

Calculation of Cable Parameters for Different Cable Shapes

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Abstract:

Efforts involving simulation of transmission line networks necessitate the accurate values of the parameters of the lines. In this paper, as an attempt in estimating such parameters, the parameters of high voltage asymmetric power line are calculated. In the process of estimation, the three phase equivalent circuit model is used. The resistance and inductance of such lines are dependent on frequency. The parameters are estimated both theoretically and using COMSOL Multiphysics Software. The results are compared and discussed with focus on field theoretical considerations.

Keywords: Partial wire technique; low voltage quad core cable; power quality calculations; equivalent pi/T-network.

1. Introduction

Increasing demand based on trends towards embedded power-generation forces scattered harmonic excitation for the electrical transmission system. Because of new methods for analysing Harmonic Resonance-Phenomenon there is a demand on detailed, all phase representation of all relevant system components like generators, transformers and lines, i.e. generation, transmission, supply and end consumption.

The manufacturers of transmission lines require also such detailed knowledge of cable parameters in conjunction with quality assurance and in meeting the needs of the customers. In addition, comes the economic consideration with respect to maximum utilisation of the copper cross section. In addition, the increasing demand for full custom design of cables meeting the customer specifications both mechanically (such as maximum tensile strength, Youngs Modulus) and electrically (such as full knowledge of the impedance, i.e. resistance, inductance and capacitance, preferably as functions of frequency).

The emphasis on the knowledge of frequency dependency of the electrical parameters, makes the COMSOL Multiphysics program an attractive tool for accomplishing the task of estimating

these parameters with ease. We present an application of COMSOL Multiphysics Program in the estimation of these parameters for a segment cable.

Transmission line models are normally based on the transmission line theory very often based on the ladder network with discrete components as shown in Figure 1.

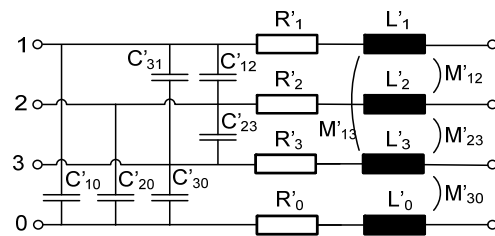


Figure 1: Ladder network of a transmission line involving discrete components

To obtain symmetry, either lateral capacitance and conductance or longitudinal resistance and inductance are divided into equal halves which lead to the common equivalent π - or T-networks. For both models the cable parameters R' , L' , C' , (G') are needed.

The „dashes“ " ' " are used to imply parameter values per unit length, for example $R' = R/l$. For coaxial cables used in communication technology or screened cables for medium voltage



Figure 2: Low voltage segment cable

power distribution, these parameters can easily be determined, if the inductive coupling is negligible. But many cables have a design for which the analytical solution cannot be applied. To show the complexity involved, the structure of a commonly used low-voltage power distribution cable is shown in Figure 2.

The manufacturers of such cables usually give the nominal inductance L_b and load current I_L . However, the need for detailed simulation demands the knowledge of all the parameters given in Figure 1. The usage of Comsol Multiphysic Program in the estimation of these parameters is discussed in the following sections.

The calculation of the cable parameters are carried out in the Multiphysic Program modules "Electrostatic" or "AC-Electromagnetic" depending on the type of parameters needed. To begin with, the cable geometry for the cable shown in Figure 1, is designed with the comfortable internal GUI. This geometry is then used in the above mentioned modules to calculate R' , C' and L' .

2. Cable capacitance

The discrete capacitance values can be calculated independent of frequency. To calculate the six partial capacitances C' of the segment cable shown in Figure 1, the voltage is applied to two consecutive conductors, repeating these connections until we have exhausted the whole cable cluster. The electric field in the isolator is calculated first. Then the energy of the electric field is determined performing the needed integration in the postprocessing of data in Multiphysic Program. By means of a linear equation system evolving out of the system shown in Figure 1, the desired capacitance values can be determined.

The cross-section of the cable shown in Figure 3, having the segments numbered from 0 to 3 is used in the simulations. As mentioned before, the consecutive segments are coupled to a voltage source of 1 kV. The conductor with "+" attached to the segment number is connected to the positive terminal of the 1kV-source. The following independent cases are studied: (1+), (2+), (1+2+), (1+2+3+), (2+3+), (1+2+3+). The unnamed cable segment in each case, is obvi-

ously earthed for the case study involving the module „Electrostatic“.

