

Computational homogenization in with an application on masonry structures

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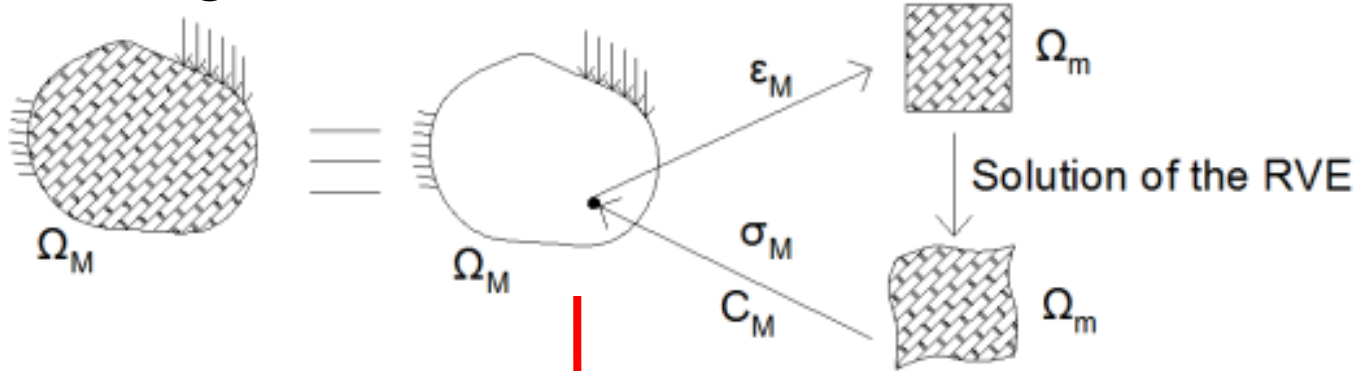
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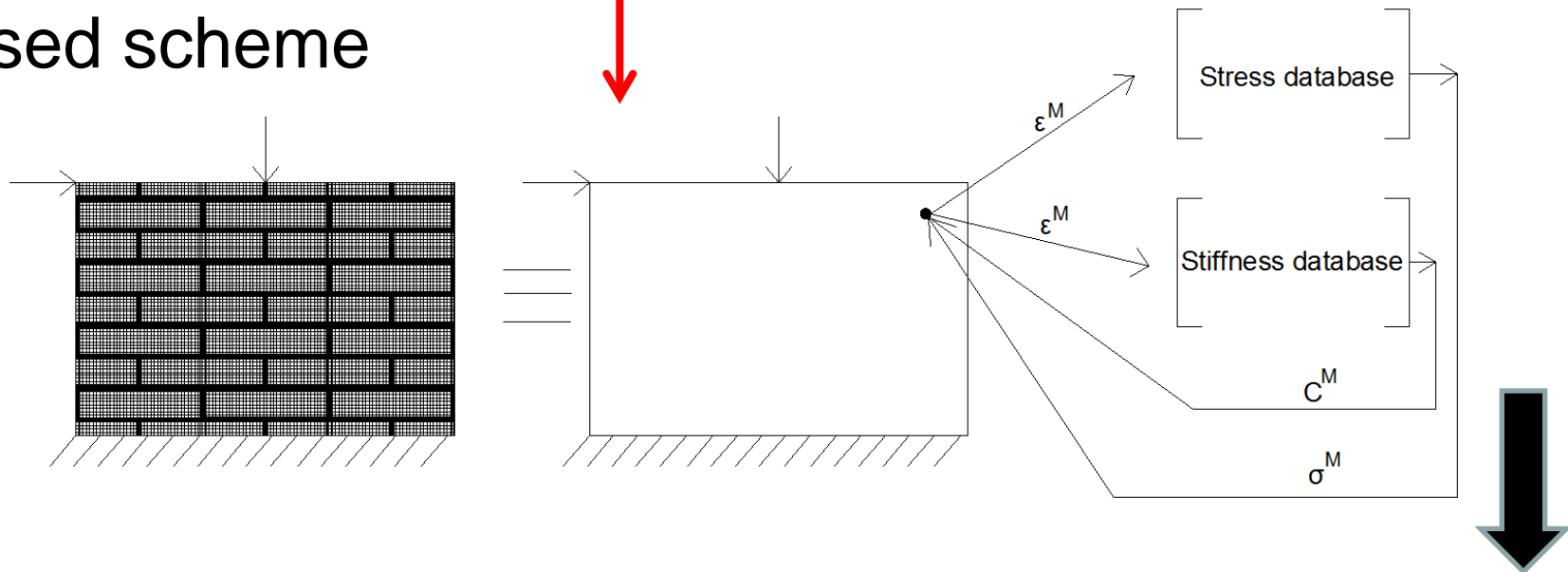
COMSOL
CONFERENCE
ROTTERDAM2013

Part A: The proposed computational homogenization method

Classical configuration

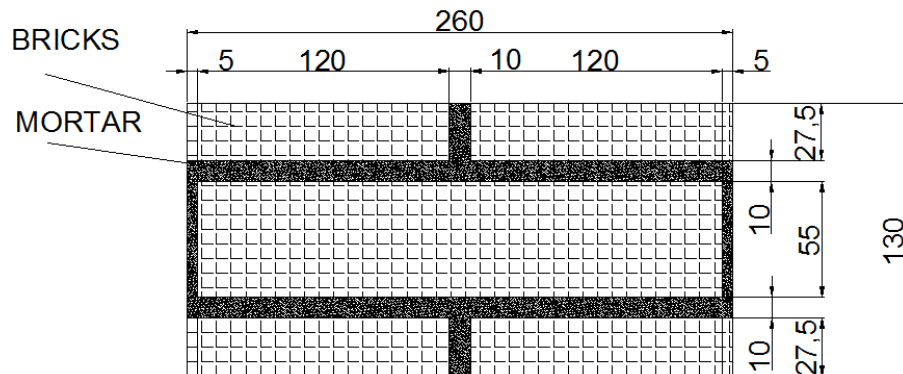


Proposed scheme



Steps

1. Consideration of a masonry RVE FEM with COMSOL Multiphysics,
 - Non-linear perfect plastic law in the mortar joints
 - Linear elastic bricks
 - Linear displacement boundary conditions loading



