Miscible viscous fingering of pushed versus pulled interface

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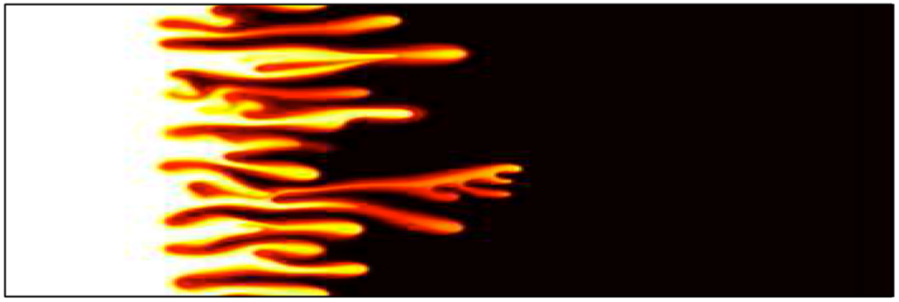
Viscous fingering (VF) instability

- Viscous fingering instability is special class of hydrodynamic instability which leads to the dispossession of the initial interfacial shape between the fluids with different viscosity.

- Less mobile fluid lag behind the high mobile fluid which penetrates through the former in a porous media.



Rectilinear displacement

Radial displacement

Viscous fingering (VF) instability

Viscous fingering can be observed in

- ❖ immiscible fluids where surface tension acts as the most important factor
- ❖ miscible fluids where the diffusion plays the key role

AIM

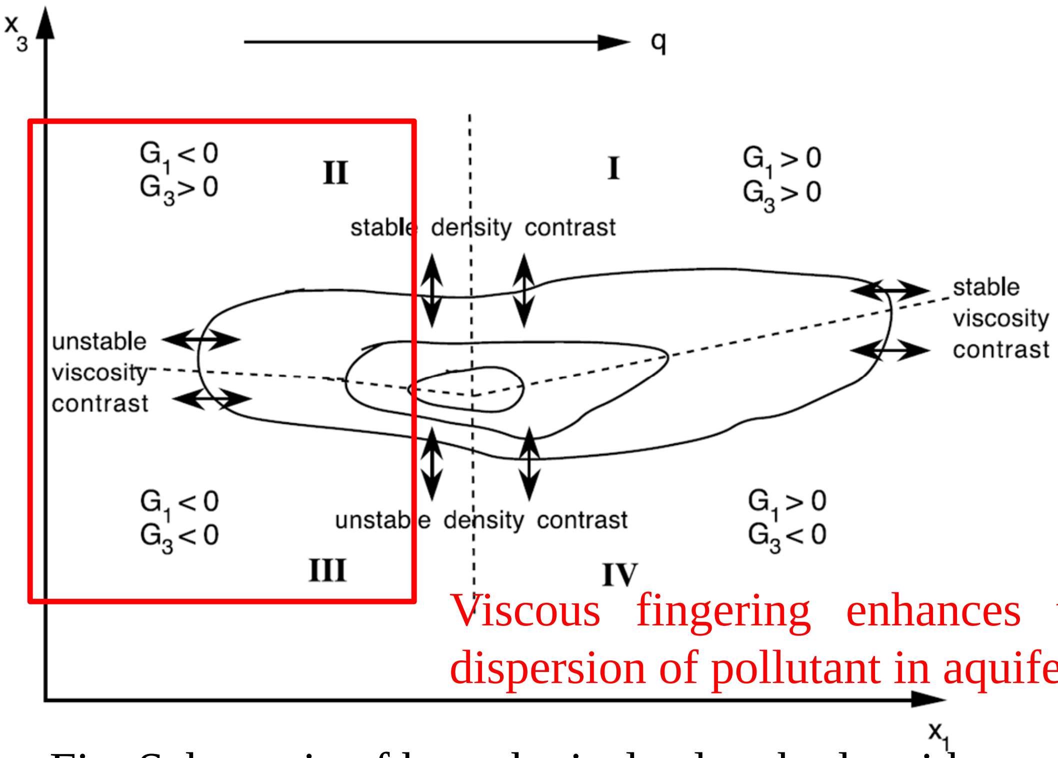
Here we focus on the rectilinear displacement of miscible solutions.

