Mixing Layer Analysis in Variable Density Turbulent Flow

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Abstract: In this study, numerical simulations of mixing in turbulent flow, subject to a change in density, are performed. Attention is focused on the binary mixing between two streams of fluid in which a variable density step are formed due to a difference in the temperature. This binary mixing problem performed by assuming low Mach number flow. The results demonstrate the variable density effects and turbulent intensity on the mixing layer between the hot and cold streams of fluids.

Keywords: Thermal mixing layer, Variable density, low Mach number, weakly compressible.

1. Introduction

In many industrial applications and fluid flows, it is common to have a variable density process. These variations in density can be caused by different sources but the most common one is the presence of an inhomogeneous temperature field. As a result of that, many scientists studied the thermal mixing layer in fluid flow.

Costa-Patry and Mydlarski [1] studied the interaction of two passive scalars, two stream of air with different temperature, in fully developed turbulent channel flow experimentally. They analyzed the evolution and interaction of the two thermal fields by taking several measurements along the downstream distances in which the thermal mixing layer evolve. In their study they looked into the mean excess temperature, the RMS of temperature fluctuations, and the correlation coefficient. They found that the temperature fluctuations profiles depend on the separation distance between the line sources, and the downstream position of the measurement along the mixing layer.

Ferchichi and Tavoularis [2] examined the thermal mixing layer in uniformly sheared turbulent flow. The high velocity-half of the sheared layer was heated electrically to a temperature slightly grater than the other half of the layer to generate a thermal mixing layer in this flow. The impact of the shear layer on the thermal mixing were manifested by an asymmetry in the mean temperature profile, which indicate a deeper penetration of the thermal layer into the lower velocity side. They concluded that the growth rate of a thermal mixing layer is higher in sheared than in unsheared turbulence, as a result of enhanced growth of turbulent velocity and length scales by the shear.

Hirota el al. [3] did an experimental study on the control of turbulent thermal mixing of hot and cold airflows in a T-junction that simulates the HVAC unit for automobile air-conditioning system. To develop the thermal mixing layer, they injected hot flow by small jets into the main channel clod flow. The difference in turbulence intensity between the two flow can induce the the turbulent mixing between the hot and cold airflows and therefore the degree of thermal mixing can be controlled by changing the jet velocity.

LaRue and Libby [4] examined experimentally the temperature in a thermal mixing layer downstream partially heated turbulence grid. Special attention was paid to the mean temperature and temperature intensity profiles. They were able to obtain additional data of the thermal mixing layer such as the skewness and kurtosis of the temperature fluctuations along with the probability density distributions of the temperature.

As noted above, the thermal mixing layer is an interesting problem to consider especially when turbulent flow exist. In this study, numerical simulations of mixing in turbulent flow, subject to a change in density, are performed. Attention is focused on the binary mixing between two streams of fluid in which a variable density step are formed due to a difference in the temperature as shown in Figure (1). This binary mixing problem performed by assuming low Mach number flow, in which low speed, non-reactive species, and no heat generation or heat releases are applied..

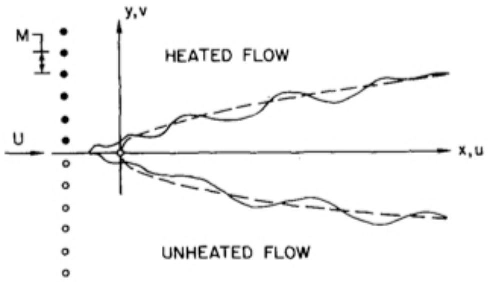

Figure 1. Schematic of the thermal mixing layer adapted from Ref. [5].

2. Use of COMSOL Multiphysics

The mixing layer will be examined in a 2-D space dimension in which the hot fluid will flow on the top of the cold fluid. The mixing layer will formed as the two streams of fluid flow from one side to anther using inflow and outflow boundary conditions. In this study I will use the turbulence flow k-ε model in additional to the variable density for non-isothermal flow model. The effect of changing the temperature ratio between the two streams and/or changing the turbulent intensity on the mixing layer will be examined.