Steps

2. Consideration of different loading paths and loading levels (parametric analysis)
3. Estimation of the average stress and strain
4. Repetition for the estimation of stiffness information

Steps

5. Creation of two databases:
 - a) Stress
 - b) Stiffness

6. Incorporation in an overall multi-scale homogenization scheme in MATLAB using interpolation (metamodel)

7. Comparison with direct heterogeneous macroscopic analysis in ABAQUS-MARC

The microscopic analysis

- Scanning the 3d space of loading stains =>
Determination of several loading paths for the RVE

Key parameter for the success of the concept

- Incorporation of a parameter in the linear displacement equations =>
Determination of several strain loads

Loading paths

- Linear displacements:

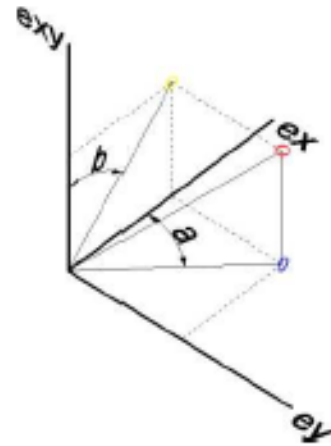
$$\mathbf{u}|_{\partial V_m} = \boldsymbol{\epsilon}^M \mathbf{X} \quad \boldsymbol{\epsilon}^M = [e_{xx} \quad e_{yy} \quad e_{xy}]^T$$

- Creation of loading paths:

- Simulation of possible combinations of $\boldsymbol{\epsilon}^M$ members =>

- Introduction of two angles, a, b =>

- 3d scanning of the strain space:



- $(a, b) = (a, 90), (a, 60), (a, 30), (a, 0), (a, -30), (a, -60), (a, -90)$, for $a=0:30:360$ => **91 loading paths**

- Incremental application of (each) loading

Averaging procedure: **strains-stresses**

- For each load path and load level:

$$\underline{\langle \epsilon \rangle_{V_m} = \epsilon^M} \quad \langle \sigma \rangle_{V_m} = \frac{1}{V_m} \int_{V_m} \sigma^m dV_m$$

- Postprocessing subdomain integration:
COMSOL
- Usage of script files to request the output quantities
- Stress database: Saving in MATLAB mat files

Averaging procedure: **effective constitutive tensor**

- Repetition of analyses for every load path and load level
- For each load path – load level:
 - Three test, incremental strain vectors are considered
 - Three incremental average stress vectors are calculated
- Estimation of the effective elasticity tensor:

Hooke's law $[\delta\epsilon^M] = [\delta\epsilon_1^M \quad \delta\epsilon_2^M \quad \delta\epsilon_3^M]$

$$[\delta\sigma^M] = [\delta\sigma_1^M \quad \delta\sigma_2^M \quad \delta\sigma_3^M]$$

$$[\delta\sigma^M] = C^M [\delta\epsilon^M] \Rightarrow C^M = [\delta\sigma^M][\delta\epsilon^M]^{-1}$$

Overall multi-scale computational homogenization scheme

- [MATLAB FEM² code: masonry structures](#)
- Plane stress, first order, full integration FE
- Obtaining macroscopic information:
 - Stress database → Macroscopic stress
 - Stiffness database → Macroscopic tangent stiffness
 - Repetition for each Gauss point and time step
- **An interpolation method is needed**
 - Simplest, easier solution:

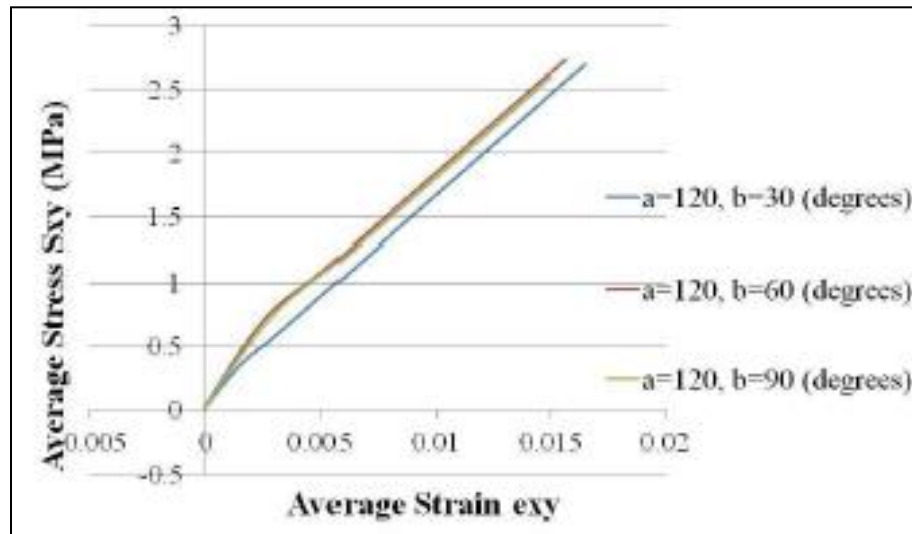
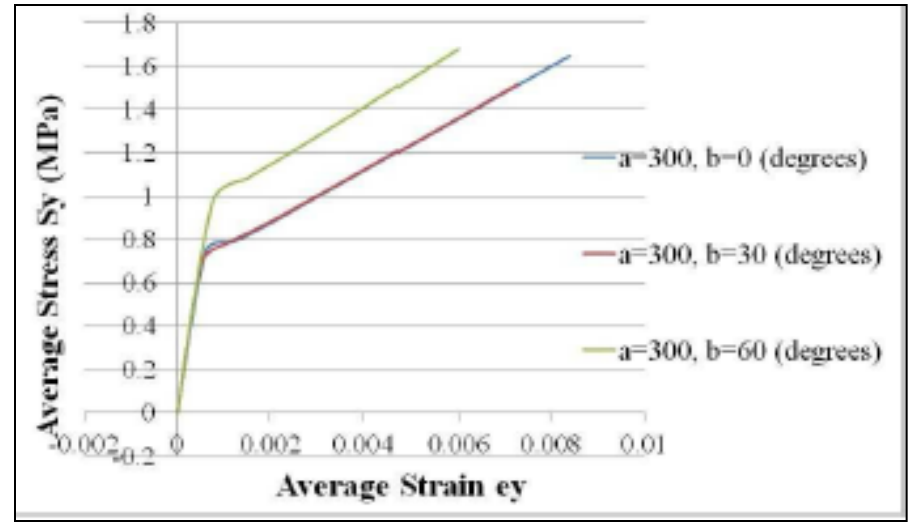
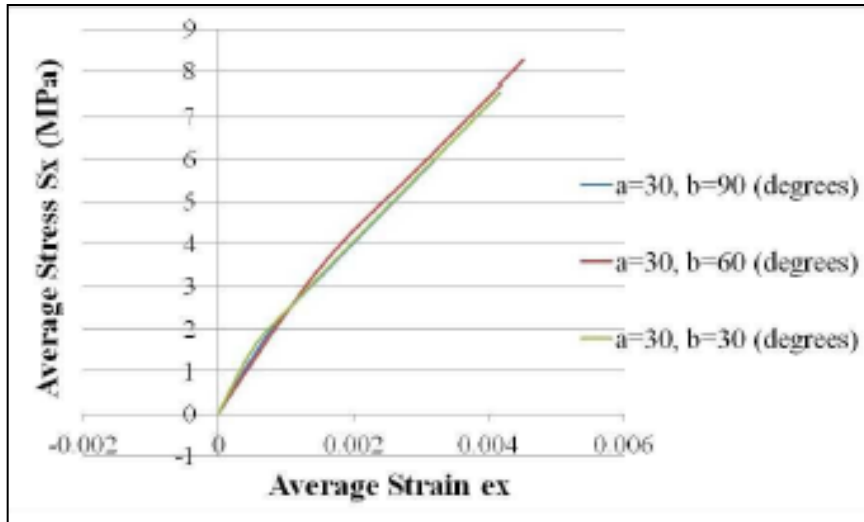
MATLAB function “TriScatteredInterp”

