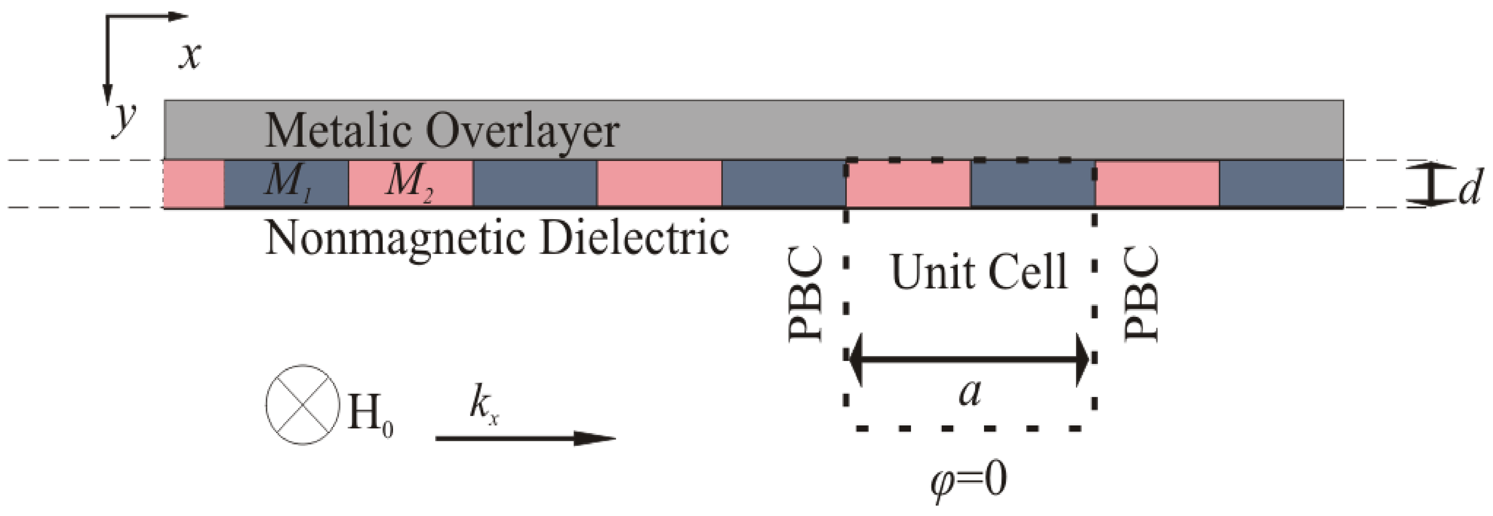
A Weak Formulations for Calculating Spin Wave Dispersion Relation in Magnonic Crystals

M. Mruczkiewicz[1]

[1]Faculty of Physics, Adam Mickiewicz University in Poznan , ul. Umultowska 85, 61-614, Poznan, Poland

Introduction: We study the spin wave excitation (coherent precession of magnetic moments) in periodically arranged magnetic stripes, i.e., in one-dimensional magnonic crystal (MC).



Computational Methods: Two approaches have been implemented [1,2,3]:

I (red points in figures)

$$\frac{\partial \boldsymbol{M}(\boldsymbol{r},t)}{\partial t}=\gamma\mu_0 \boldsymbol{M}(\boldsymbol{r},t)\times \boldsymbol{H}_{eff}(\boldsymbol{r},t)$$

$$\nabla\times\boldsymbol{h}=\sigma\boldsymbol{E}$$

$$\nabla\times\boldsymbol{E}=-\mu_0\frac{\partial(\boldsymbol{h}+\boldsymbol{m})}{\partial t}$$