Two most important applications of viscous fingering instability are

1. **Chromatographic separation and**
2. **Pollutants dispersion of in aquifers.**

- ➢In both the cases one *fluid is localized within a finite region* and is displaced by another carrying fluid.
- ➢ Sometimes, the finite fluid can be confined within a *circular region* and hence single interface model is not appropriate for these cases.

Dispersion pollutant in aquifers



Viscous fingering enhances the dispersion of pollutant in aquifers

Fig. Schematic of hypothetical solute body with signs of concentration gradients indicated in each quadrant.

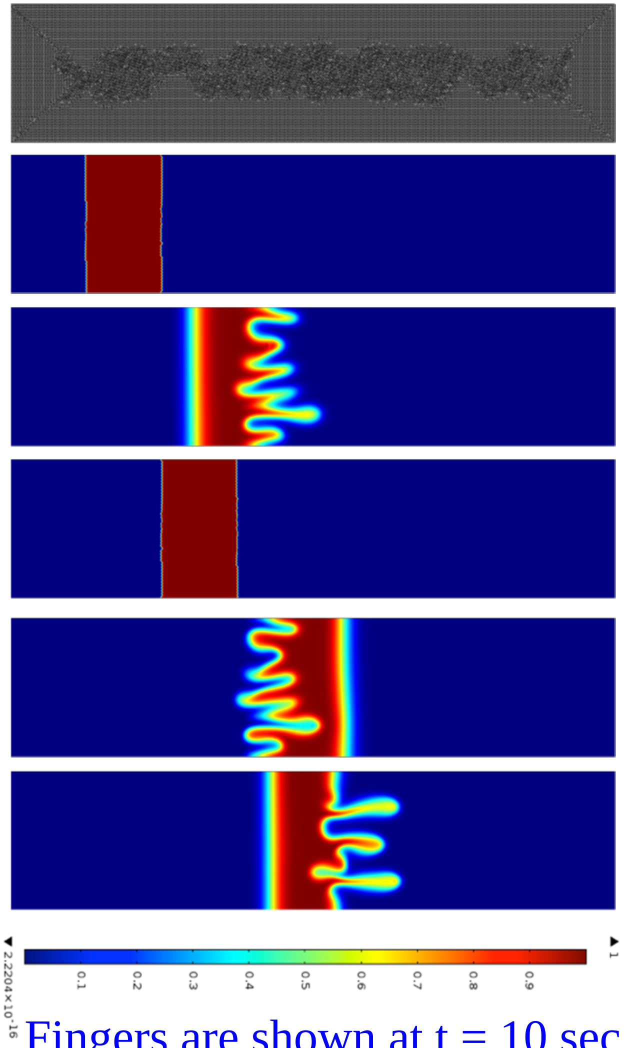
Heavy sample of higher viscosity is displaced by water in a porous media

g

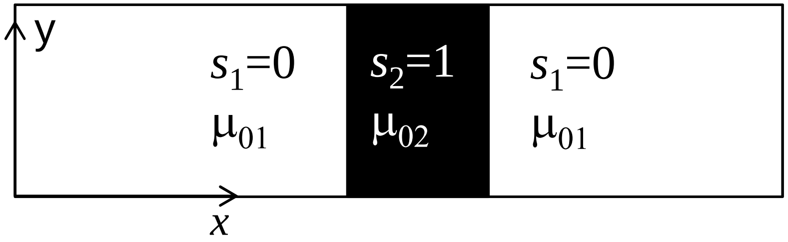
$$G_i = -\frac{\partial c}{\partial x_i}, i = 1, 2$$