Overall multi-scale computational homogenization scheme

- Stress interpolation:
 - Each strain vector (3×1) corresponds to one average stress value
 - 3 repetitions to obtain the (3×1) stress vector
- Effective elasticity tensor interpolation:
 - Each strain vector (3×1) corresponds to one value of the tensor
 - 9 repetitions to obtain the (3×3) elasticity tensor

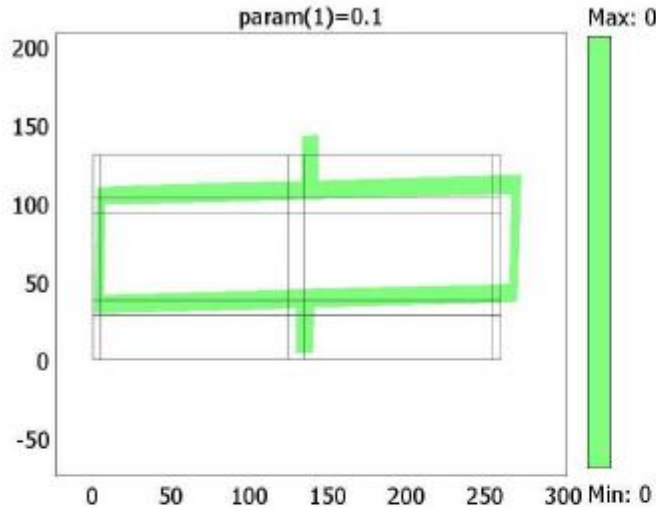
Results: micro simulations

Non-linear average stress-strain behaviour

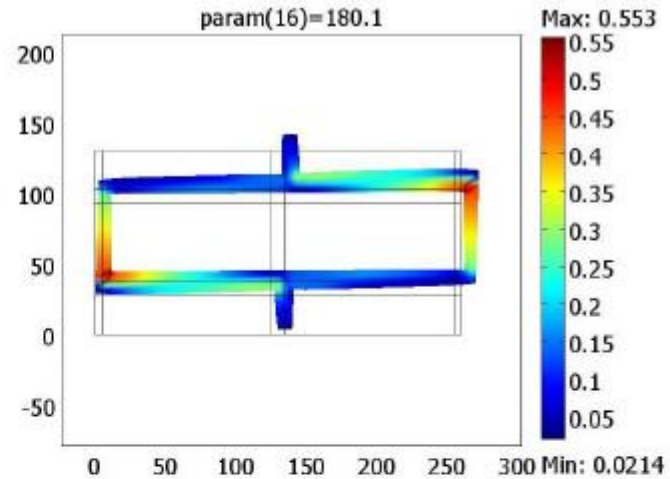


Results: micro simulations

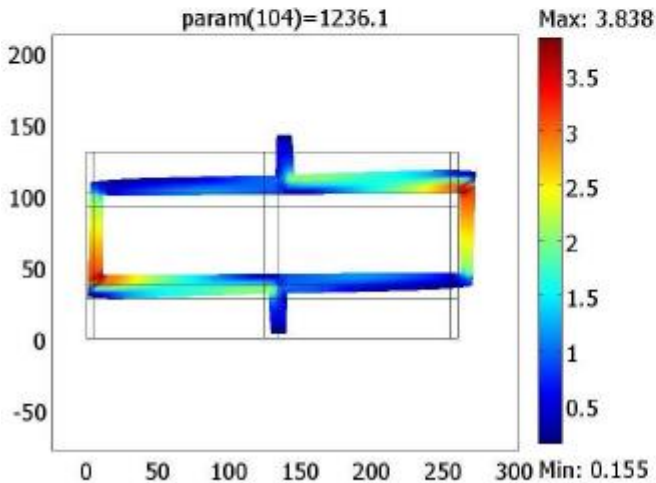
Plastic strain: gradually increased, in the mortar



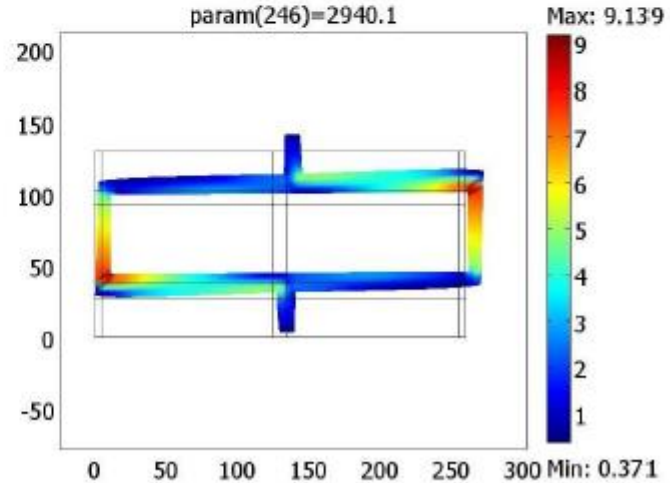
(a)