II (black points in figures)

$$\nabla\times(\mu_r^{-1}\nabla\times\boldsymbol{E})-\omega^2\sqrt{\epsilon_0}\mu_0(\epsilon_r-\frac{i\sigma}{\omega\epsilon_o})\boldsymbol{E}=0$$

$$\mu_r(f)=\begin{pmatrix}\mu(f) & i\mu_a(f) & 0\\ -i\mu_a(f) & \mu(f) & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\mu_r(f)=\begin{pmatrix}\frac{(\gamma\mu_0H_0(\gamma\mu_0H_0+M_S\gamma\mu_0)-(2\pi f)^2)}{(\gamma\mu_0H_0)^2-(2\pi f)^2} & i\frac{M_S\gamma\mu_0 2\pi f}{(\gamma\mu_0H_0)^2-(2\pi f)^2} & 0\\ -i\frac{M_S\gamma\mu_0 2\pi f}{(\gamma\mu_0H_0)^2-(2\pi f)^2} & \frac{(\gamma\mu_0H_0(\gamma\mu_0H_0+M_S\gamma\mu_0)-(2\pi f)^2)}{(\gamma\mu_0H_0)^2-(2\pi f)^2} & 0\\ 0 & 0 & 1\end{pmatrix}$$

Bloch Thorem $\varphi=\varphi' e^{i\cdot k_x x}$

φ' *is periodic function*, $\varphi=m_x, m_y, h_x, h_y, e_z$

Weak formulation $0=\int_\Omega F_i\,dxdy$

I

$$F_1=\frac{-2A}{M_S\mu_0}\frac{\partial m_y}{\partial x}\frac{\partial v}{\partial x}-\frac{2A}{M_S\mu_0}\frac{\partial m_y}{\partial y}\frac{\partial v}{\partial y}-\frac{2A}{M_S\mu_0}k_x^2 m_y v+i\frac{4A}{M_S\mu_0}k_x\frac{\partial m_y}{\partial x}v-Hm_y v+M_S h_y v+\frac{4\pi}{\gamma\mu_0}f m_x v$$

$$F_2=\frac{2A}{M_S\mu_0}\frac{\partial m_x}{\partial x}\frac{\partial v}{\partial x}+\frac{2A}{M_S\mu_0}\frac{\partial m_x}{\partial y}\frac{\partial v}{\partial y}+\frac{2A}{M_S\mu_0}k_x^2 m_x v-i\frac{4A}{M_S\mu_0}k_x\frac{\partial m_x}{\partial x}v+Hm_x v+M_S h_y v+\frac{4\pi}{\gamma\mu_0}f m_y v$$

$$F_3=i2\pi f\mu_0(m_x+h_x)v+\frac{\partial e_z}{\partial y}$$

$$F_4=i2\pi f\mu_0(m_y+h_y)v-\frac{\partial e_z}{\partial x}-ik_x e_z v$$

$$F_5=\frac{-\partial h_y}{\partial x}v+ik_x h_y v+\frac{\partial h_x}{\partial y}v-\sigma e_z v$$