Welty et al., Water Resour. Res. 39(6), 2003

Modeling VF with COMSOL Multiphysics



Fingers are shown at t = 10 sec



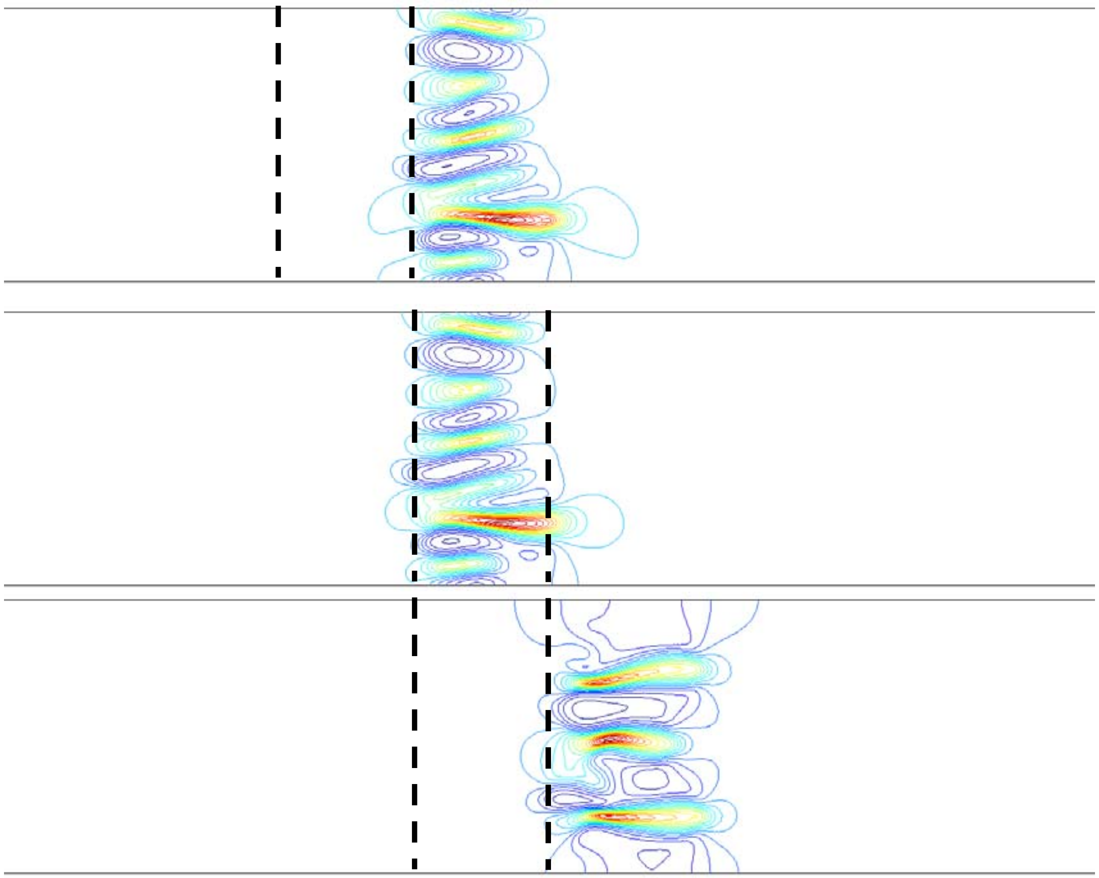
Pramanik *et al.* "Miscible viscous fingering: Application in chromatographic columns and aquifers", Proceedings of the COMSOL conference 2012 Bangalore.

In version 4.2a:

- Two-phase Darcy's law
- Free triangular meshing
- Reproduce the work of Mishra *et al.* by keeping the unstable interface at same position for both $R > 0$ and $R < 0$.