(b)



(c)

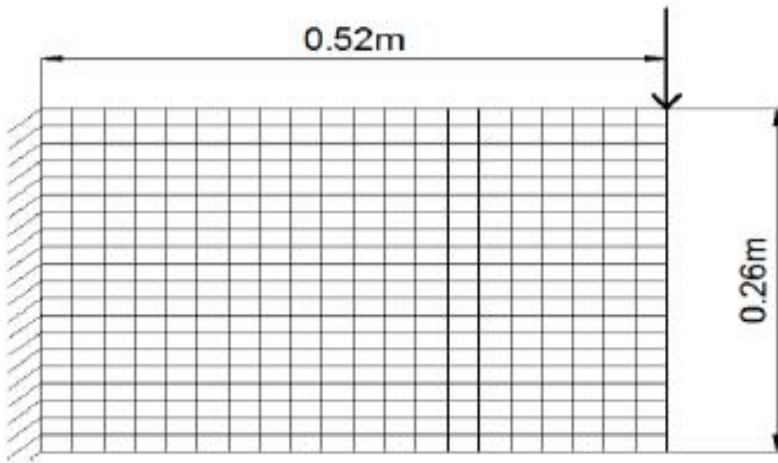


(d)

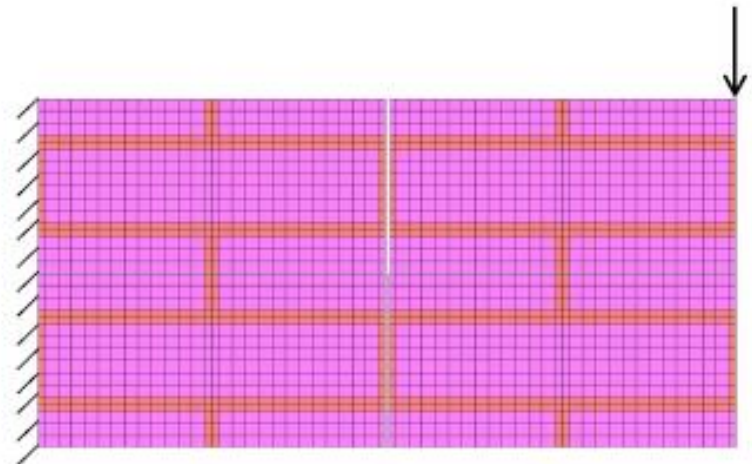
Results: overall homogenization scheme

Application 1: small masonry wall

Homogeneous model:
Proposed approach
(20x20 elements)



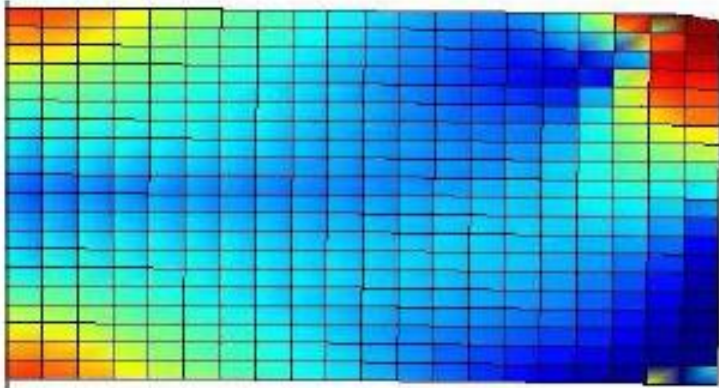
Direct heterogeneous model:
ABAQUS/MARC software