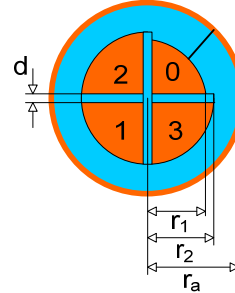


Figure 3: Cross-section of the cable with the numbered cable sections. Selection of cable segments for applying voltages discussed in text.

For all the cases mentioned above, the energy W_e of the electrostatic field is calculated. Then the well known equation (1) between electrostatic energy W_e , voltage U and capacitance value C is used to estimate the cable capacitance.

$$W_e = \frac{1}{2} CU^2 \quad (1)$$

The conducting earth is assumed to be at $r_a = 3.5 r_2$.

2.1 Energy of the field with applied voltage on segment 1

In the case of segment 1 being supplied with 1kV, i.e. (1+), all the other conductors and the screening conductor are earthed. For the electrostatic case, the simplified equivalent circuit for the case of segment 1 connected to 1 kV, is given in Figure 4.

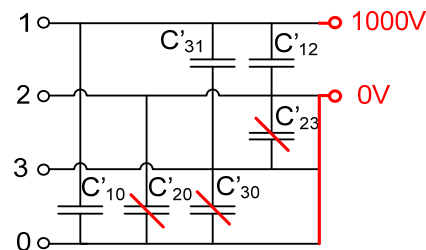


Figure 4: Voltage on conductor segment 1 with all the other conductors and screen earthed

In the case of (1+), the capacitances $C'_{10}, C'_{31}, C'_{12}$ are in parallel connection, giving the following equation (2) for the energy

$$W_{e(1+)} = \frac{1}{2} (C'_{10} + C'_{31} + C'_{12}) U^2 \quad (2)$$

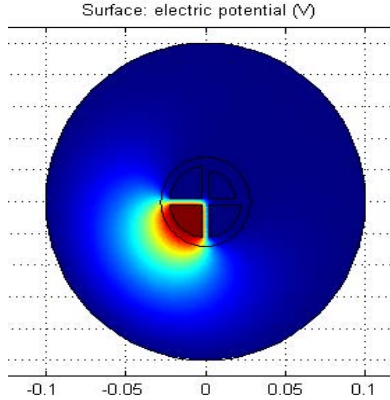


Figure 5: Voltage distribution for the case with 1kV on conductor segment 1, and all the other parts earthed. Red implying high end and blue the low end of the voltage scale.

It is of interest to observe the voltage distribution of the case under study, as shown in Figure 5. In the Postprocessing stage of the Comsol Multiphysic, the energy intensity is evaluated to be

$$W_{e(1+)} = 1.558726 \cdot 10^{-4} \text{ Ws/m}$$

2.2 Electrostatic energy in the case of voltage applied to conductor segments 1 and 2

In the next simulation, the case of (1+2+), i.e. the application of 1kV on conductor segments, is studied. In the equivalent circuit, the capacitances $C'_{10}, C'_{20}, C'_{31}, C'_{23}$ are now coupled as known in Figure 6.

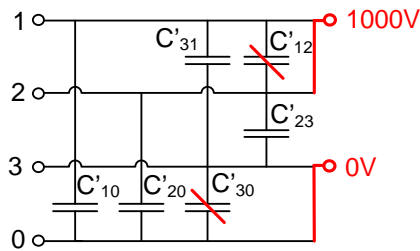


Figure 6: Voltage on conductor segments 1 and 2, with screen and conductor 3 earthed

As discussed before, the electrostatic energy intensity in the case of conductor segments 1 and 2 being energized is given by

$$W_{e(1+2+)} = \frac{1}{2} (C'_{10} + C'_{20} + C'_{31} + C'_{23}) U^2 \quad (3)$$

$$= 1.589495 \cdot 10^{-4} \text{ Ws/m}$$

and the corresponding voltage distribution is shown in Figure 7.

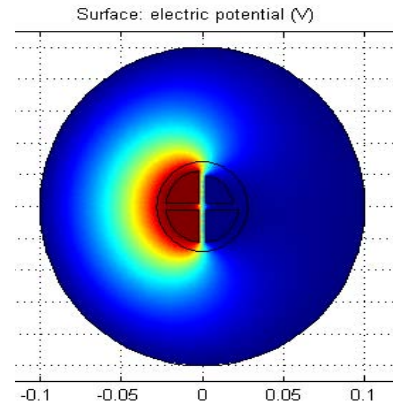


Figure 7: Voltage distribution with 1kV on conductor segments 1 and 2 with the other parts earthed. Red implying high end and blue the low end of the voltage scale

2.3 Complete set of capacitances

After performing the simulations involving the six different cases, the various capacitance values are connected to each other and the supplied voltage through a linear system of equations, as given in equation (4).

