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Mathematical Modelling and Simulation of Magnetostrictive Materials by Comsol Multiphysics

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Outline

- Motivation
- History and Physical behavior
- Functions and applications
  - Sensors
  - Actuators
  - Energy conversion
- Mathematical modelling
- Modelling in Comsol Multiphysics
- Problems
- Conclusion

Motivation

- **make tool to achieve for a quantitativies analysis**
- **the relation magneto-mechanical**

Particuliarities of the material:

- high density of available energy
- long life time
- prospective for new type of energy conversion

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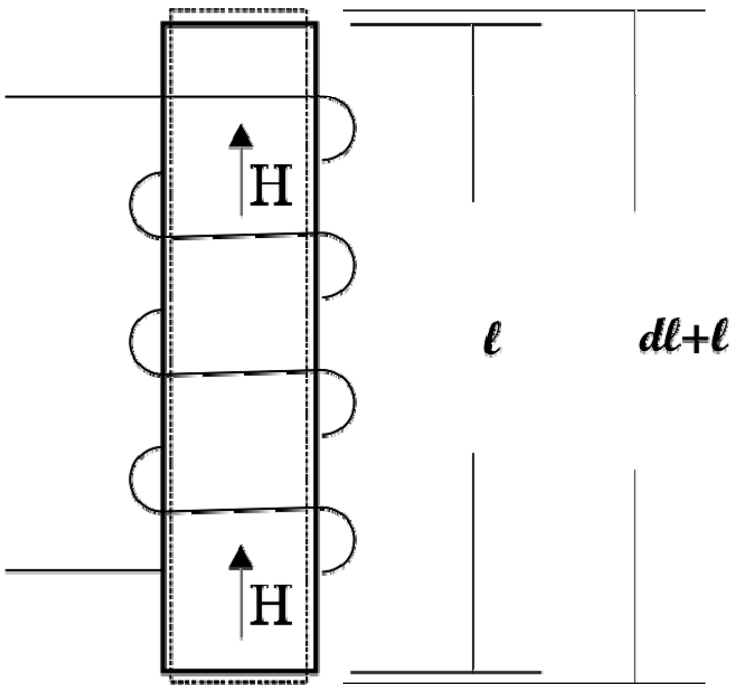
History and Physical behavior

History:

- Discovered on 1842 James Joule

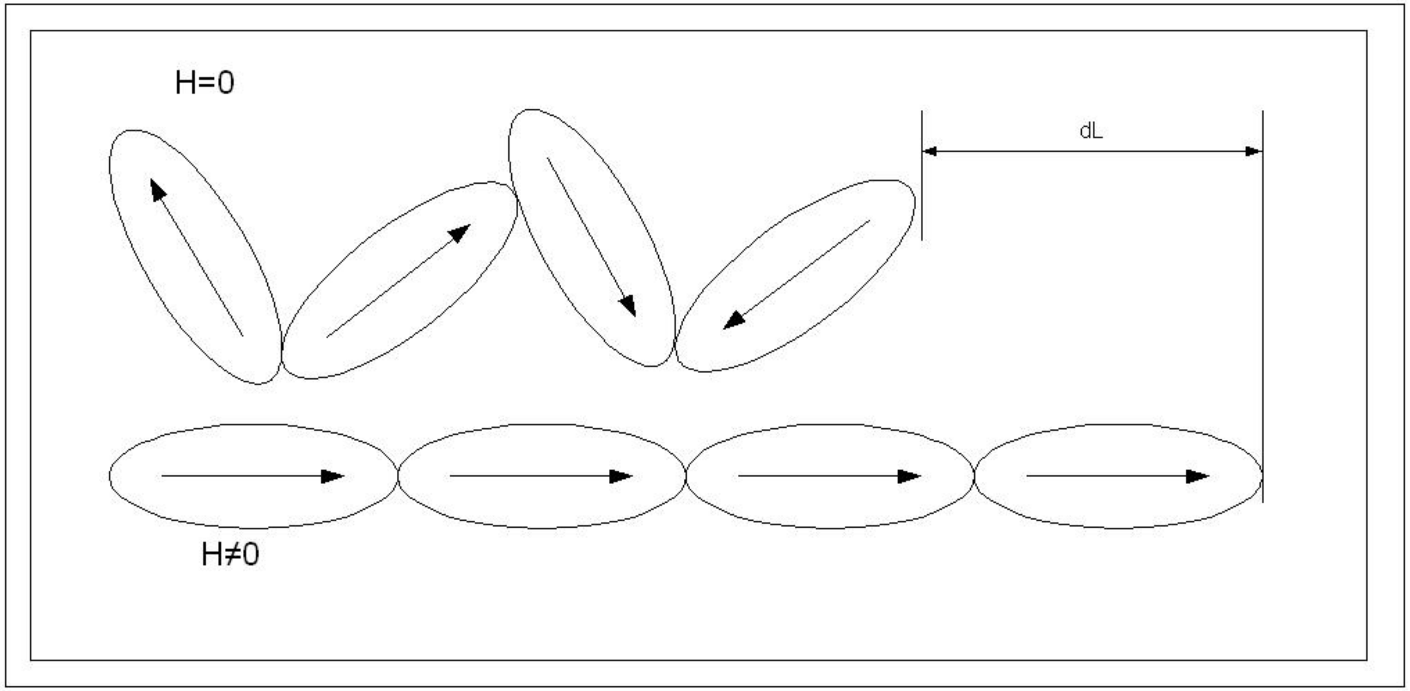
- Giant magnetostrictive 1960 USA Navy



Physical behaviour:

- Apply magnetic field H

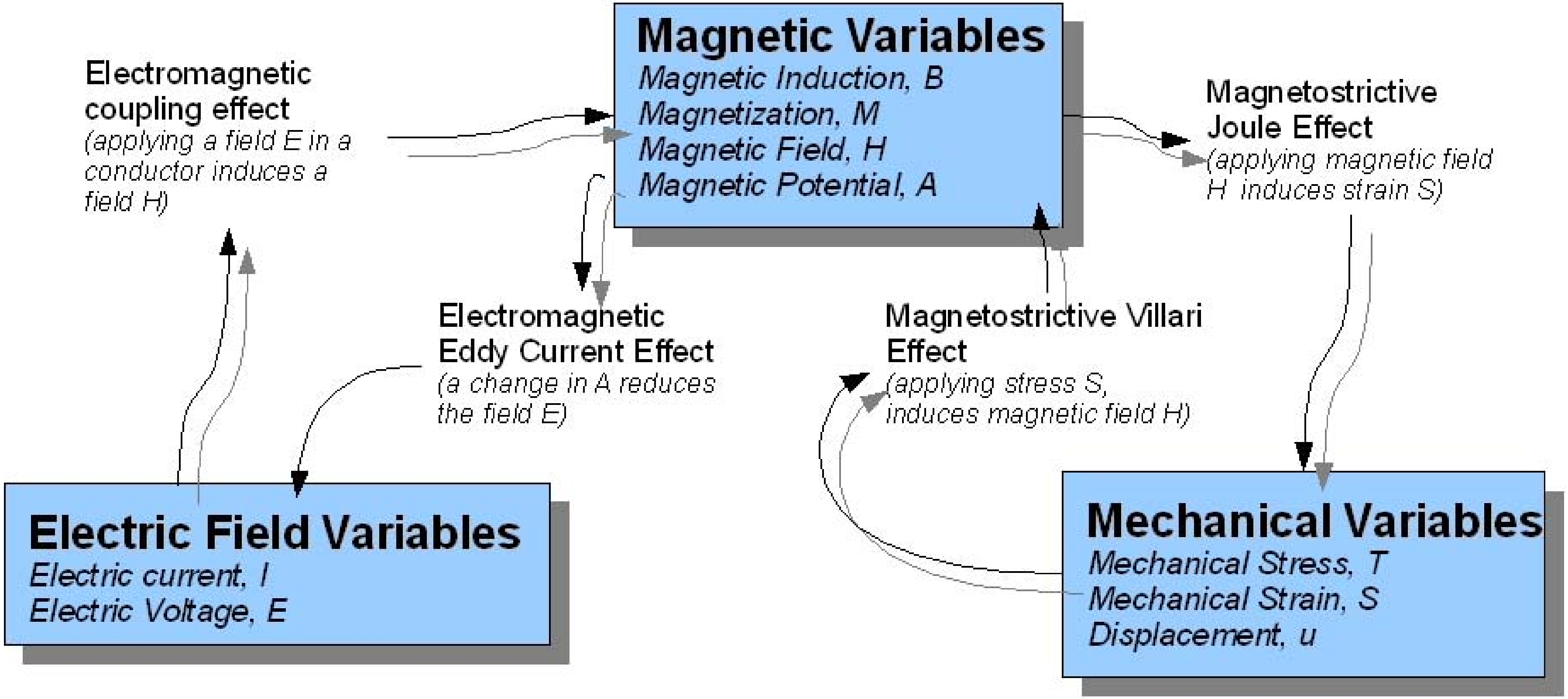
History and Physical behavior



- Internal domains alignment
- Changing in shape ⟶ changing in magnetic field

History and Physical behavior

- Reversible cycle and magneto-mechanical coupling diagram



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Functions and applications

Sensors and actuators:

Coil

electrical energy ⇄ magnetic energy

Magnetostrictive rod

magnetic energy ⇄ mechanical energy.

Possibility to recover energy from waste

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Mathematical modelling

The constitutive magnetostriction equations are:

$$S = \frac{\Delta l}{l}$$

relative deformation of the shape

the compliance at constant magnetic field

$$S = \eta_{(H)} \cdot T + d \cdot H$$

stress *T* is the forces per unit of area

piezomagnetic strain constant

magnetic field (induction)

$$B = d \cdot T + \mu_{(T)} \cdot H$$

permeability at constant stress

is the magnetic filed (intensity)

Mathematical modelling

By making some mathematical steps the model will be:

$$\begin{cases} \nabla^2 H = \sigma d \dfrac{\partial T}{\partial t} + \dfrac{d}{\mu}(\mu\varepsilon - \rho\eta)\dfrac{\partial^2 T}{\partial t^2} + \mu\sigma \dfrac{\partial H}{\partial t} + \dfrac{\left(\mu^2\varepsilon - \rho d^2\right)}{\mu}\dfrac{\partial^2 H}{\partial t^2} \\ \dfrac{\partial^2 T}{\partial x^2} = \rho\eta \dfrac{\partial^2 T}{\partial t^2} + \rho d \dfrac{\partial^2 H}{\partial t^2} \end{cases}$$

Considering harmonic analysis:

$$\begin{cases} \nabla^2 H = \omega^2 \tilde{k}_1 T + \omega^2 \tilde{k}_2 H \\ \dfrac{\partial^2 T}{\partial x^2} = \omega^2 \tilde{G}_1 T + \omega^2 \tilde{G}_2 H \end{cases}$$

$$\tilde{k}_1 = \frac{\left(d\left(\rho\eta - \mu\varepsilon\right) + \dfrac{i\mu\sigma d}{\omega}\right)}{\mu}$$

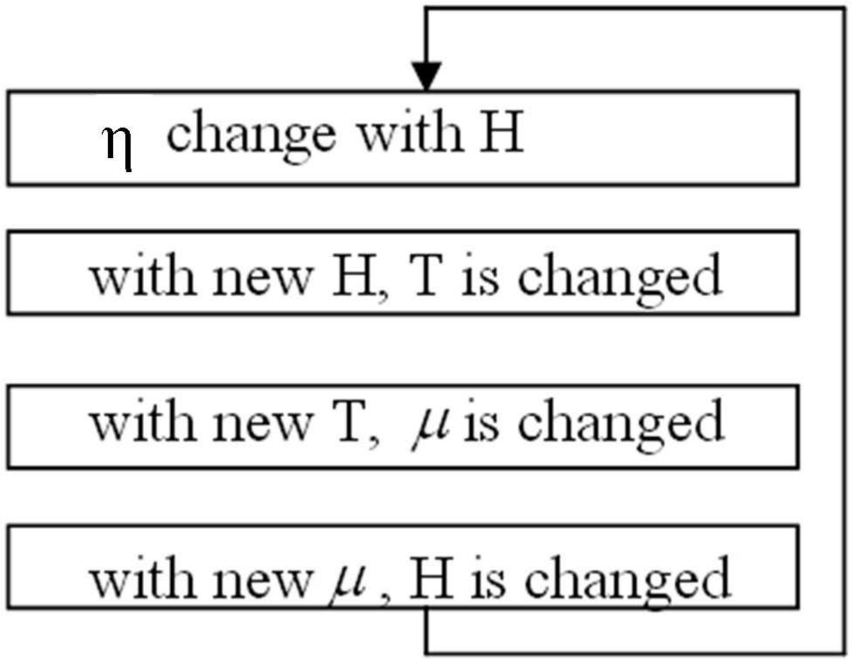
$$\tilde{k}_2 = \frac{\left(\left(\rho d^2 - \mu^2\varepsilon\right) + \dfrac{i\mu^2\sigma}{\omega}\right)}{\mu}$$