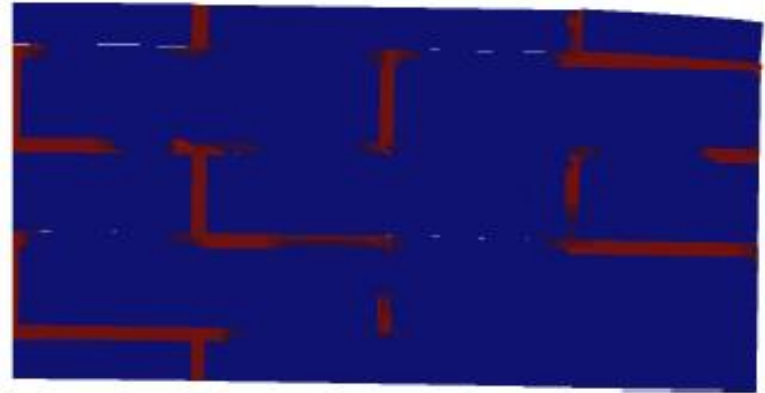
Results: overall homogenization scheme

Degradation of strength

Homogeneous model:
Proposed approach



Direct heterogeneous model:
ABAQUS

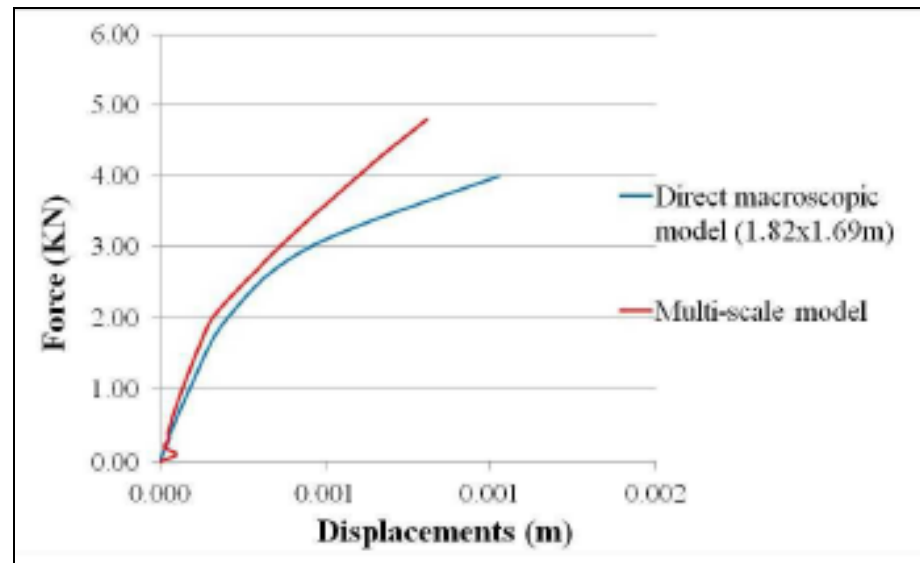
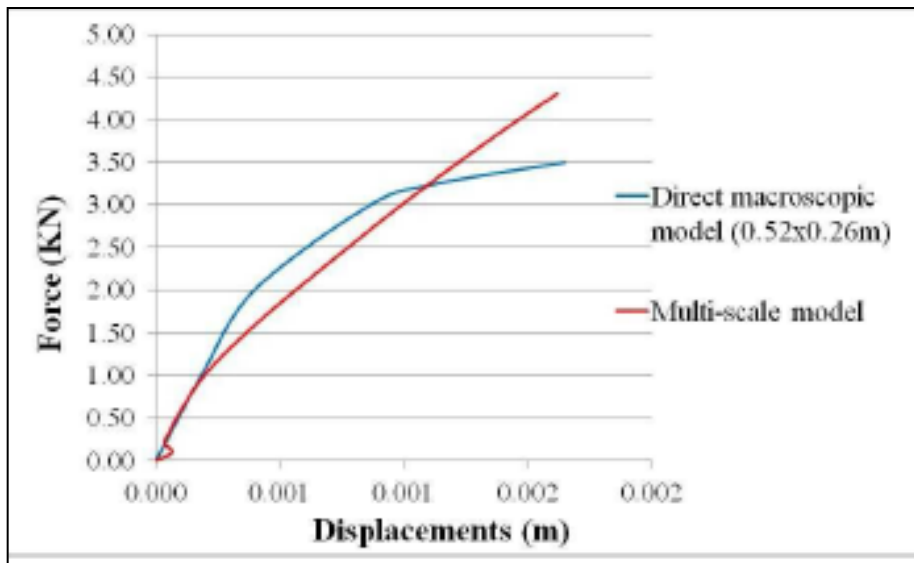