The Chemical Engineering Module provides the Weakly Compressible Navier-Stokes application mode, which can be used to simulate variable density flows. By using the predefined multiphysics coupling we can combine the Weakly Compressible Navier-Stokes mode, along with the Convection and Conduction application modes. Using the Non-Isothermal Flow predefined coupling in the The Chemical Engineering Module, can help to simulate variable density flows due a deferential step in temperature. When the density become a function of the temperature, the momentum equations, continuity equation, and temperature equation can form a closely coupled system of equations. The Weakly Compressible Navier-Stokes application mode contains the fully compressible formulation of the continuity equation and momentum equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

$$\begin{aligned} \rho \frac{\partial \mathbf{u}}{\partial t} &+ \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p \\ &+ \nabla \cdot \left(\mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) - \left(\frac{2}{3} \mu \right) (\nabla \cdot \mathbf{u}) \mathbf{I} \right) \end{aligned}$$

The full compressible Navier-Stokes can be simplified and used to solve non-isothermal flow only if we assume flow with low Mach number. Mach number define the ratio between the flow speed to the speed of sound. In incompressible flow, the speed of sound tend to infinity and the Mach number become zero, which allow the Navier-Stokes equations to have a special numerical property that make any disturbance in fluid system to spread instantaneously thought the computation domain. As the Mach number approach unity, the solution for the compressible Navier-Stokes equations become complicated and hard to solve since the simple boundary conditions used for incompressible Navier-Stokes equations become invalid.

The Favre-Averaging method can be used with the Weakly Compressible Navier-Stokes mode to simulate turbulent in Non-Isothermal flows. The Favre-Average, which is a density based average, can be applied to Equation (1) to get:

$$\begin{aligned} &\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{u}_i) = 0 \\ &\bar{\rho} \frac{\partial \tilde{u}_i}{\partial t} + \bar{\rho} \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \\ &\frac{\partial}{\partial x_j} \left(\mu \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} - \overline{\rho u_j'' u_i''} \right) \end{aligned} \tag{2}$$

In Equation (2) the Favre-Averaged Reynolds stress tensor is modeled using the standard k-ε turbulent model

$$-\overline{\rho u_j'' u_i''} = \mu_T \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{2}{3} \left(\mu_T \frac{\partial \tilde{u}_k}{\partial x_k} + \bar{\rho} k \right) \delta_{ij} \tag{3}$$

where *k* is the turbulent kinetic energy of the flow. In this simulation the turbulent intensity will be controlled instead of the turbulent kinetic energy to examine the effect of the turbulent flow on the thermal mixing layer.

3. Approach

In this numerical experiment, we focused on examining the effect of increasing the initial temperature step between the hot and cold streams, and varying the inlet turbulent intensity for the flow. To do that we made seven numerical simulations as noted in Table (1). The variation in temperature steps help us to examine the effect of the variable density in this analysis. Since both streams of fluid were chosen to be air, the variations in the temperature steps considered in our simulation can help us to examine variable density flows with up to a density ratio of 10.

The turbulent intensity of the fluid flow can also play an important role in developing the mixing layer. Turbulent intensity basically define the ratio of the velocity fluctuation to the mean flow velocity. Usually 10-20% turbulent intensity represent moderate turbulent flow, while 1-10% represent low to medium strength turbulent flow.

Table 1: Numerical simulations description.

Case Number	Turbulent Intensity	Temperature Step	Density Ratio
1	10%	250 [K]	2
2	10%	700 [K]	4
3	10%	1250 [K]	7
4	10%	1650 [K]	10
5	1%	250 [K]	2
6	5%	250 [K]	2
7	20%	250 [K]	2

4. Results and Discussion

To analyze the results for the above cases on the thermal mixing layer development, we need first to defined a normalized temperature as:

$$T_{\text{Normalized}} = \frac{(T - T_{\text{High}})}{(T_{\text{High}} - T_{\text{Low}})} \tag{4}$$

The effect of the change of temperature steps are shown in Figures (2), (3), and (4). The contour plots for the mixing layer development, Figure (2), shows that the increase in the temperature step force the mixing layer to shift toward the hotter stream side. Figures (3), and (4) show the development of the mixing layer by examining the layer at different cross section locations. By comparing these two figures we can see the impact of increasing the temperature step on the spreading rate and shifting of the mixing layer.

On the other hand, the turbulent intensity can also affect the development of the thermal mixing layer as shown in Figures (5), (6), and (7). By increasing the turbulent intensity, the layer width become quickly wider since the high velocity fluctuation helps to spread the temperature faster between the hot and the cold side of the thermal mixing layer.