$$\tilde{G}_1 = -\rho\sigma$$

$$\tilde{G}_2 = -\rho d$$

Mathematical modelling

Iterative evaluation for the parameters



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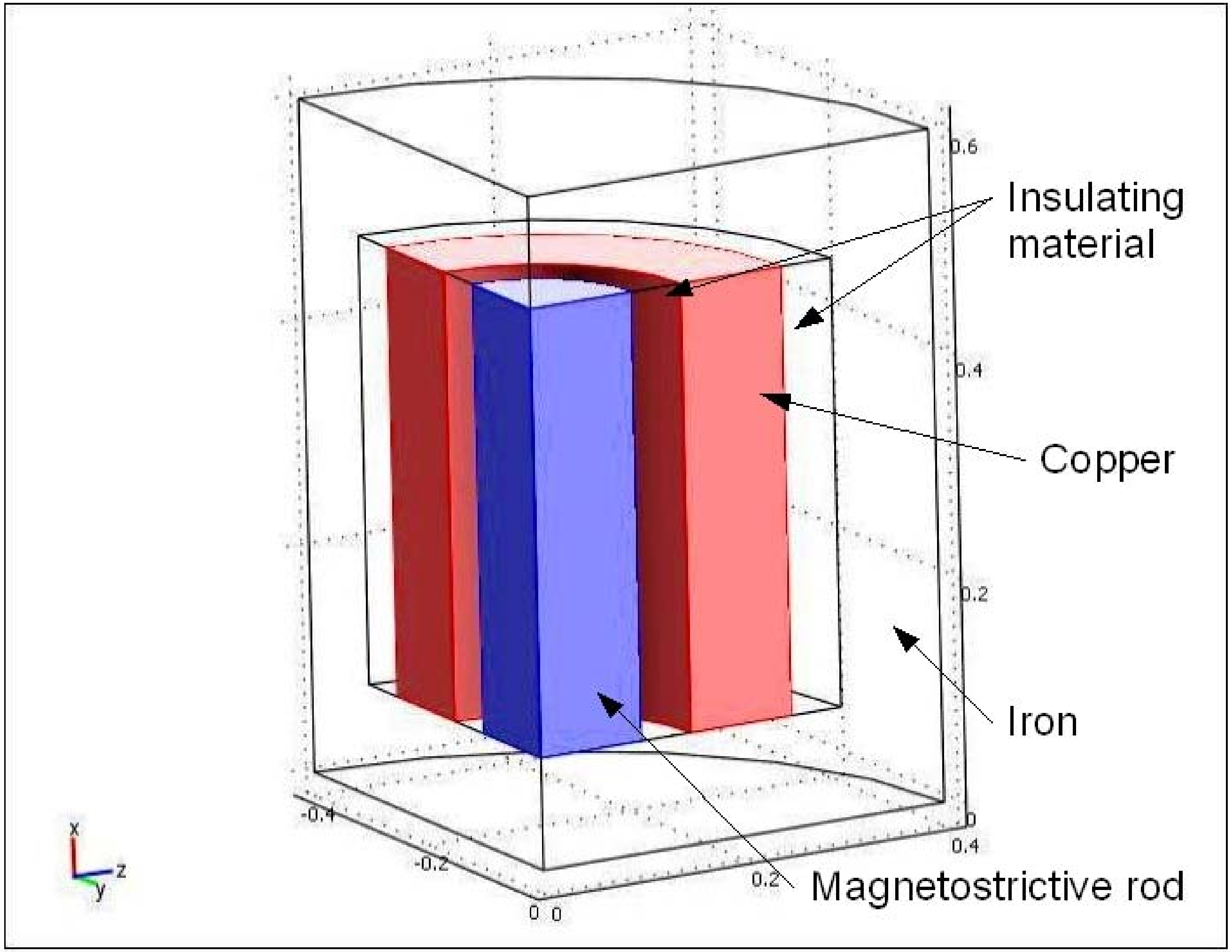
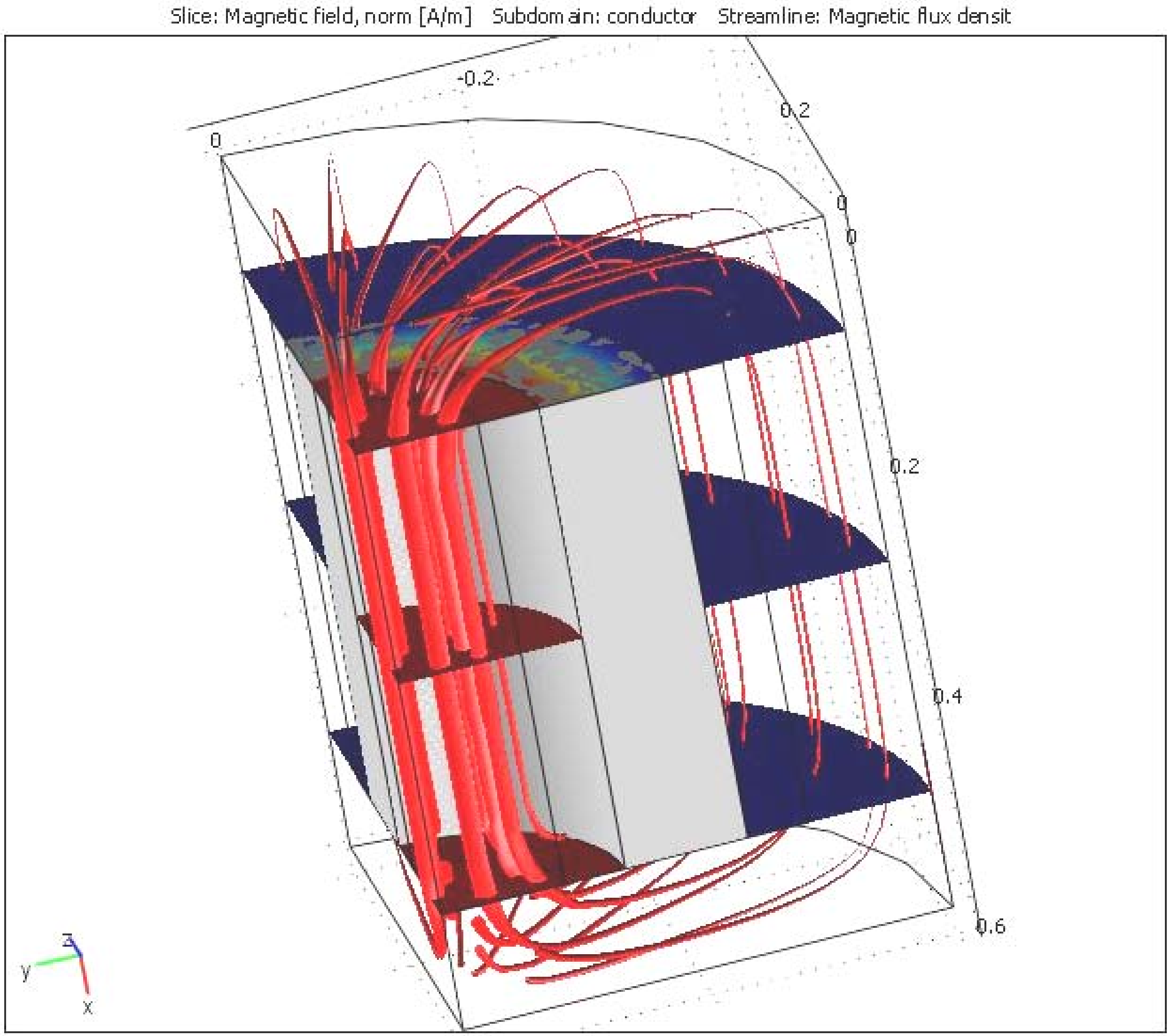
Modelling in Comsol Multiphysics