Pramanik *et al.* Proceeding of the COMSOL conference 2012, Bangalore

Velocity contours

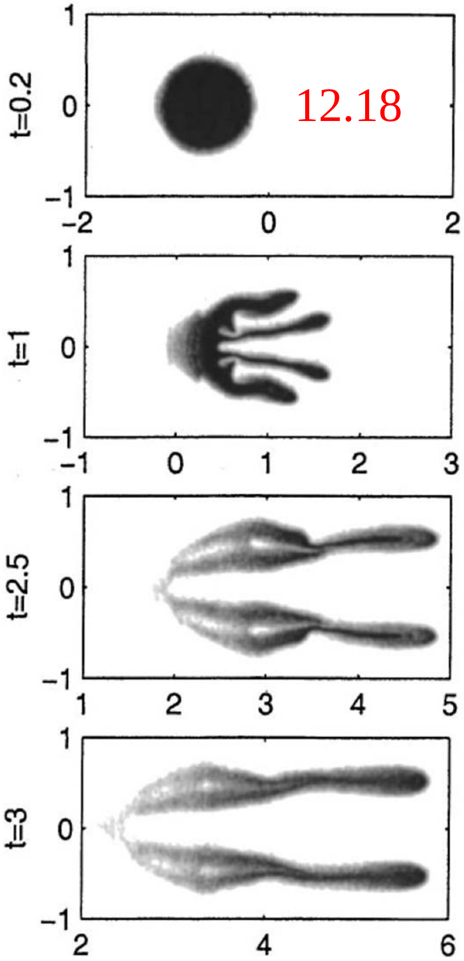


Axial velocity contours at time $t = 10$ sec

Pramanik *et al*. Proceeding of the COMSOL conference 2012, Bangalore

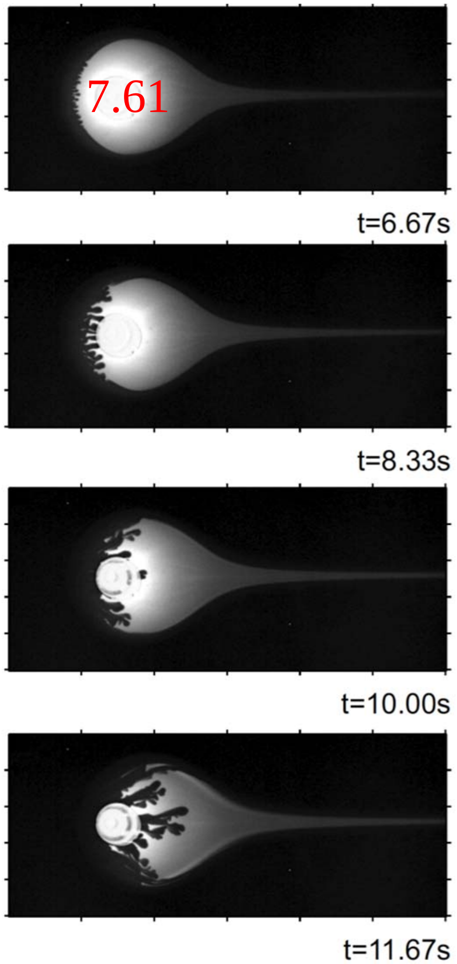
VF with circular sample

Theoretical modeling, less viscous sample



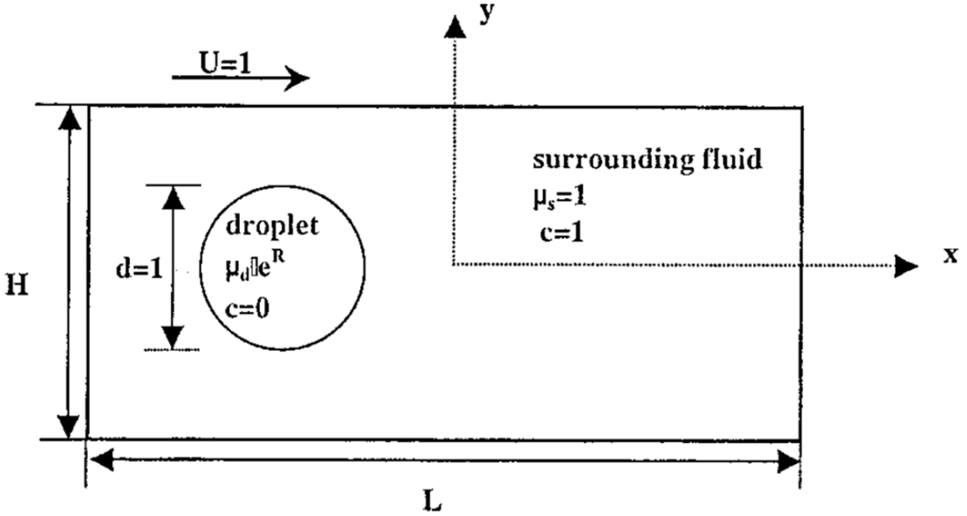
Chen *et al*. Phys. Fluids 13, 2001

Experimental investigation, more viscous sample



Maes *et al*. Phys. Fluids 22, 2010

Theoretical model



Governing equations

$$\nabla \cdot \vec{u} = 0,$$

$$\nabla p = -\frac{\mu(c)}{k}\vec{u},$$

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = \nabla \cdot (D \nabla c),$$