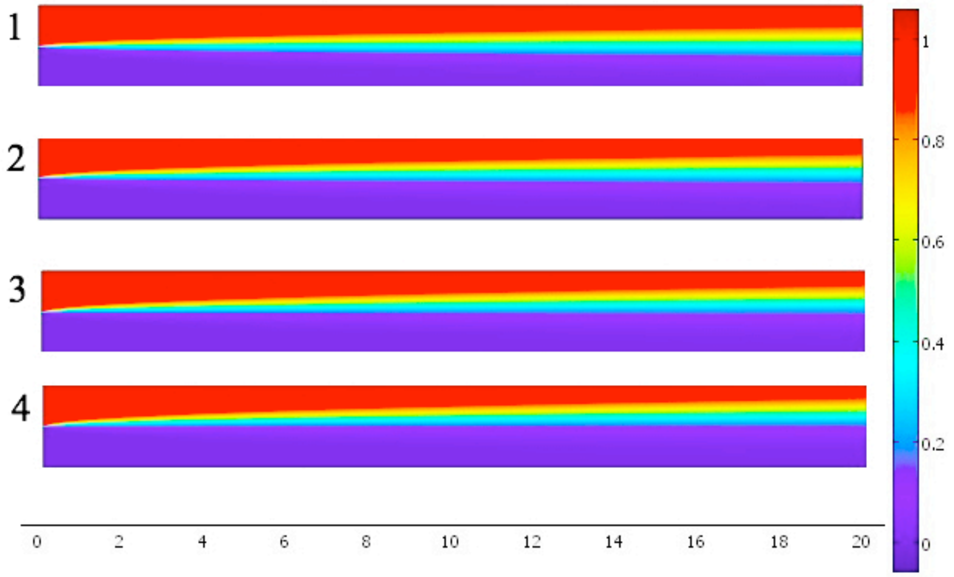

Figure 2. Contours plots for the normalized temperature for cases #1, 2, 3, and 4.

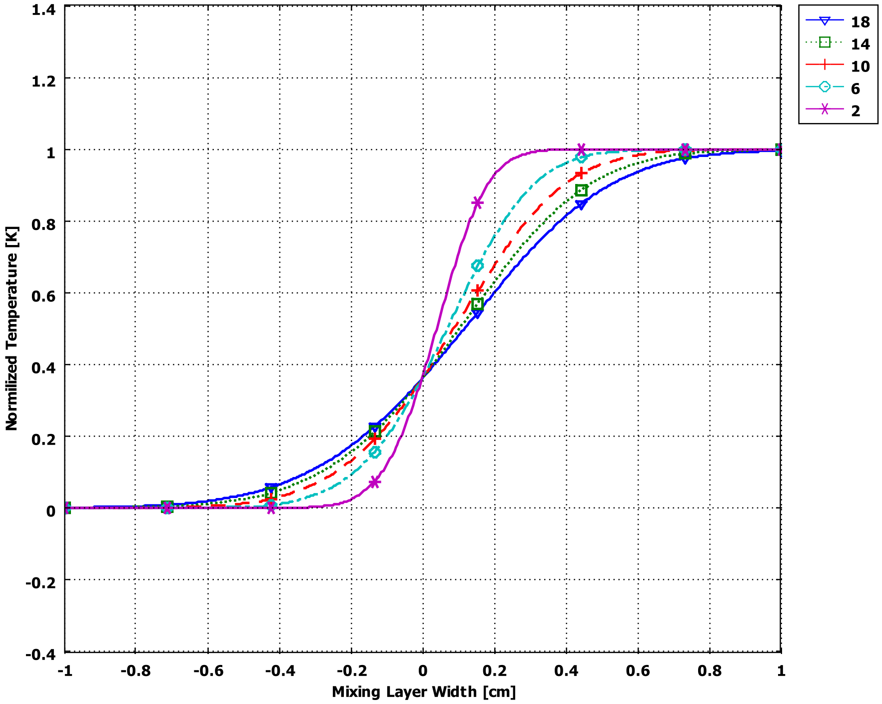

Figure 3. Thermal mixing layer development at several down steam locations for case #1.