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} C'_{10} \\ C'_{20} \\ C'_{30} \\ C'_{12} \\ C'_{23} \\ C'_{31} \end{pmatrix} = \frac{1}{U^2} \begin{pmatrix} W_{e(1+)} \\ W_{e(1+2+)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{pmatrix} \quad (4)$$

For the cable geometry shown in Figure 1, the capacitance values are hence given by,

$$\begin{pmatrix} C'_{10} \\ C'_{20} \\ C'_{30} \\ C'_{12} \\ C'_{23} \\ C'_{31} \end{pmatrix} = \begin{pmatrix} 146 \\ 17.4 \\ 146 \\ 147 \\ 147 \\ 7.24 \end{pmatrix} \frac{nF}{km}$$

These values of the cable capacitance values and their validity is discussed in section 4.

3. Equivalent resistance and inductance

The inductance L' and the resistance R' in high voltage cables can be calculated using the module "AC-electromagnetic", which helps to account for skin-effect, as the cable consists of large diameter conductors and carries currents with high harmonics. The current density is hence not homogeneous. The cable parameters, depending on the frequency, are graphically represented usually. The case of thin screen surrounding the cable segments is not addressed in the present simulations.

The equivalent circuit is now reduced to a system containing capacitances and inductances, as shown in Figure 8.

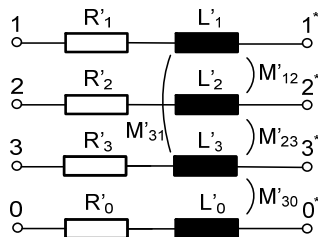


Figure 8: Equivalent circuit showing discrete values of longitudinal resistances and inductances

The equivalent circuit shown in Figure 1 is described by the system equations, given by (5):

$$U_k = (R_k + j\omega L_k)I_k + \sum_{\substack{l=0 \\ l \neq k}}^3 M_{kl}I_l \quad (5)$$

with $k = 0 \dots 3$, U_k the applied voltage on conductor k , resistance and inductance of conductor k being R_k and L_k , and the mutual inductances between conductor k and l being $M_{kl} = M_{lk}$, with $k, l = 0, 1, \dots, 3$.

Then the sequential supply of currents to the different conductor segments is carried out to estimate the voltages thus generated in the circuit given in Figure 8. The values of current is given using the menu „Options/Constants“ as $I_{_soll}$. With the additional equation, available under the menu „Physics/GlobalEquations“, the applied voltages $U_{_err2}$ and $U_{_err3}$ gives rise to the currents:

$$I_{_ist2} = \iint_{A_{ir}} S \, dA = -I_{_ist3} \quad (6)$$

Name (u)	Equation f(u,ut,utt,t)	Init
U_err3	-I_ist3-I_soll	0
U_err2	I_ist2-I_soll	0

Figure 9: Global Equations as used in the simulation of the circuit shown in Figure 8 for the cable configuration shown in Figure 5.

3.1 Currents in 2 and 3

The cable segments 2 and 3 are supplied with currents, so that $I_2 = I_{_soll}$. The equivalent circuit is shown in Figure 10.

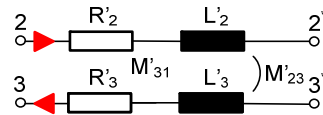


Figure 10: Equivalent circuit for the case involving current supply to conductors 1 and 2.

The current density field distribution for the case of segments 2 and 3 being supplied with currents of frequency 200 kHz with the cable geometry shown in Figure 3 is now shown in Figure 11. The current density over the cross section in the direction of the x-axis (red line) of the cable section 2 is also shown in the plot given Figure 11. As expected, the skin-effect is clearly seen as one move towards the edges of the conductor segment.

