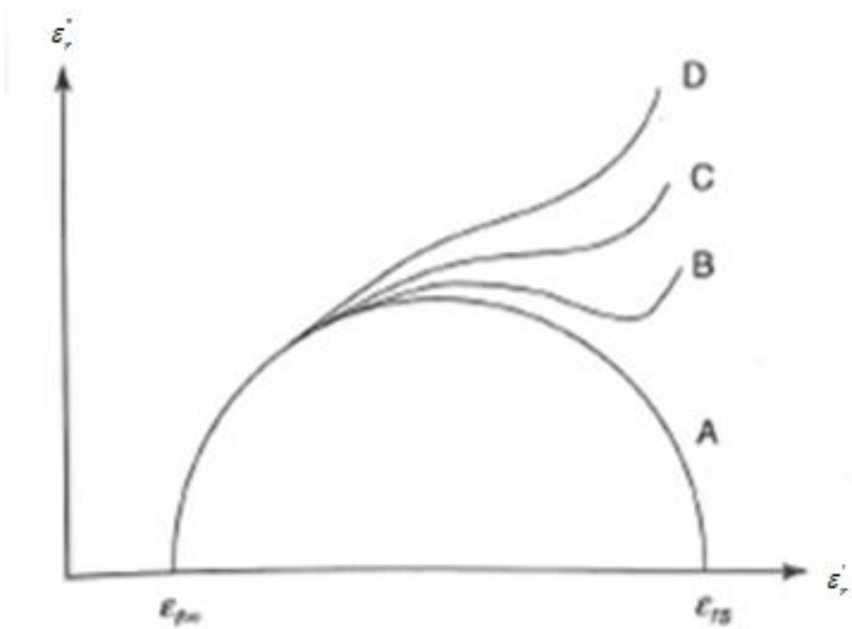
'k' is the Botzmann Constant





Dielectric Response of Real Fluids





The effect of dc conductivity on $\varepsilon_r' - \varepsilon_r''$ arc.
(A) $\sigma = 0$ (B) $\sigma = \sigma_1 > 0$ (C) $\sigma_2 > \sigma_1$ (D) $\sigma_3 > \sigma_2$

Havrikiak Negami Expression [1]

$$\chi_r(\omega) = A\left[1 + \frac{i\omega}{\omega_p}\right]^{\frac{-(1-n)}{m}}$$

High Frequecny [1]

$$\frac{\chi_r''(\omega)}{\chi_r'(\omega)} = \cot\left(\frac{n\pi}{2}\right)$$

Low Frequecny [1]

$$\frac{\chi_r''(\omega)}{\delta\chi_r'(\omega)} = \tan\left(\frac{m\pi}{2}\right)$$

Universal Dielectric Response

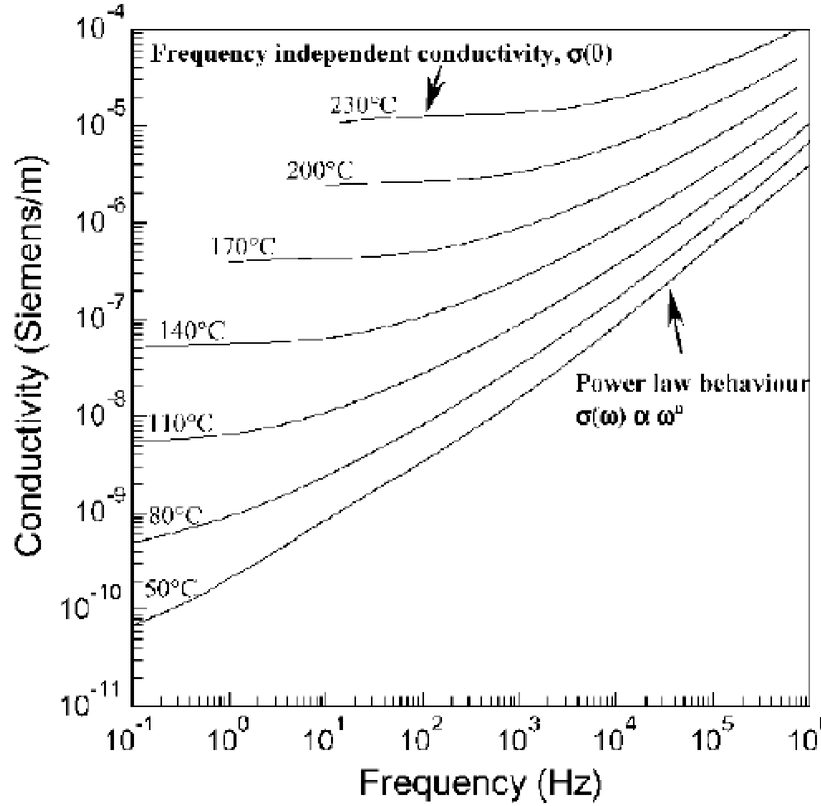
$$\sigma(\omega) = \sigma_0 + \varepsilon_0 \omega \chi_r''(\omega)$$ Ref. [2]