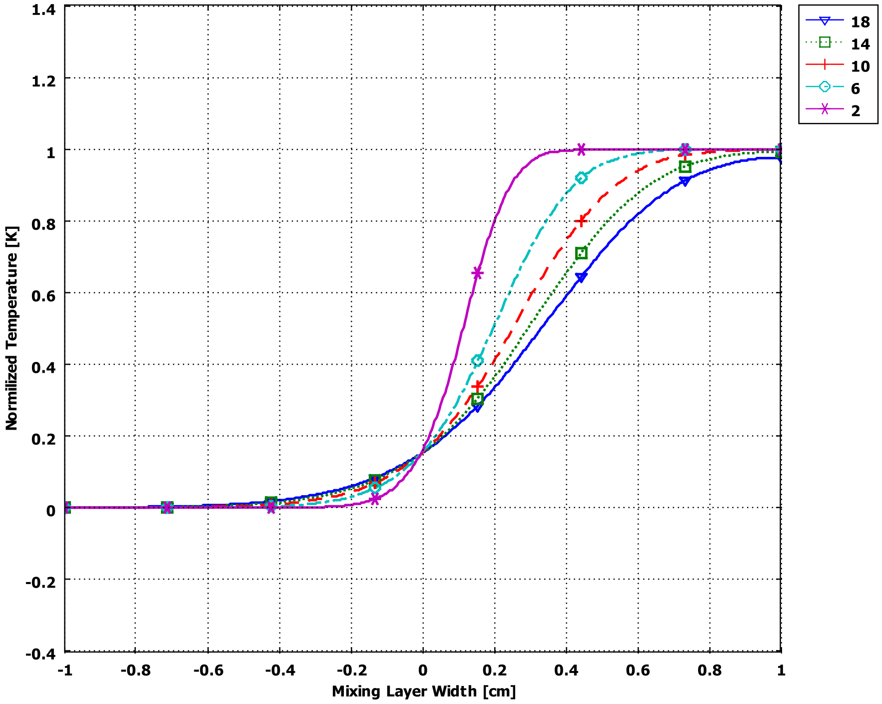

Figure 4. Thermal mixing layer development at several down steam locations for case #4.

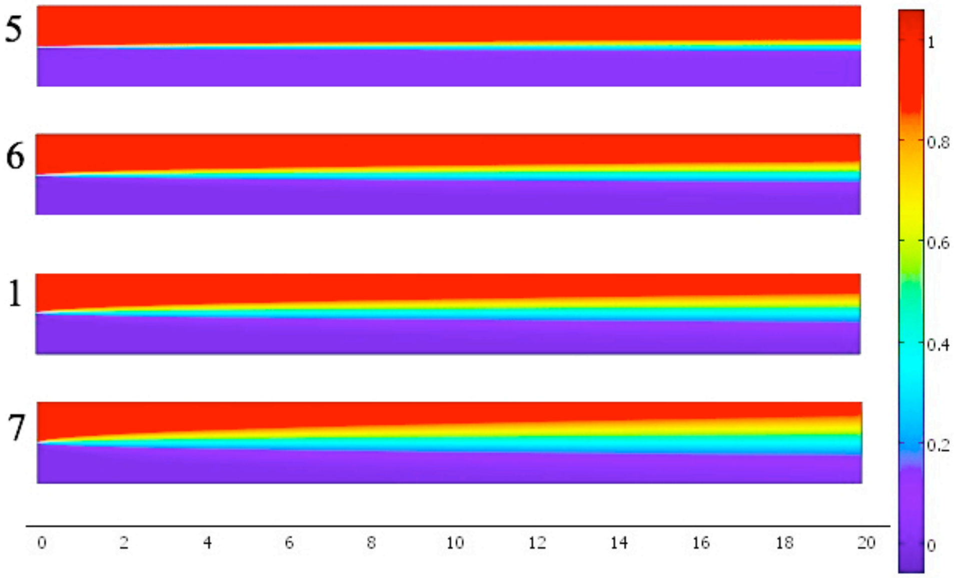

Figure 5. Contours plots for the normalized temperature for cases #5, 6, 1, and 7.