$$\mu(c) = e^{R(1-c)}.$$

$\vec{u}$: 2D velocity field

k : permeability

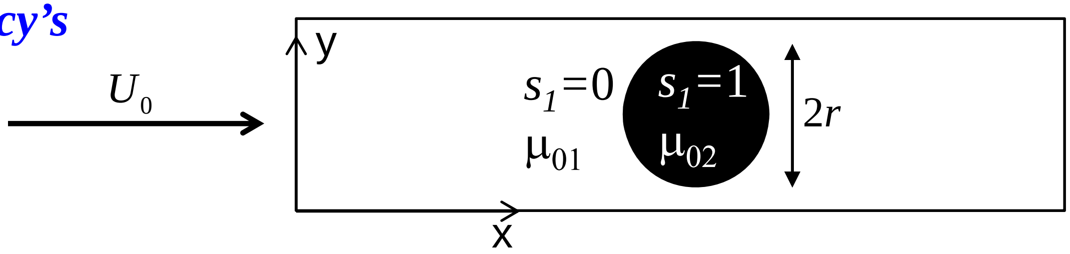
p : pressure

c : concentration of the solute driving viscosity

D : dispersion coefficient

COMSOL Multiphysics modeling

Two-phase Darcy's law



Governing equations

$$\frac{\partial \varepsilon_p \rho}{\partial t} + \nabla \cdot \rho \vec{u} = 0, \quad \vec{u} = -\frac{\kappa}{\mu} \nabla p$$

$$\rho = \rho_1 s_1 + \rho_2 s_2, \quad \frac{1}{\mu} = s_1 \frac{\kappa_{r1}}{\mu_1} + s_2 \frac{\kappa_{r2}}{\mu_2},$$

$$\frac{\partial \varepsilon_p \rho}{\partial t} + \nabla \cdot c_1 \vec{u} = \nabla \cdot D_c \nabla c_1, \quad c_1 = \rho_1 s_1$$

$$\mu_1 = \mu_2 = \mu_{01} e^{R s_1}$$

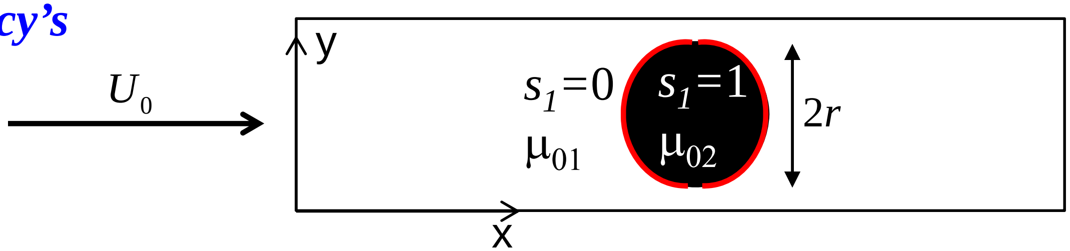
$$\rho = \rho_1 = \rho_2 = \text{constant}$$