$$\chi_r''(\omega) \propto \omega^{n-1} \quad @ \quad n < 1$$

A More General Relation for Universal Dielectric Response, Ref. [3]

$$\sigma(\omega) = \sigma_0 + A\omega^n$$

Where
'σ_0,' 'A' and 'n' are assumed to be constant
ω is the sweeping frequency.



Resistor – Capacitor Networks for a Doped Zirconia [3]

Analytical Model - Conductivity Relation for Atmospheric Ice

Parameter	Value	Reference
C_∞ E_∞	$1.6\text{x}10^6$ S/m 0.57 eV	[5]
σ_∞ (@ 0^0C, 10kHz) E_∞ (@ 0^0C, 10kHz)	$(4.47\pm0.14)\ \text{x}10^{-5}$ S/m (0.56 ± 0.004) eV	[6]
σ_s (@ 10^0C) E_s (@ T > 40^0C)	$(1.1\pm0.5)\text{x}10^{-8}$ S/m (0.34 ± 0.02) eV	[4]

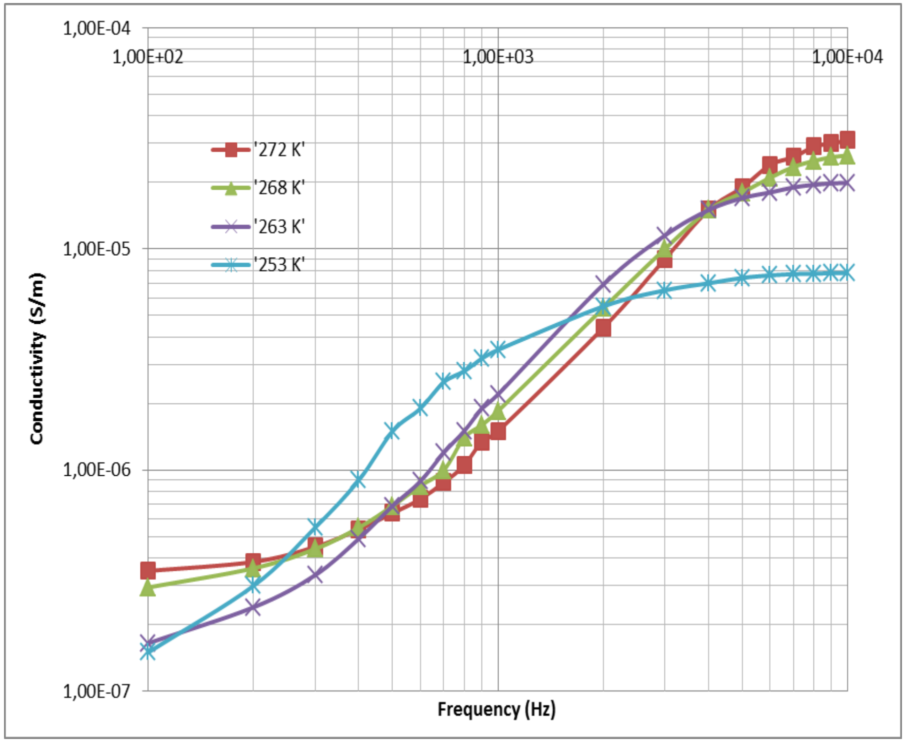
$$\sigma(\omega) = \sigma_0 + A\omega^n$$