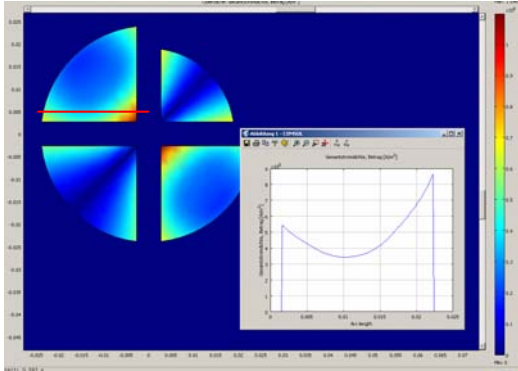


Figure 11: Magnetic field distribution and current distribution for the case of conductor 2 and 3 fed with currents

The results are obtained using the menu „Physics/SubmainSettings“ and the menu „Postprocessing/GlobalVariables“. The voltages U_{err2} und U_{err3} are hence obtained. The impedance of the conductor segment 2 is then calculated using

$$Z_{2-2^*} = \frac{U_{2-2^*}}{I_2} = R_2 + j\omega(L_2 - M_{23}) = \frac{U_{err2}}{I_{ist2}} \quad (7)$$

3.2 All inductances

For the system under study, we have 3 unknown resistances

$$R^T = (R_1 \quad R_2 \quad R_3)$$

And 10 unknown inductances,

$$L^T = (L_1 \quad L_2 \quad L_3 \quad M_{12} \quad M_{13} \quad M_{13} \quad \dots \quad M_{30})$$

Using 10 different current loadings, the impedances can be coupled to a linear system of equations for a given frequency.

4. Verification using analytical field theoretical methods

When the focus is on the verification of the results, independent theoretical estimates are useful. Simulation tools such as COMSOL Multiphysics are easy to handle and can help us to estimate and visualize various physical characteristics/phenomena, and deliver the results very fast, even for very for complex geometries. However results produced using such versatile tools should be verified using independent methods. The results of simulation using COMSOL

Multiphysics are analytically verified with different field theoretical considerations. The frequency dependent capacitance values are estimated using coupled capacitance models. The frequency dependent inductance values involving skin effect of the cable is estimated using partial wire method.

4.1 The capacitance Values

In an idealised version of the geometry, the cable segments could be represented by parallel plate capacitors, as the parallel surfaces of the segments facilitate such a simplification of the geometry. With the original dimensions of Figure 3, the twin capacitor system shown in Figure 12 evolves.

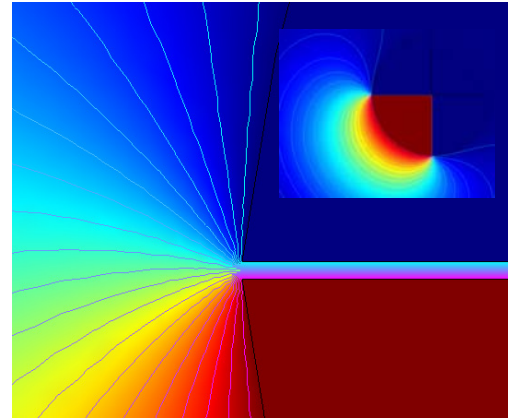


Figure 12: Idealised geometry with distance between segments 0.2 mm

The distance between the plates is then $d = 5.7mm$ with an area of $A = 21.2mm \times 1m$. The capacitance values for Comsol Multiphysics can be estimated using $W = \frac{1}{2}CU^2$. Fringe

fields and fields outside the plates in the simplified model, lead to a difference of -26% compared to capacitance values obtained with simulations. To compensate these effects, the distance between cable segments 2, 3, 4 is reduced to $d_{kontr} = 0.2mm$. The estimate is then made using the parallel plate capacitor model.

The energy intensity for the actual model WFemLab and parallel capacitor model WKont are found to be in the ratio

$$\frac{W_{Kontr}}{W_{FemLab}} = \frac{2.150 \cdot 10^{-3} \text{ Ws/m}}{2.205 \cdot 10^{-3} \text{ Ws/m}},$$

implying a good set of tallying values.

4.2 The Inductance Values

The verification of the frequency dependent inductance values follows using partial wire method [3], [4]. The bulk conductor is considered as a cluster of closely lying conductors as shown in Figure 13. Each conductor in such a set of clustered conductors has its own resistance and inductance and in addition mutual inductances to each and every other conductor in the cluster. The verification is now limited to conductor segments 2 and 3 only.

