# Degeneracy Breaking, Modal Symmetry and MEMS Biosensors

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**Introduction**: Modes of vibration in cyclic systems occur in degenerate, orthogonal pairs. Degenerate-mode sensors exploit the special relationship between the modes. In this work, we compare our original analytical work to COMSOL Multiphysics®.

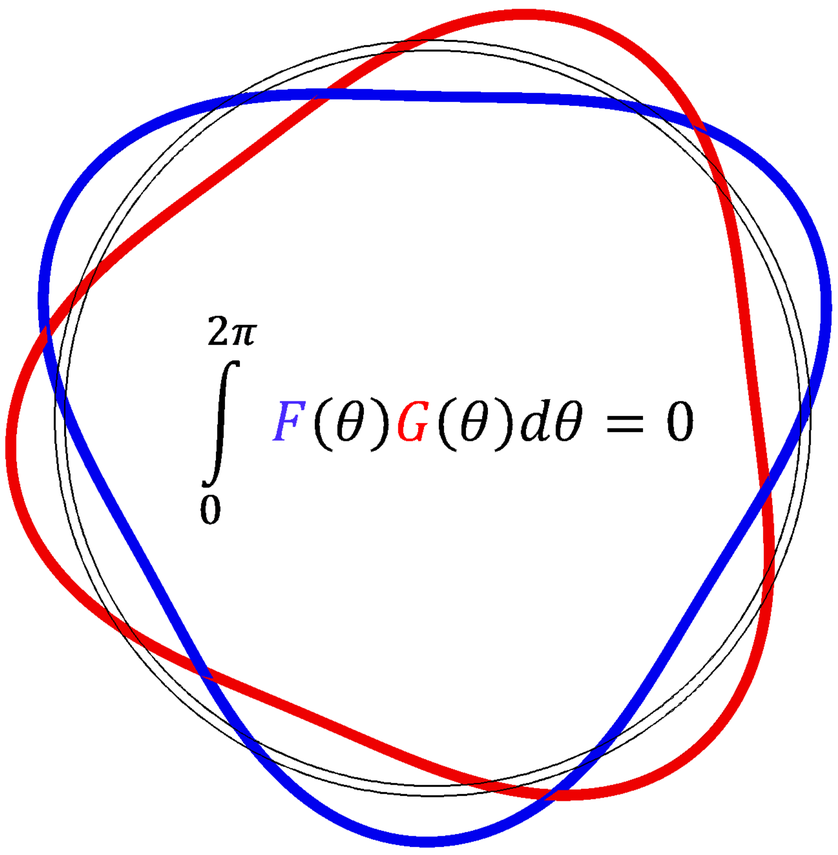


**Figure 1**. N=3 degenerate in-plane flexural modes of a ring gyroscope. The modes are coupled by the Coriolis force.

**Method**: A $\infty$ –cyclically symmetric mode can be described by a multipole series as:

$$Y(r,z,\theta) = \sum_{n=i}^{\infty} e^{in\theta} \int_0^{\infty} F(r,z)\, J_n(kr)dr$$

If the underlying symmetry of the system is broken to cyclic order $m$, then projection operator methods of functional analysis can be used to understand the effect on the resulting modes. Previous approximations[1] assume no change in the mode shapes, which implies the modes remain degenerate unless $2n = m$, which follows from Rayleigh's Quotient. Our analytical work predicts an $O(\varepsilon)$ split for $2n = m$, but an $O(\varepsilon^2)$ split for $2n \neq m$ with a smaller coefficient.