It is found that

$\sigma_0 = f(\sigma_s, \sigma_\infty)$

$A = g(\sigma_s, \sigma_\infty)$

$n = f(T)$ [0,1]



Experimental results of Fujino [10] for Conductivity variation with frequency and temperature for pure ice

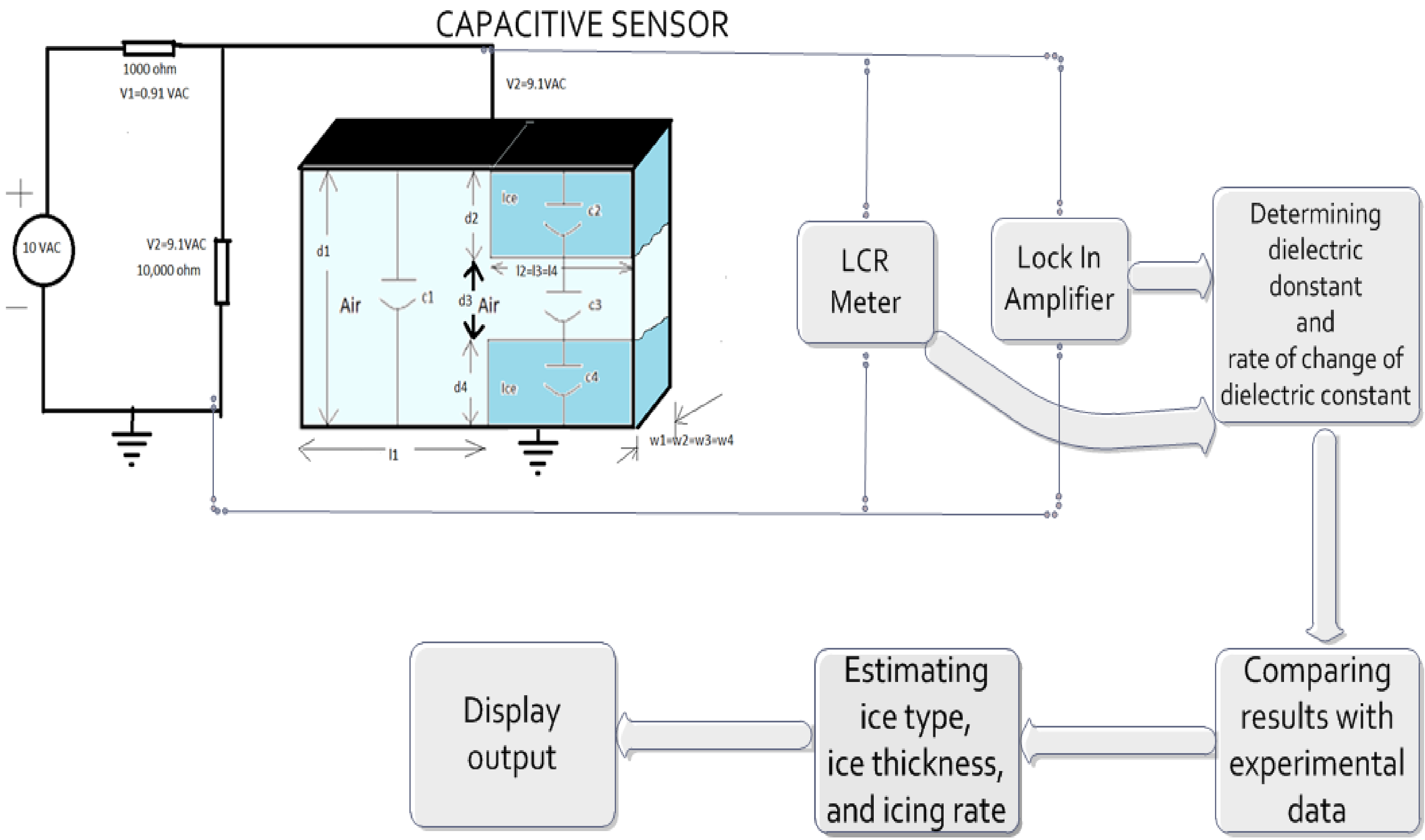