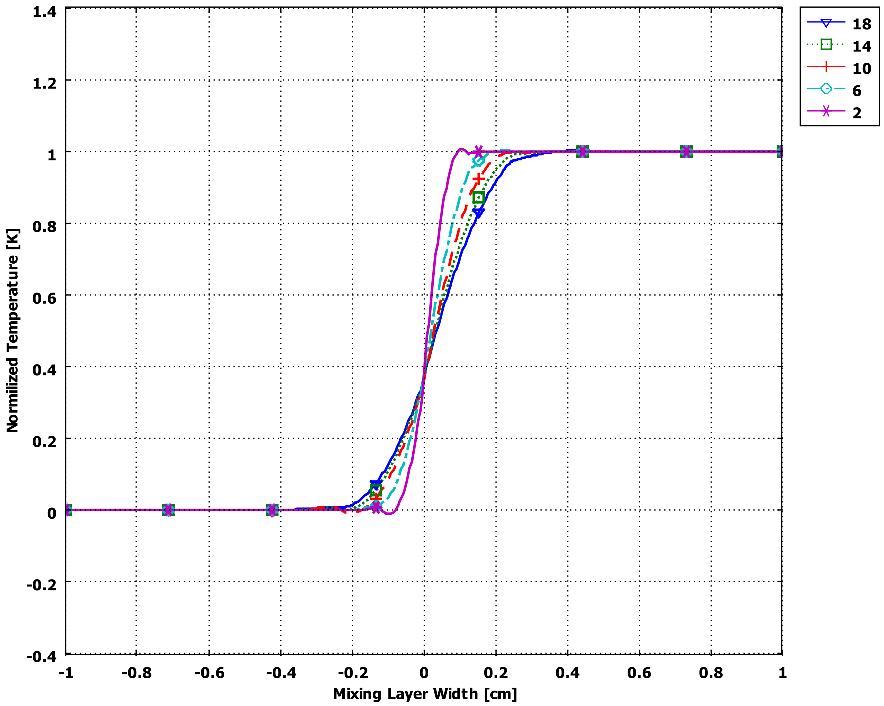

Figure 6. Thermal mixing layer development at several down steam locations for case #5.

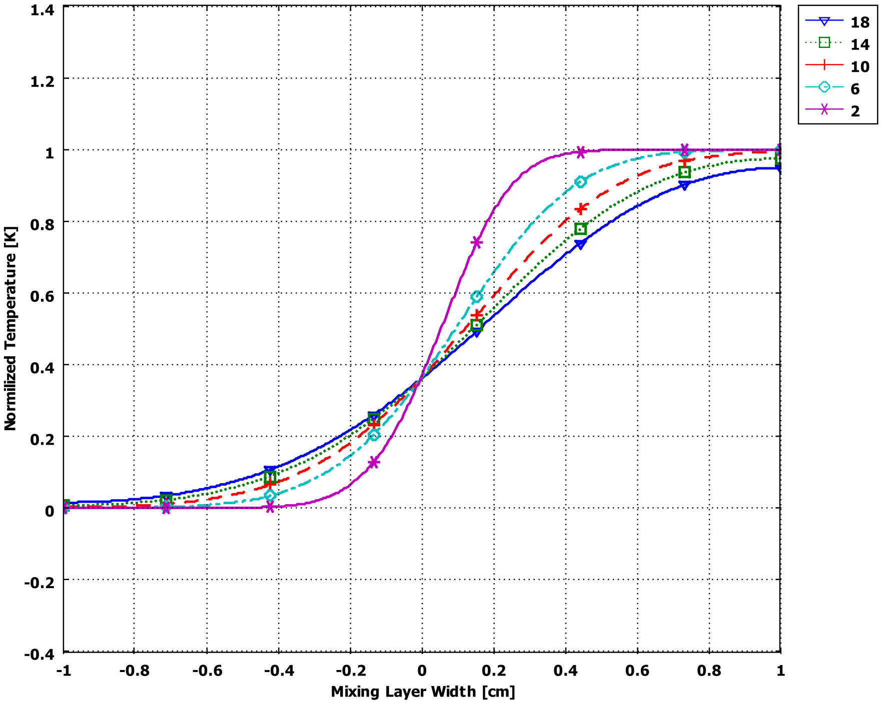

Figure 7. Thermal mixing layer development at several down steam locations for case #7.

5. Conclusions

Simulations for thermal mixing layer was successfully computed by using the Weakly Compressible Navier-Stokes application mode in COMSOL Multiphysics. Simulations with low Mach number enable us to simplify the compressible Navier-Stokes equations to simulate the variable density turbulent flow. The Favre-Averaging method was used with the Weakly Compressible Navier-Stokes mode to simulate turbulent in Non-Isothermal flows and the standard k-ε turbulent model helped to model the Favre-Averaged Reynolds stress tensor.

The temperature step difference between the hot and cold air streams, and the turbulent intensity can significantly affect the spreading rate of the thermal mixing layer. Increasing the temperature step lead the mixing layer to shift toward the hot stream, while increasing the turbulent intensity make the layer width spread faster.

6. References

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