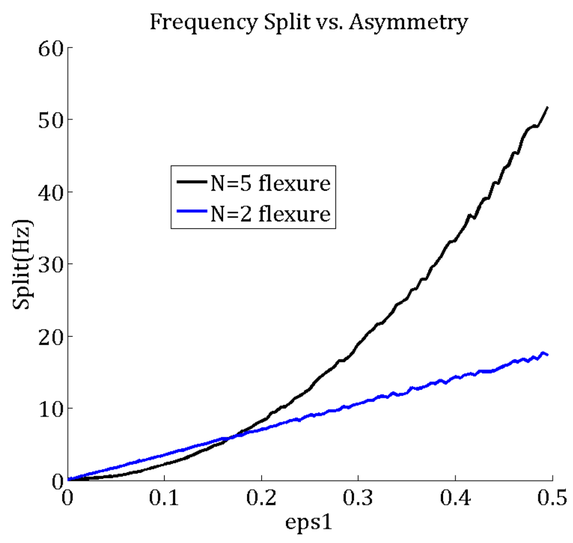


**Figure 2**. COMSOL results for a ring with degeneracy broken by an m=4 variation in the Young's Modulus.

**Results:** The ideas and insight from the analysis has been used to develop new concepts for degenerate and near-degenerate sensors in anisotropic crystalline materials like LiNbO3 and SiO2. COMSOL has expedited the detailed design of the devices, which are currently being fabricated in-house.

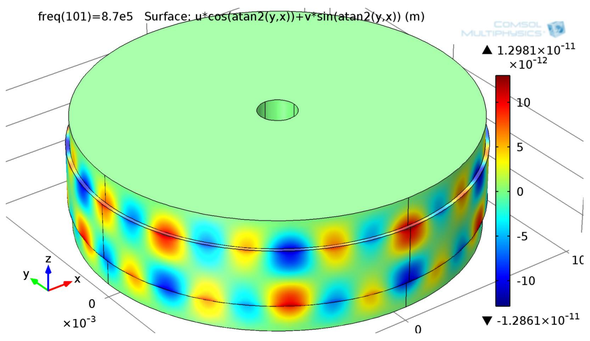


**Figure 3**. Quasi-Love mode barrel..

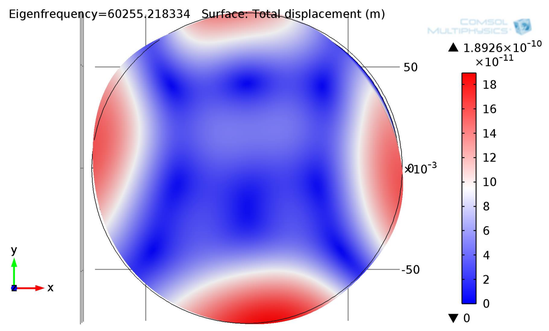


**Figure 4**. Broken degeneracy In LiNbO3.

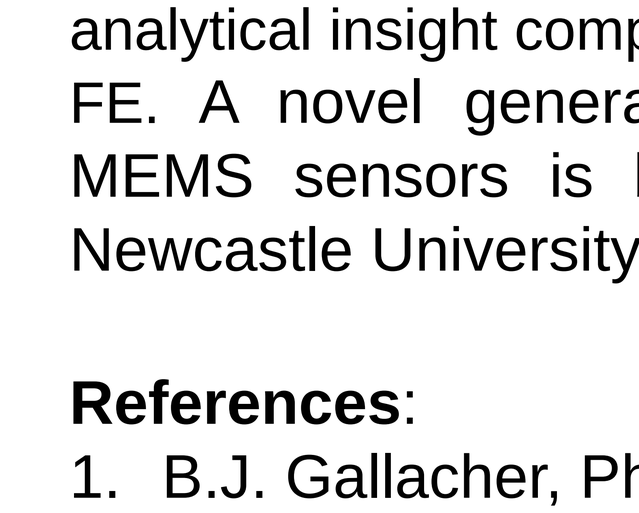


**Figure 4**. High order degenerate mode of a cylinder

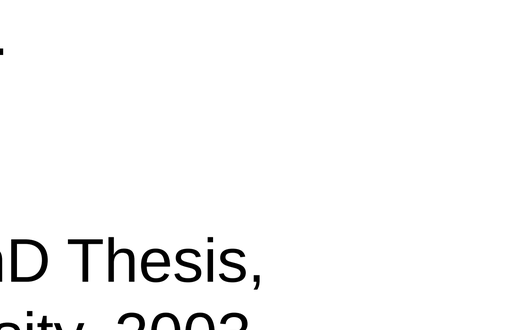


**Figure 5**. Degenerate Rayleigh Wave mass sensor

**Conclusions**: The analytical work and COMSOL are found to agree. The analytical insight complements the power of FE. A novel generation of degenerate MEMS sensors is being developed at Newcastle University.

**References**:

1. B.J. Gallacher, PhD Thesis, Newcastle University, 2003.

Excerpt from the Proceedings of the 2013 COMSOL Conference in Rotterdam