@ Low Frequecny

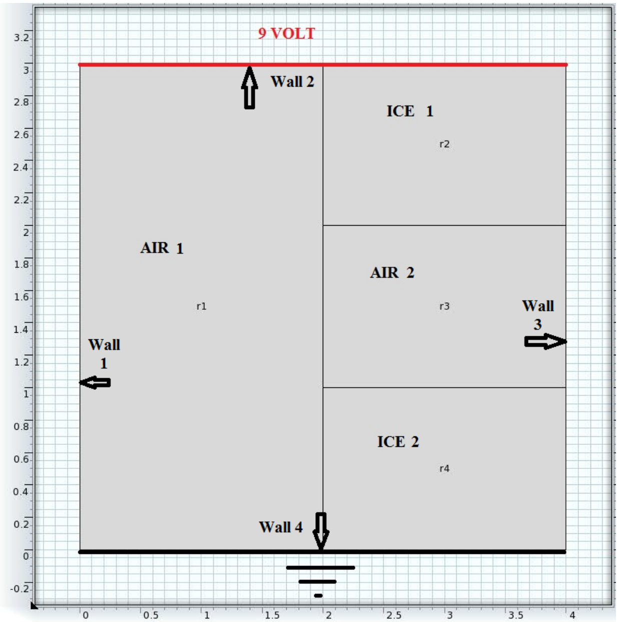
$$\sigma_s = C_s e^{\left(\frac{-E_s}{kT}\right)}$$

@ High Frequecny

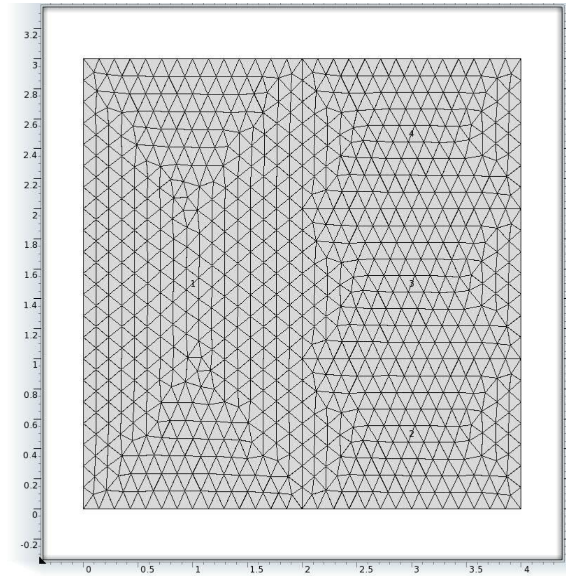
$$\sigma_\infty = C_\infty e^{\left(\frac{-E_\infty}{kT}\right)}$$