- **Model related to the mathematical one for the simulation**

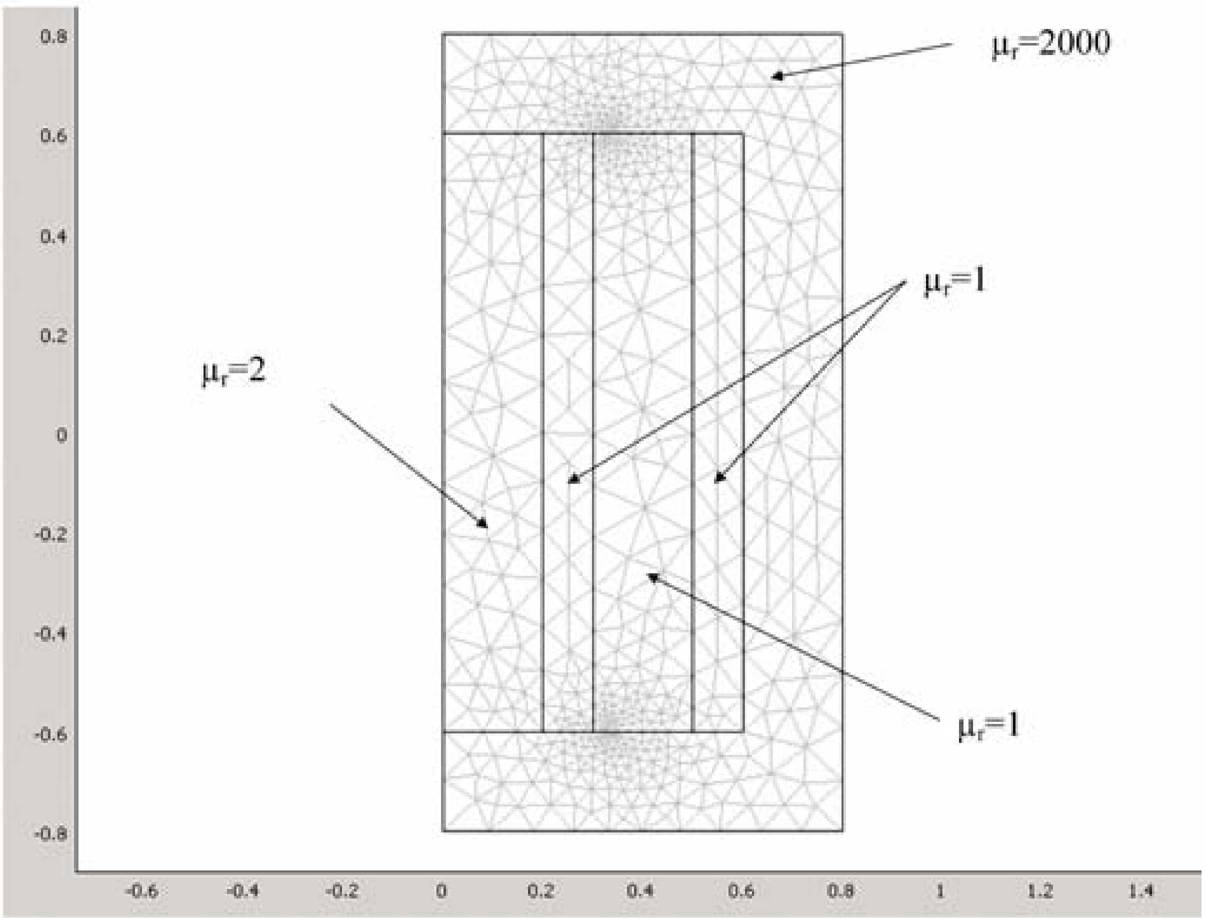
Modelling in Comsol Multiphysics



Induction flux lines

Modelling in Comsol Multiphysics

Auxiliary 2D geometry and meshing



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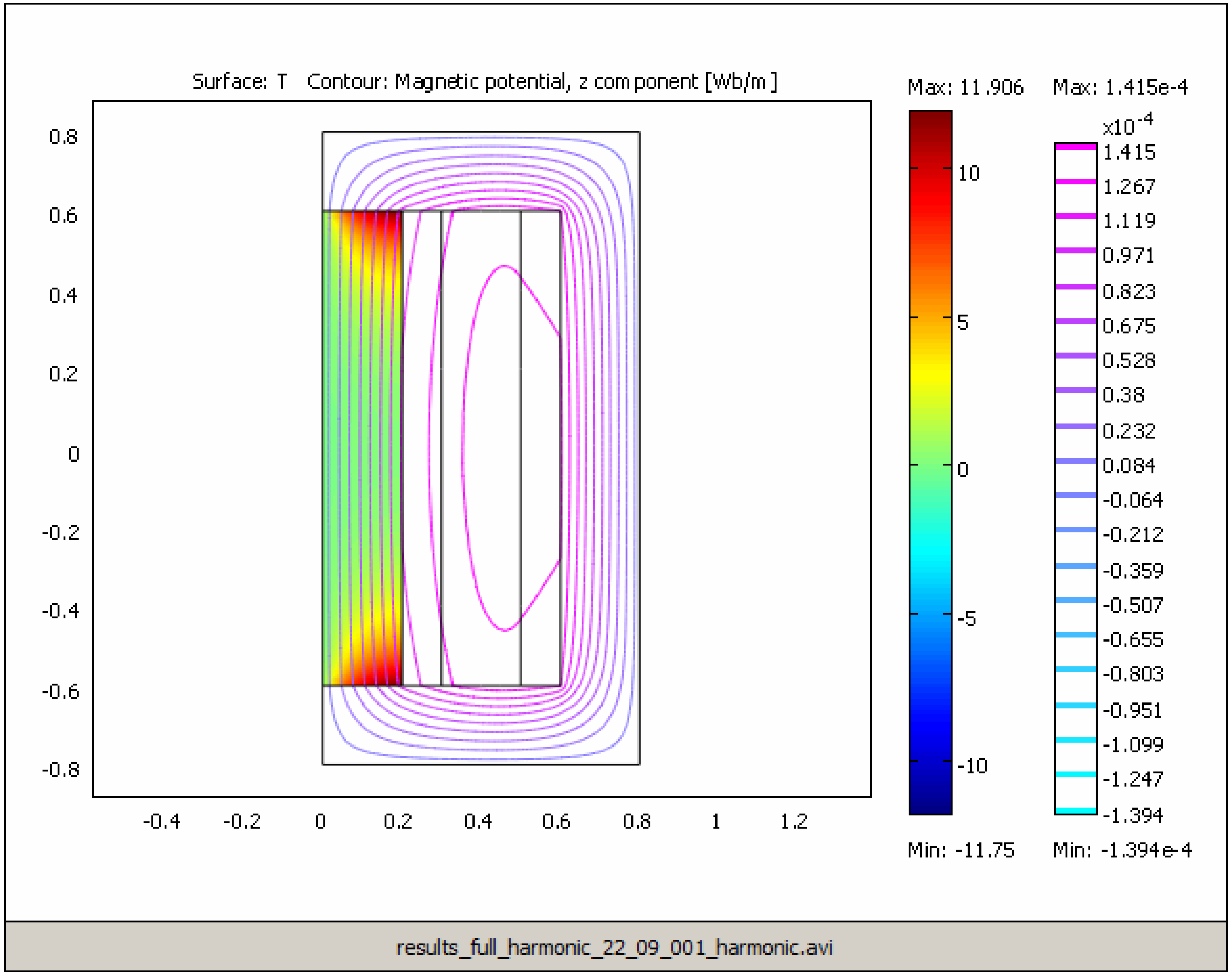
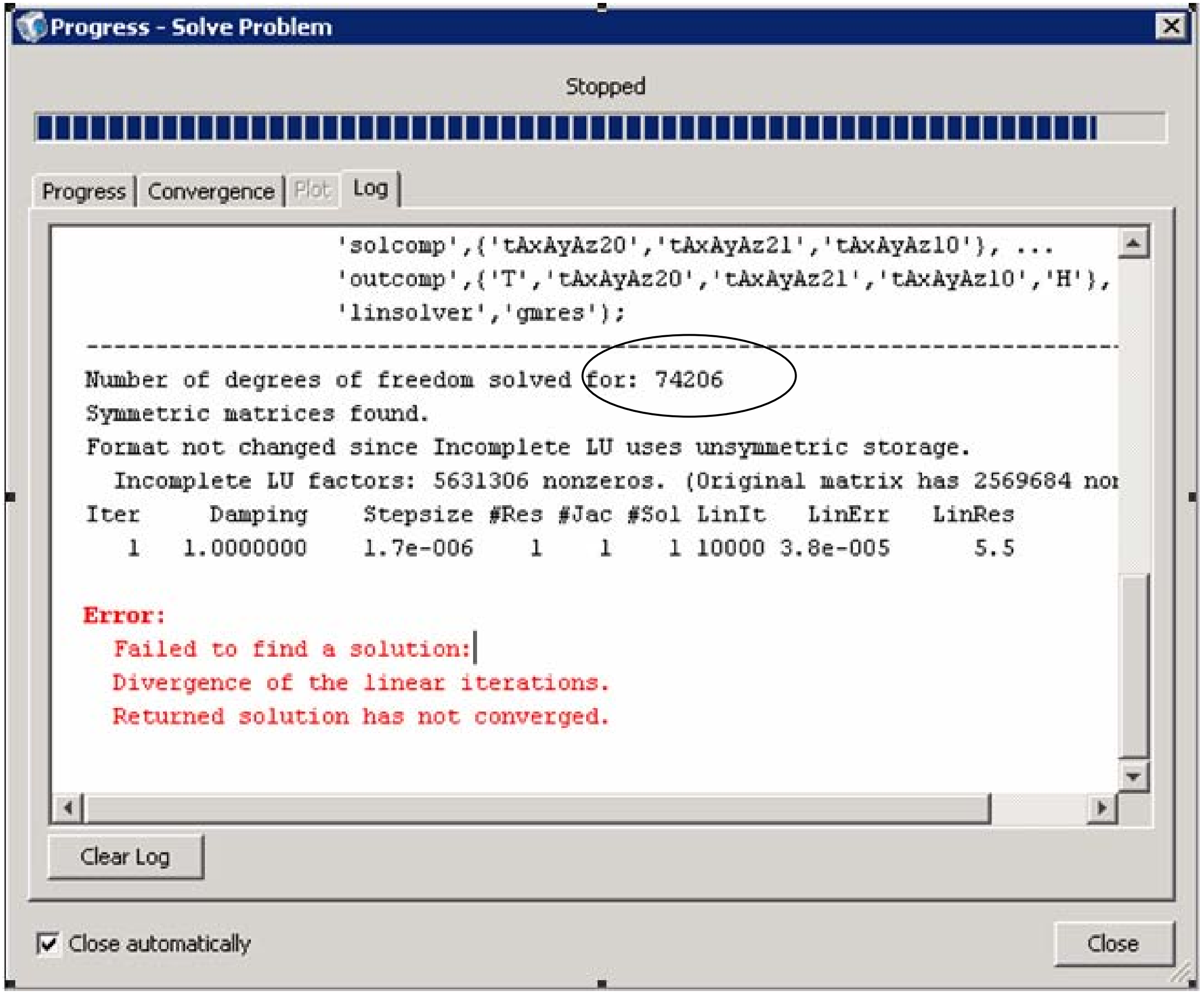
Results and problems

Full harmonic excitation simulation



Results and problems



3D model problem

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Conclusion

- Comsol© can solve this problem
- Stress depend of H and time
- Tractive and compressive efforts located top and bottom

Future work:

-transient analysis

-complete and detailed simulation

Thank you!

Mathematical modelling

Mathematical modelling

- Making time-derivative and the second derivative on the equations (2.1) and (2.2):

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial t^2}\right) = \eta \frac{\partial^2 T}{\partial t^2} + d\frac{\partial^2 H}{\partial t^2} \qquad (2.4)$$

- Consider the second Newton's law:

$$F = ma \qquad (2.5)$$

Then obtaining:

$$\frac{\partial T}{\partial x} = \frac{\partial\left(\frac{F}{A}\right)}{\partial x} = \frac{\partial F}{\partial V} = \frac{\partial(m \cdot a)}{\partial V} = \rho \cdot a = \rho \frac{\partial^2 u}{\partial t^2} \qquad (2.6)$$