Results: overall homogenization scheme

Force – displacement diagrams

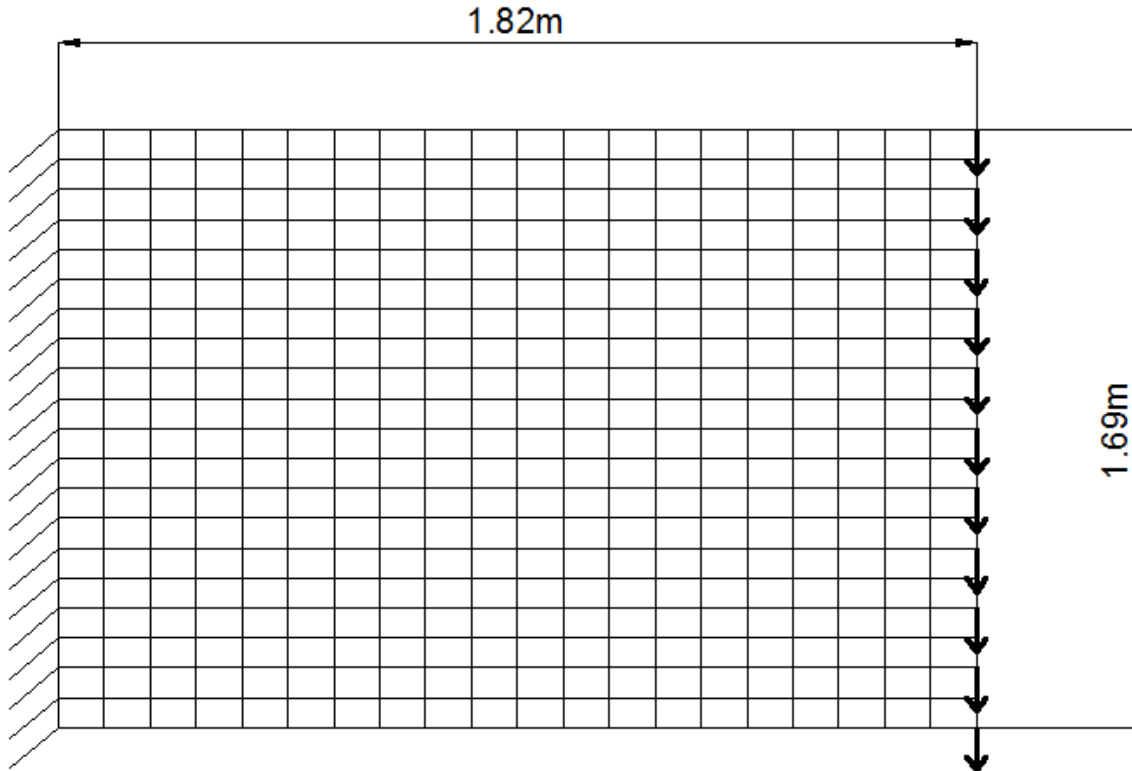
The previous small wall:
0.52x0.26m

A new, bigger masonry wall:
1.82x1.69m



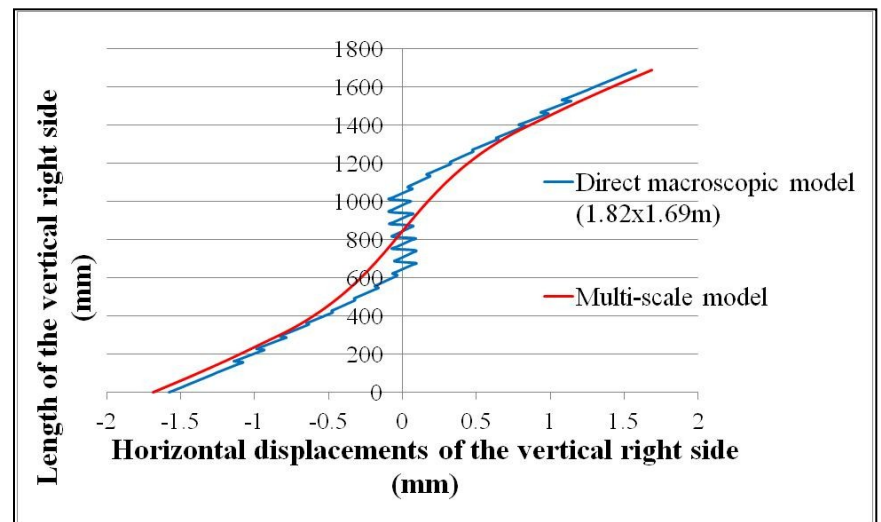
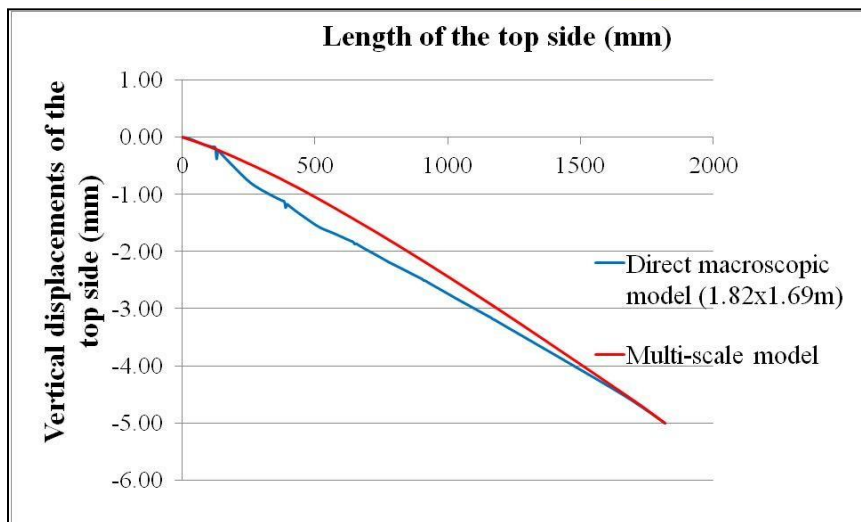
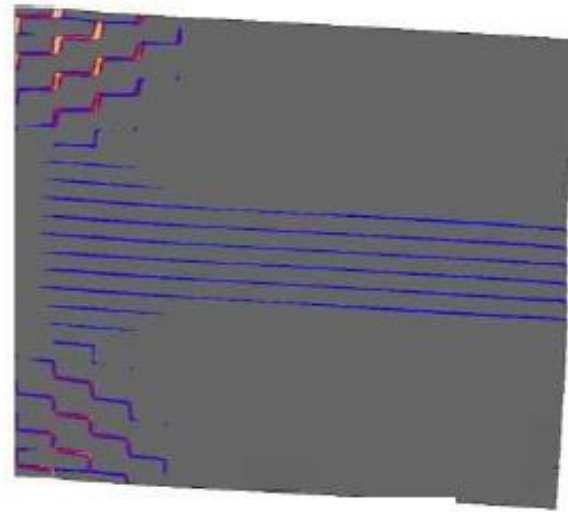
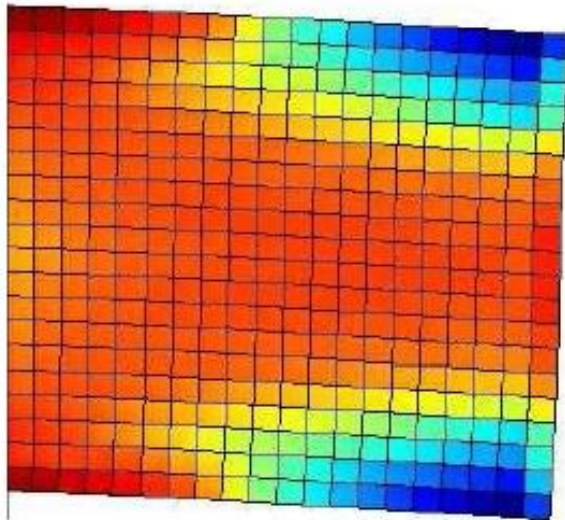