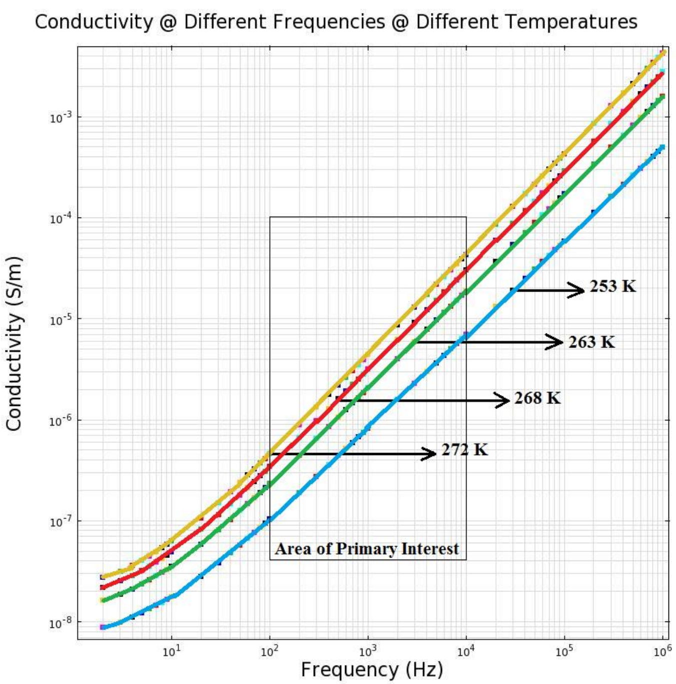
Experimental Model

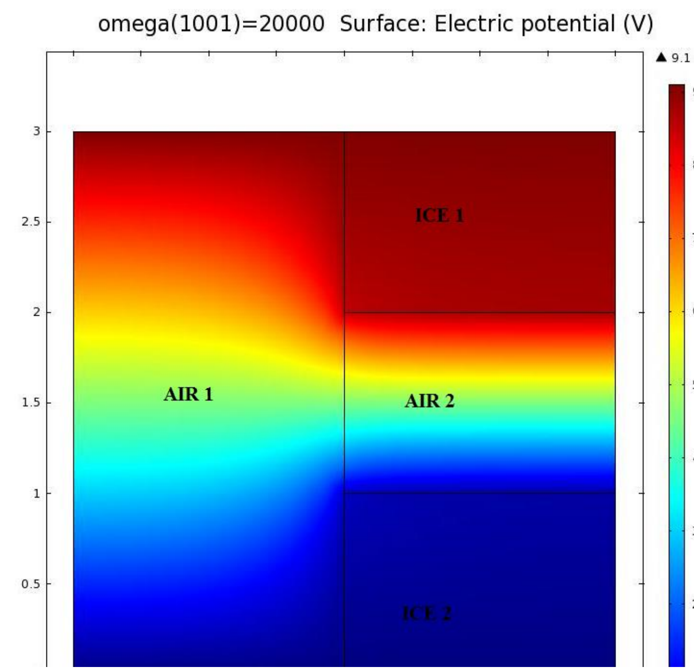




Numerical Model

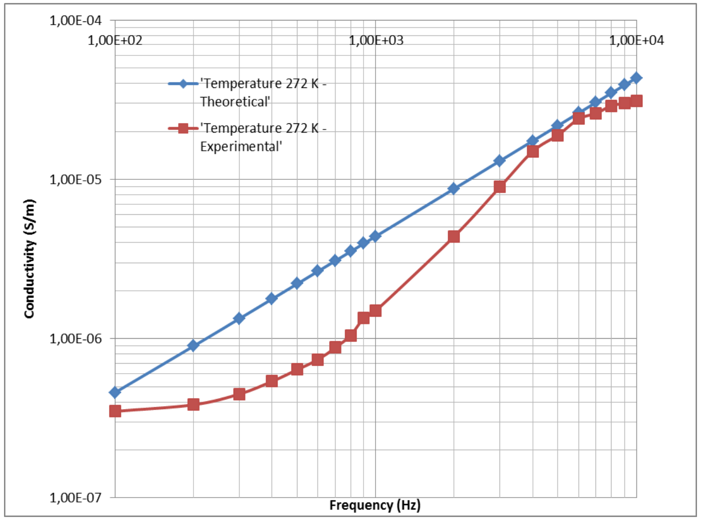




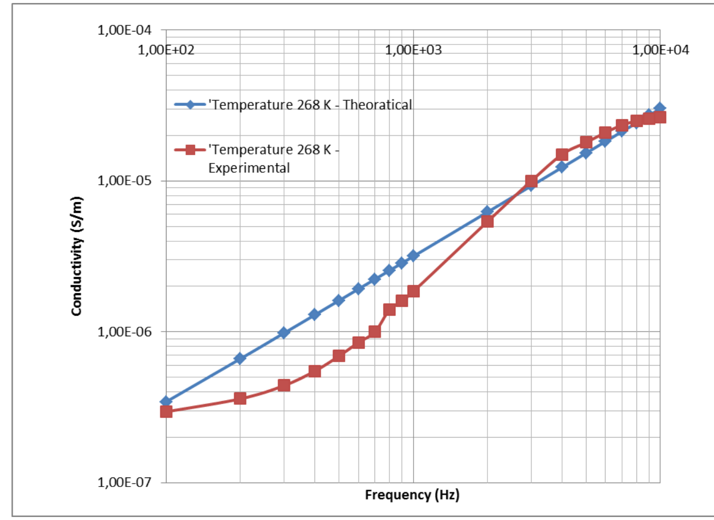


Results

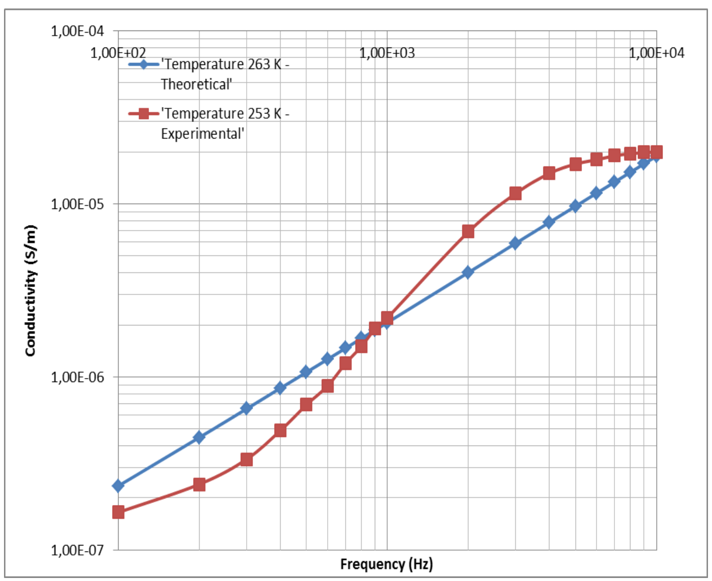
Temperature = 272 K, R^2 = 0.856



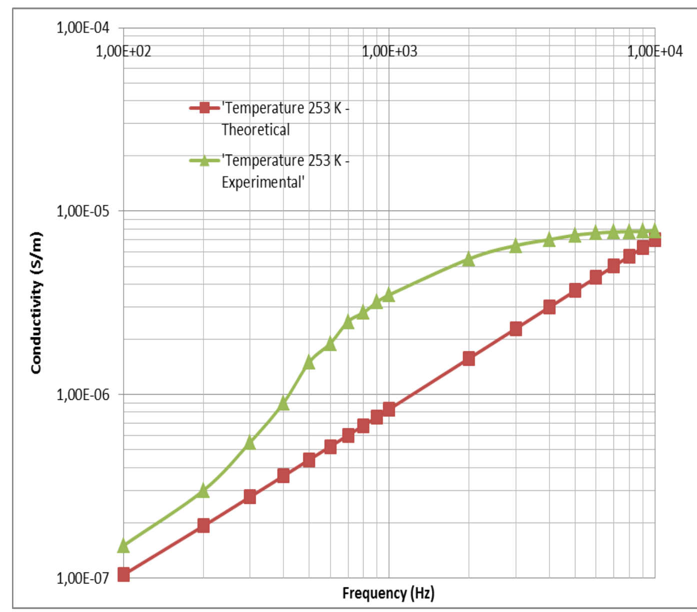
Temperature = 268 K, R^2 = 0.973



Temperature = 263 K, R^2 = 0.808



Temperature = 253 K, R^2 = 0.305



Conclusion and Discussion

- UDR forms the basis of the conductivity variation with a fractional power of excitation frequency but it does not relate temperature with the power law.

- In this paper the Maxwell Boltzmann statistics is used for thermal excitation of proton jump for atmospheric ice and is used in the conductivity relation Eq. 4 which adequately supports the experimental results of Fujino [10]. Similarly the power exponent 'n' also varies from 0 to 1 and is also used as temperature dependent.

- At some temperatures the conductivity dependent on frequency and temperature shows more deviation e.g. 253 K (Fig. 12) which may be due to the nonlinear exponential interaction between the molecules but it's not clear.

- This study reflects that Universal Dielectric Response as proposed by Jonscher need some additional explanations of the assumed constants which in conductivity relation Eq. (4) are termed as 'σ_0', 'A' and 'n'. In this paper these all constants are found to be explicitly dependent on the temperature.

References

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I appreciate your attention

I am now open for all questions

ACKNOWLEDGMENT

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