$$\varepsilon_p, \kappa = \text{constant}$$

$$\kappa_{r1} = \kappa_{r2} = 1$$

COMSOL Multiphysics modeling

Two-phase Darcy's law



Boundary conditions

$$-\vec{n}\cdot\rho\vec{u}=0$$ No flux

$$-\vec{n}\cdot D_c\nabla c_1=0$$ Outflow

$$-\vec{n}\cdot\rho\vec{u}=(s_1\rho_1+s_2\rho_2)U_0$$ Inflow

$$s_2=1-s_1$$

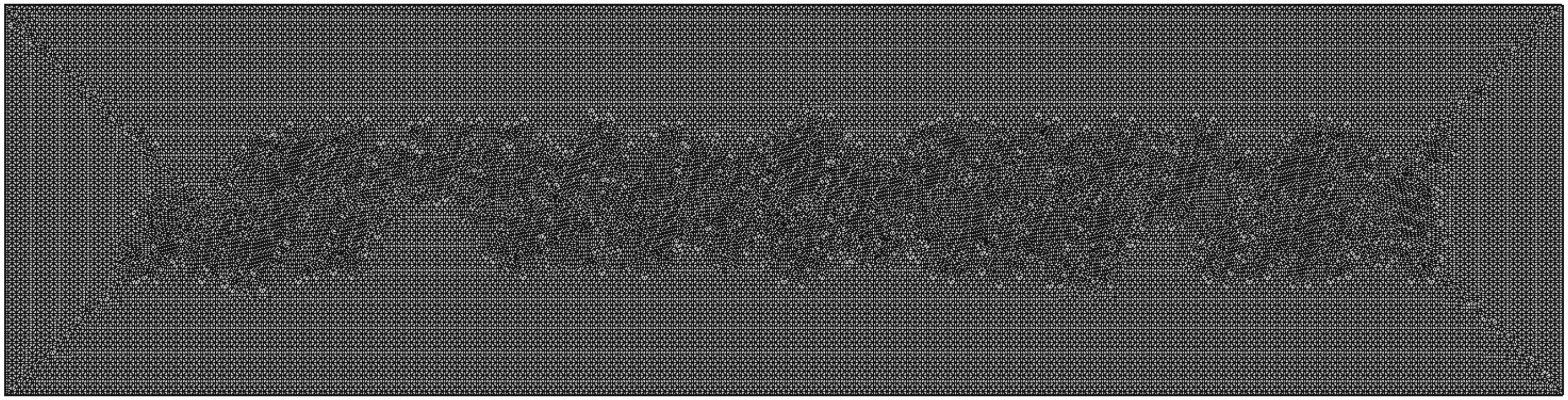
Initial condition

$$s_1=\begin{cases}1, & (x-x_0)^2+(y-y_0)^2\leq r^2\\ 0, & (x-x_0)^2+(y-y_0)^2>r^2\end{cases}$$

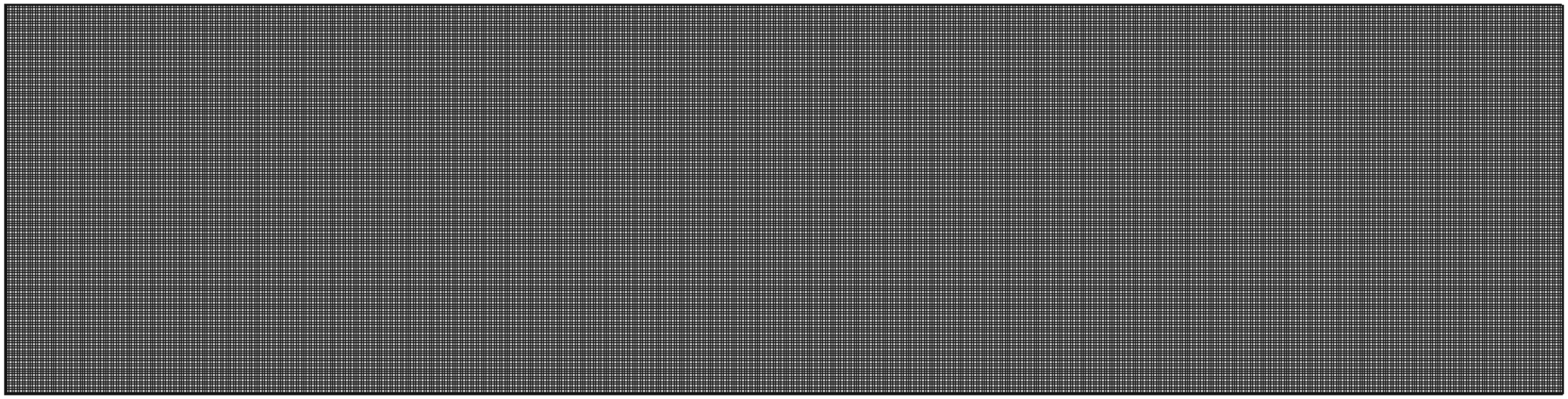
(x_0,y_0) being the center of the circle