II

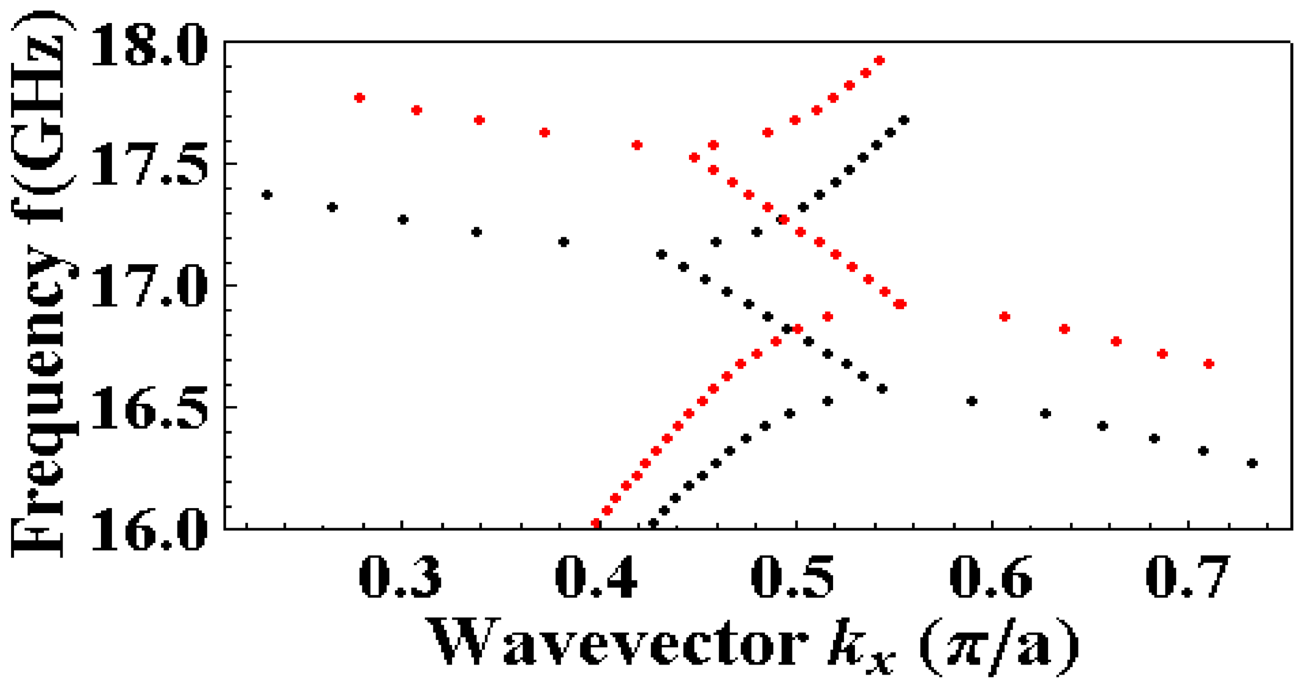
$$F_1=-\frac{\mu}{\mu^2-\mu_a^2}\frac{\partial e_z}{\partial y}\frac{\partial v}{\partial y}-\frac{i\mu_a}{\mu^2-\mu_a^2}\frac{\partial e_z}{\partial x}\frac{\partial v}{\partial y}+\frac{i\mu_a}{\mu^2-\mu_a^2}\frac{\partial e_z}{\partial y}\frac{\partial v}{\partial x}+\frac{\partial e_z}{\partial x}\frac{\mu}{\mu^2-\mu_a^2}\frac{\partial v}{\partial x}$$
$$+\epsilon_0\mu_0(2\pi f)^2 e_z v-k_x^2 e_z\frac{\mu}{\mu^2-\mu_a^2}v+k_x i\frac{\partial e_z}{\partial x}\frac{\mu}{\mu^2-\mu_a^2}v-k_x i e_x\frac{\mu}{\mu^2-\mu_a^2}\frac{\partial v}{\partial x}$$
$$+k_x\frac{\partial e_z}{\partial y}\frac{\mu_a}{\mu^2-\mu_a^2}v+k_x e_z\frac{\mu_a}{\mu^2-\mu_a^2}\frac{\partial v}{\partial y}-i\sigma\mu_0 2\pi f e_z v$$

where $\boldsymbol{M}(\boldsymbol{r},t)$ *is magnetization vector*, $H_{eff}(\boldsymbol{r},t)$, *effective magnetic field*, H_0 *external magnetic field* m_x, m_y *dynamic components of the magnetization vector*, h_x, h_y, e_z *dynamic field components*, M_S *static component of the magnetization vector (subtituted by* M_1, M_2*)*, A *exchange constant*, k_x *wavevector*, f *frequency*, v *test function*

Results:

a) *d*<100nm, small exchange interactions:

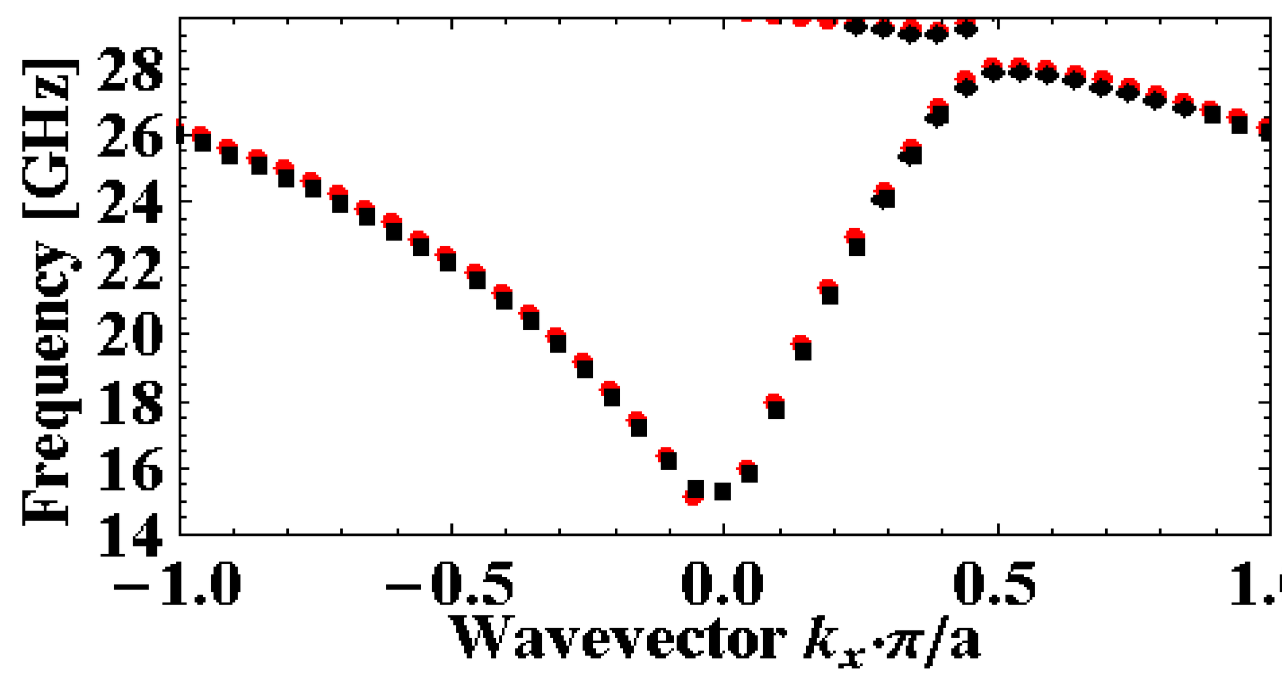
A=500 nm, *d*=40 nm, $\sigma=10*10^{15}$S/m, H_0=0.1 T, $M_1=1.7*10^6$A/m, $M_2=1.7*10^6$ A/m



	Mesh points	No of degrees of freedom	time	Exchange Interactions	d>100nm
Approach I (direct solver)	7410	60565	28 s	✓	x
Approach II (iterative solver)	5670	12139	26 s	x	✓

b) *d*<100nm, large exchange interactions:

A=500 nm, *d*=40 nm, $\sigma=10*10^{15}$S/m, H_0=0.1 T, $M_1=0.95*10^6$A/m, $M_2=1.05*10^6$ A/m, $A=6*10^{-11}$J/m



Conclusions: We have presented two approaches to calculate spin wave dispersion relation in magnonic crystals. We have defined a structure that dispersion relation can be obtained using both approaches and compared them. In general, the approach I have to be use for MCs where the exchange interactions are important (small lattice constant), while the approach II is useful in the structures with large thickness and low exchange coefficient.

Acknowledgments:
We acknowledge the financial assistance from the NCN of Poland project DEC-2-12/07/E/ST3/00538, Magnowa.

References:

1. T. Wolfram and R. E. De Wames, Phys. Rev. B 4, 3125 (1971)
2. M. Mruczkiewicz et al., J. Appl. Phys. 113, 093908 (2013)
3. C. Fietz, Y. Urzhumov, and G. Shvets, Optics Express, 19, 20, (2011)