Results: overall homogenization scheme

*Application 2: a bigger masonry wall +
distributed displacement of 5mm
(20x20 elements)*



Results: overall homogenization scheme

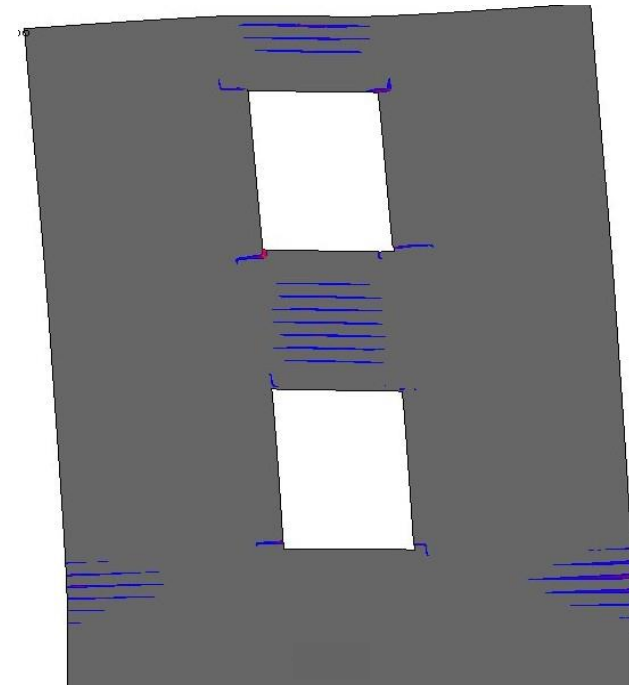
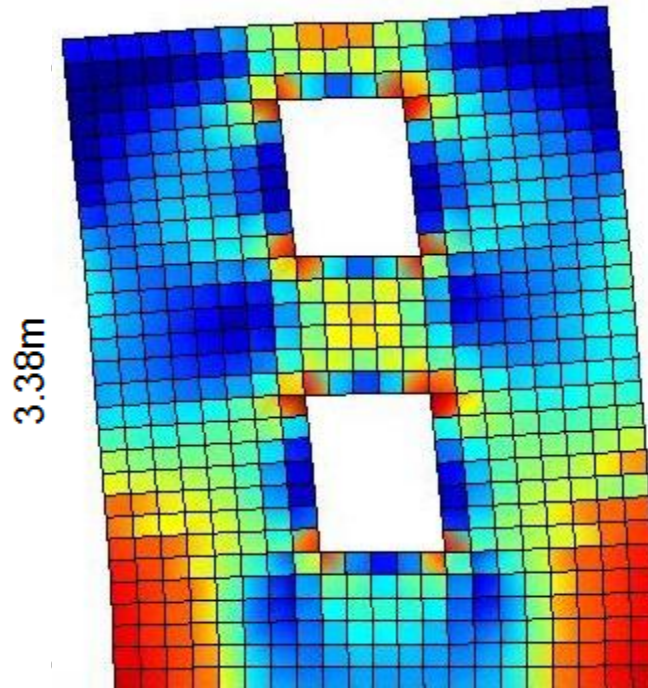
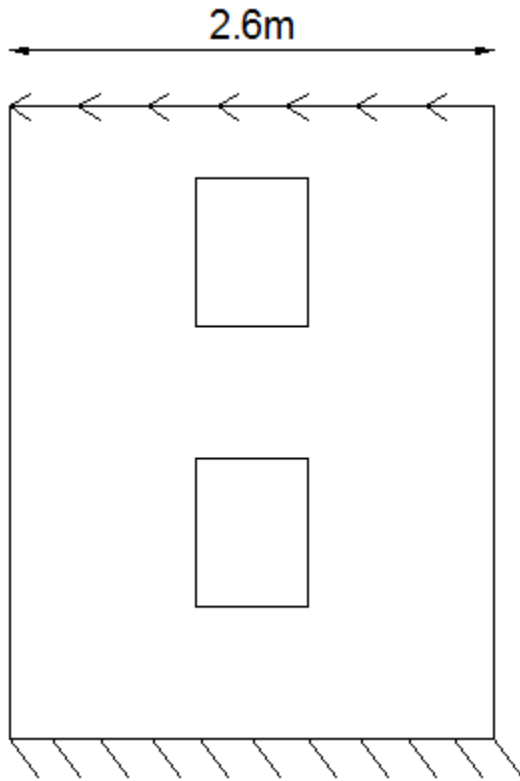
Degradation of strength – Displacement distribution