Normal inflow velocity U_0

COMSOL Multiphysics modeling



Extra fine free triangular mesh (# of elements 64852)



Extremely fine mapped mesh (# of elements 91196)

COMSOL Multiphysics modeling

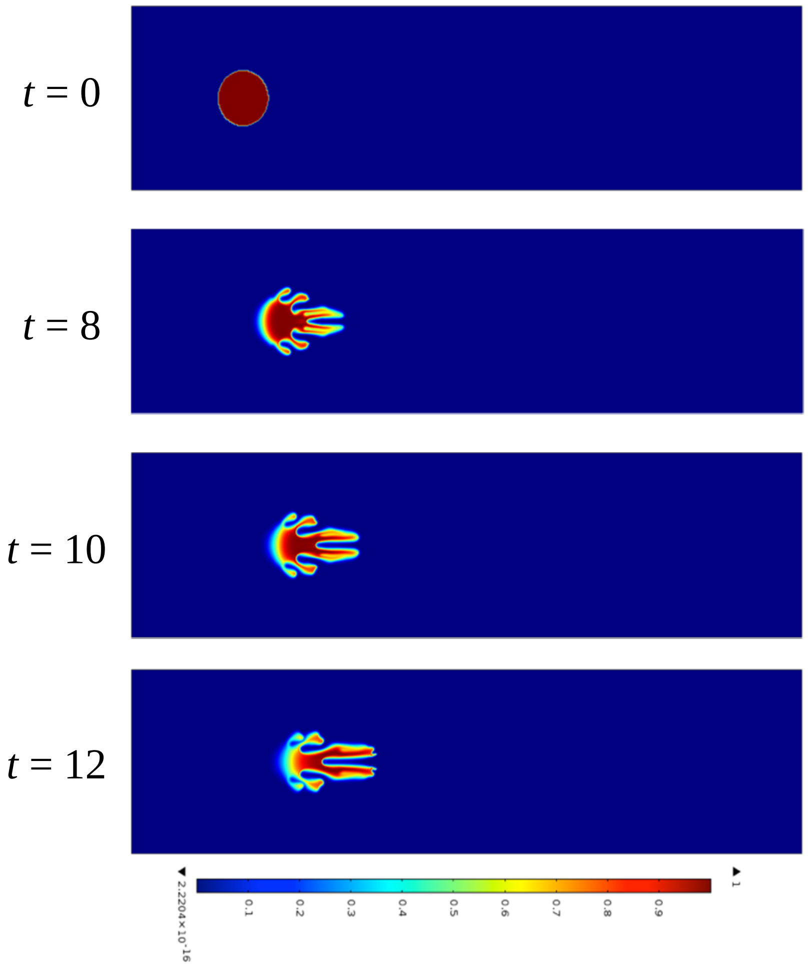
Parameters	Symbols	Value & Unit
Length of the domain	L_x	0.32 mm
Width of the domain	L_y	0.08 mm
Log-mobility ratio	R	-3, 0, 3
Injection speed	U_0	1 mm/s
Viscosity of the displacing fluid	μ_{01}	10^{-3} Pa-s
Aspect ratio	A	4
Radius of the circular sample	$r = 0.15 \times L_y$ $r = 0.3 \times L_y$ $r = 0.45 \times L_y$	0.012 mm 0.024 mm 0.036 mm
Center of the circle	$x_0 = L_x/9$, $y_0 = L_y/2$	(0.0356,0.04) (mm)

Comparison of two linear solvers MUMPS and PARDISO

Parameter set	Computational time	
	MUMPS	**PARDISO**
$R = -2$, $L_y = 0.08$ mm, $r = 0.45 \times L_y$, $A = 4$	8319 seconds	21345 seconds
$R = 2$, $L_y = 0.08$ mm, $r = 0.45 \times L_y$, $A = 4$	2479 seconds	1996 seconds
$R = 0$, $L_y = 0.08$ mm, $r = 0.3 \times L_y$, $A = 4$	1912 seconds	466 seconds