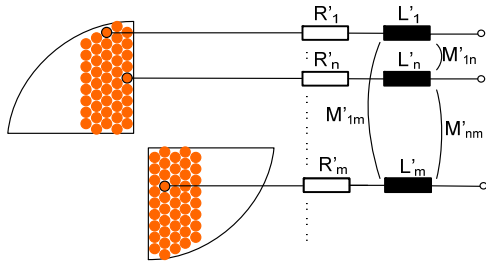


Figure 13: Bulk of conductor viewed as a cluster of snugly lying set of single conductors in line with partial wire techniques

The equivalent electrical circuit for the system shown in Figure 13 is described by the system equations below, with the standard notations adopted before, but modified to the partial wire system under discussion now..

$$U_k = (R_k + j\omega L_k)I_k + \sum_{\substack{l=1 \\ l \neq k}}^m M_{kl} I_l \quad (8)$$

with $k = 1 \dots m$ and $M_{kl} = M_{lk}$, written in matrix form as

$$U = Z I \quad \text{bzw.} \quad I = Z^{-1} U \quad (9)$$

The impedance matrix is then given by

$$Z = \begin{pmatrix} R_{11} + j\omega L_{11} & M_{12} & M_{13} & \vdots \\ M_{21} & R_{22} + j\omega L_{22} & M_{23} & \vdots \\ M_{31} & M_{32} & R_{33} + j\omega L_{33} & \vdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (10)$$

The inductance and mutual inductance for parallel thin conductors can now be estimated using well known formulae, e.g. [1], [2]. The necessary

calculations involving the matrices defined above can be done using MATLAB. The partial wire system for 2×93 conductors is given in Figure 14.

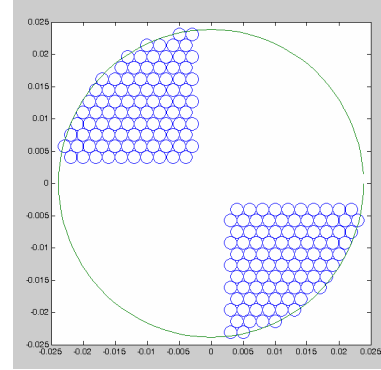


Figure 14: Distribution of the cluster of conductors for segments 2 and 3 of cable shown in Figure 3.

The current distribution at $f = 50\text{Hz}$ is given in Figure 15. For the sake of comparison, the conductivity of segment 0 and 1 is set to zero.

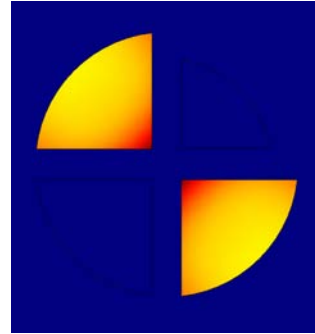


Figure 15: Current distribution when only conductor segments 2 and 3 are fed

Figure 16 shows the resistance and inductance values of the segments based on partial wire method and the values obtained with Comsol Multiphysics for the actual cable geometry (shown by the blue circles).

The difference in the resistance values can be attributed to non-conducting areas (filling factor of the circles with respect to the bulk conductor). This can be improved by improving the model.

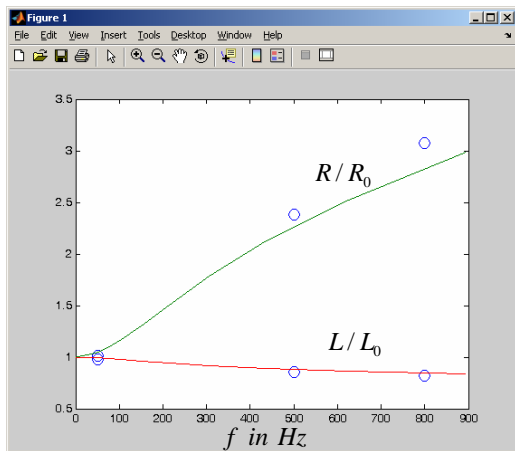


Figure 16: Inductance and resistance values estimated (continuous lines) using partial wire method compared to estimates using Comsol Multiphysics (circles)

5. Conclusions

Using Comsol Multiphysics program and the modules „Electrostatic“ and „AC Electromagnetic“ the impedance values of a three segment power cable are estimated for different frequencies. The estimated values are then compared with values obtained using partial wire methods for inductances and simplified ideal models for capacitances. The results obtained for the selected cable show that these values tally well. The results show that the need for electrical characterisation of the cables produced by cable manufacturers can be addressed to some extent satisfactorily using the techniques described in the paper, although some modifications and additional work are required to improve performance.

6. References

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