Results: overall homogenization scheme

Application 3: masonry wall + openings

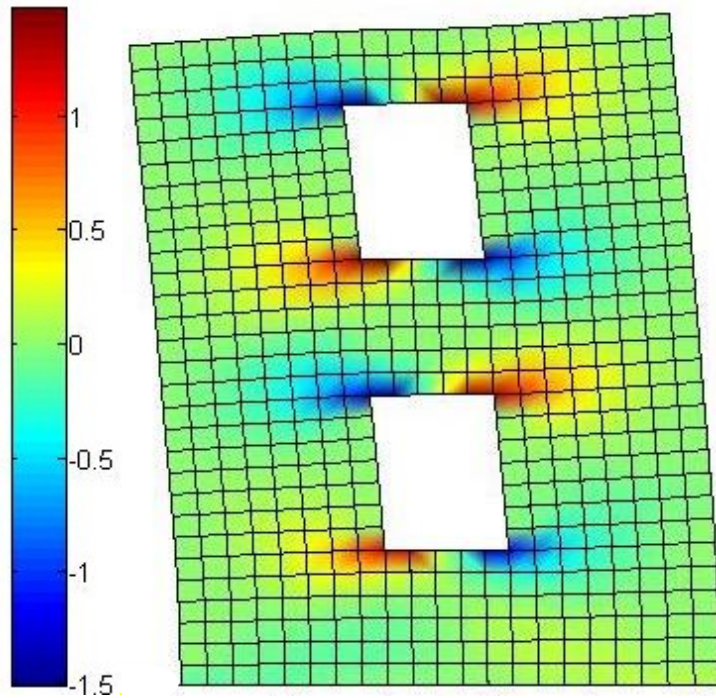
Degradation of strength



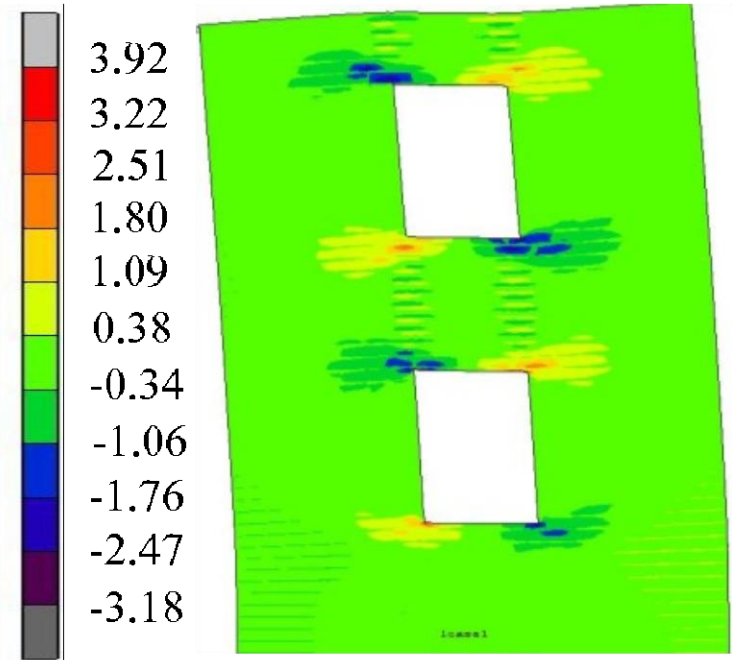
Results: overall homogenization scheme

Stresses S_{xx}

Multi-scale homogenization



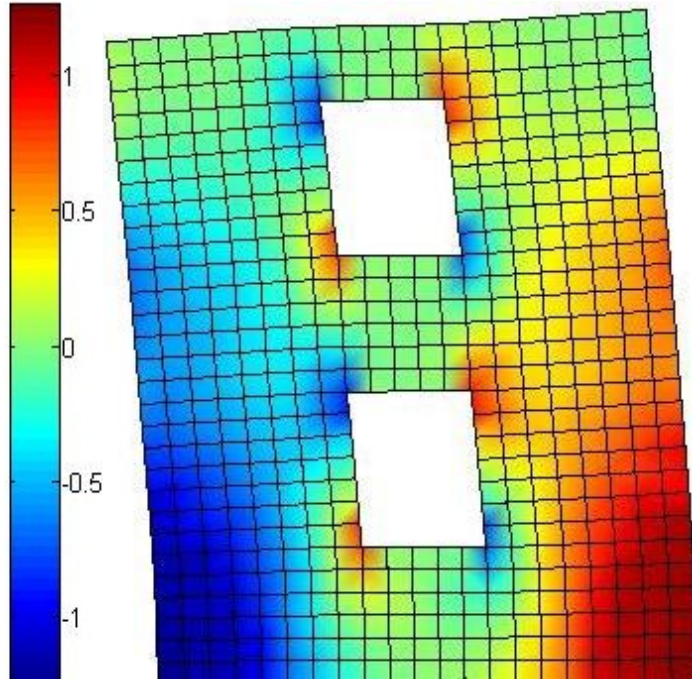
DNS model



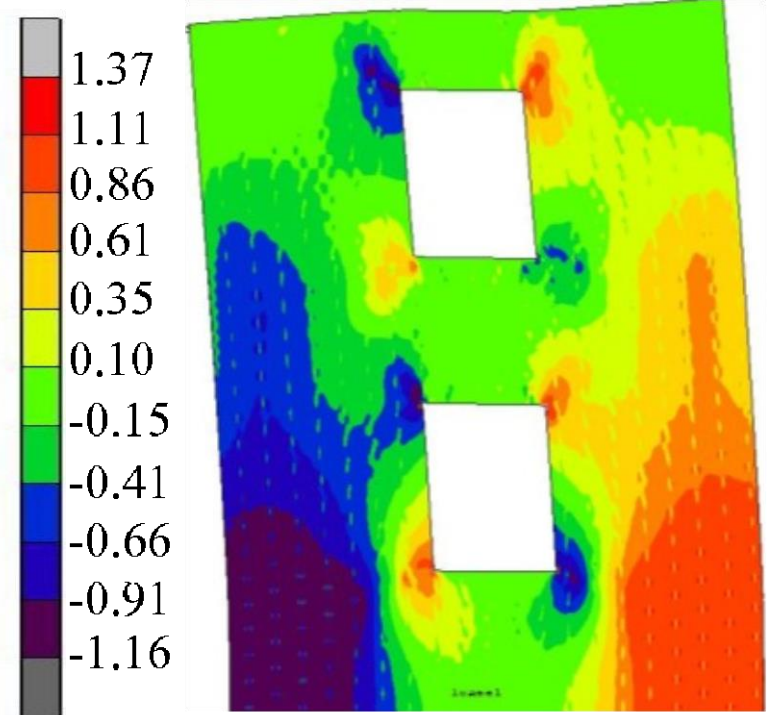
Results: overall homogenization scheme

Stresses S_{yy}

Multi-scale homogenization



DNS model

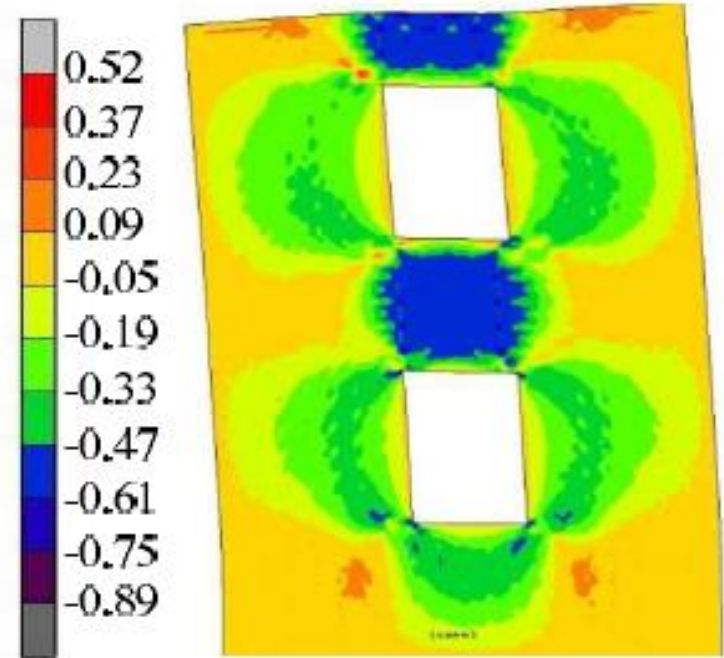
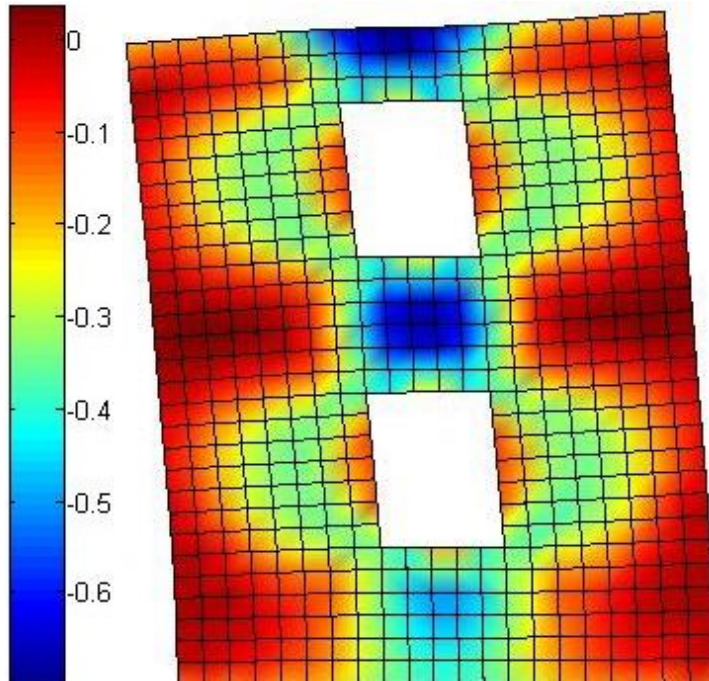


Results: overall homogenization scheme

Stresses S_{xy}

Multi-scale homogenization

DNS model



Conclusions

- A method for non-linear homogenization
- Good convergence with direct macroscopic analysis
- General method:
 - 1) Can be applied to other RVEs
 - 2) Can be applied to different constitutive RVEs laws
- Future study:
 - 1) Application to more complex constitutive laws / different RVEs
 - 2) Different interpolation methods (Neural Networks)



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