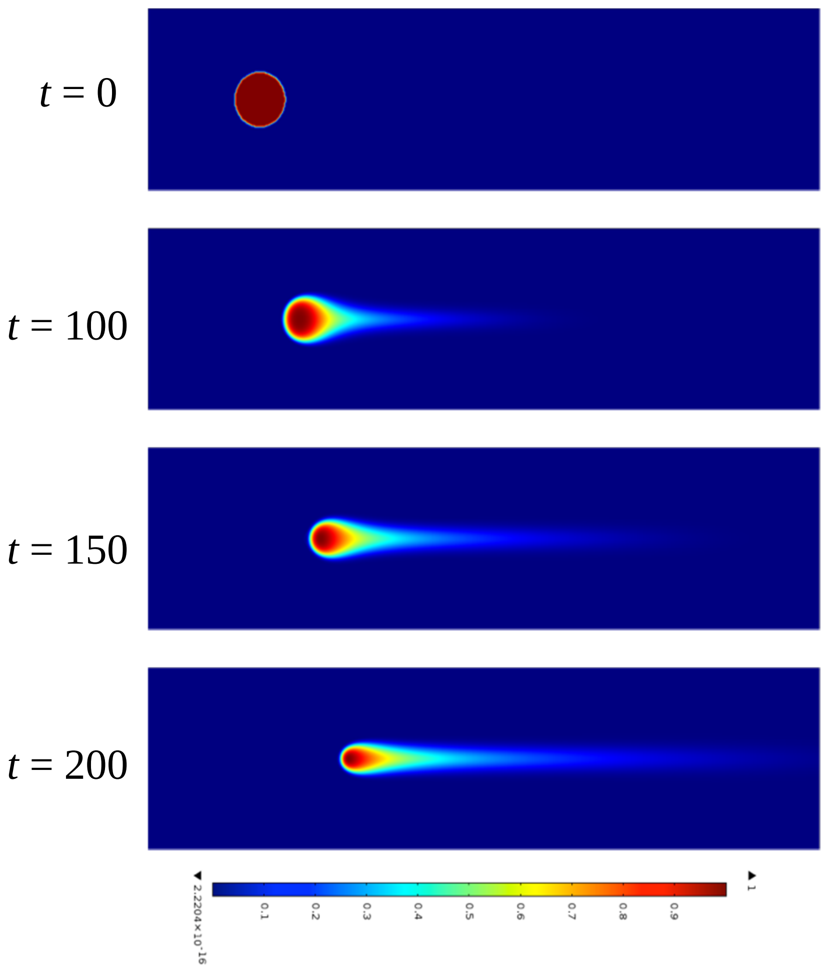
Results

VF at pushed interface



U_0 =1 × 10^{-3} *m/s*, *R* = -3, L_y = 0.08 mm. and r = 0.15 × L_y.

VF at pulled interface



U_0 =1 × 10^{-3} *m/s*, *R* = 3, L_y = 0.08 mm. and r = 0.15 × L_y.

Results

$U_0 = 1 \times 10^{-3}$ m/s, $R = -3$, $L_y = 0.08$ mm. and r = $0.15 \times L_y$

Results

$U_0 = 1 \times 10^{-3}\ m/s$, $R = 3$, $L_y = 0.08$ mm. and $r = 0.15 \times L_y$

Conclusions

- COMSOL Multiphysics has been used to model miscible viscous fingering of *pushed and pulled interfaces*.
- The classical fingering phenomena like tip splitting, merging etc. are captured through COMSOL Multiphysics.
- Results are *reproducible* and are in well *accordance with the experimental findings*.
- Differences in the fingering patterns for *pushed* and *pulled interfaces* are explained through the *streamlines*.
- VF modeling with COMSOL Multiphysics *strongly depends on the meshing*.

Acknowledgements

- COMSOL Multiphysics

COMSOL MULTIPHYSICS®

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  - DST Govt. of India

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