



**POLITECNICO DI BARI**  
**I FACOLTA' DI INGEGNERIA**  
**DIPARTIMENTO DI INGEGNERIA MECCANICA E GESTIONALE**  
**SEZIONE MACCHINE ED ENERGETICA**

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***A NOVEL FEM METHOD FOR PREDICTING  
THERMOACOUSTIC COMBUSTION  
INSTABILITY***

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# Combustion Instability

Improvement of gas turbine performance

**Efficiency**

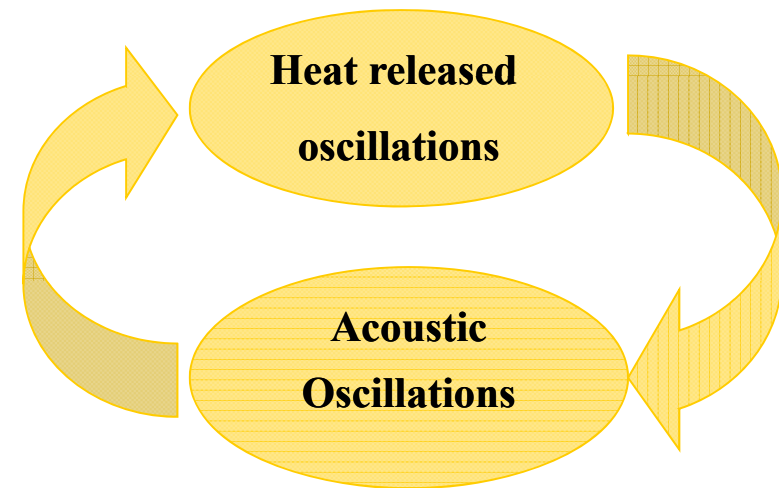
**Emissions**

annular combustors

fuel premix

Larger tendency to the creation of thermoacoustic instability

- excessive noise
- mechanical failures
- flame detaching and misfiring
- production of emissions





# Objectives

- Introduction of a mathematical model able to study acoustics and heat released oscillations;
- Resolution of this mathematical model through FEM technique in order to identify the instability conditions of the system examined;
- Demonstrate the capability of COMSOL Multiphysics to accurately solve eigenvalue problems in the analysis of combustion instability applied to complex geometries.



# Mathematical Model

With regards to a compressible and viscous fluid, the conservation equations of mass, momentum and energy can be written as follows:

$$\frac{D\rho}{Dt} + \rho \nabla \mathbf{u} = 0 \qquad \rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \frac{\partial \sigma_{i,j}}{\partial x_j} \mathbf{e}_i$$

$$\rho \frac{D}{Dt} \left( e + \frac{1}{2} u^2 \right) = -\nabla \cdot (p\mathbf{u}) + q + \nabla \cdot (k\nabla T) + \frac{\partial}{\partial x_j} (\sigma_{i,j} u_i)$$

When a fluid is considered non viscous, a gas perfect and mean flow negligible, the linear forms of the previous equation brings to the inhomogeneous wave equation:

$$\frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} - \nabla \cdot \left( \frac{1}{\rho} \nabla p' \right) = \frac{\gamma - 1}{c^2} \frac{\partial q'}{\partial t}$$

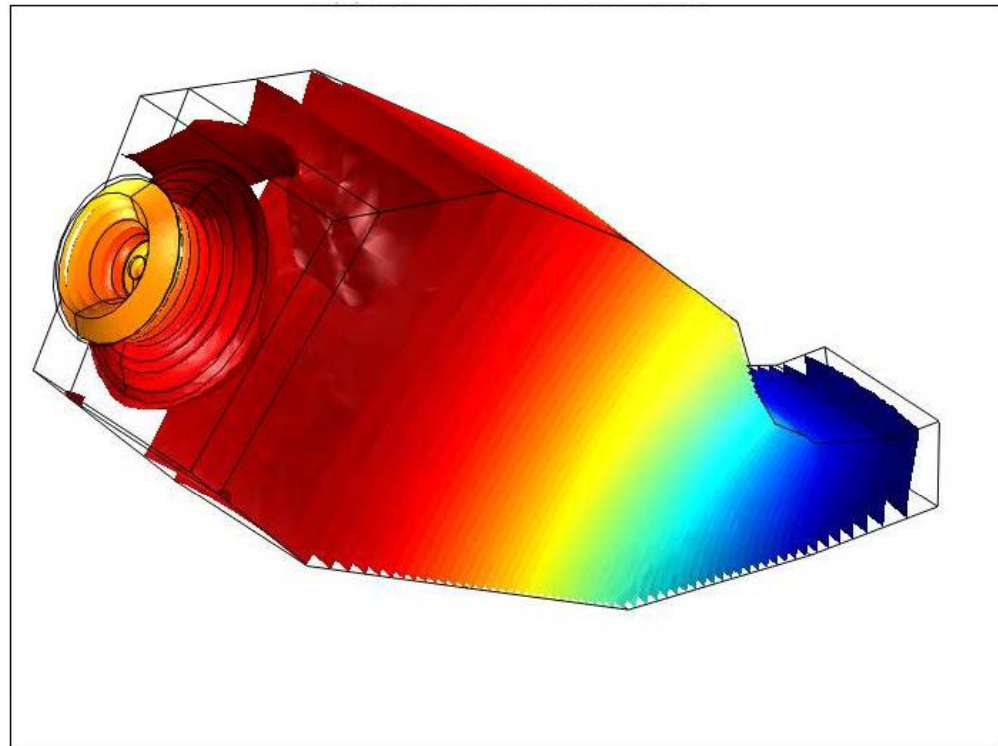
Heat release rate per volume unit [W/m<sup>3</sup>].



# COMSOL Multiphysics

In COMSOL Multiphysics the *Acoustics Module* is the module adopted in the present analysis.

The application mode used, *Pressure Acoustics*, is able to find the complex eigenfrequencies of the system.





# Subdomain Settings

Air has been described introducing:

- *Fluid density;*
- *Speed of sound.*

Heat release has been described introducing:

- *Monopole source*

in agreement with the heat release law imposed.



# Boundary Settings

Solid walls are modelled with *Sound Hard Boundary*.

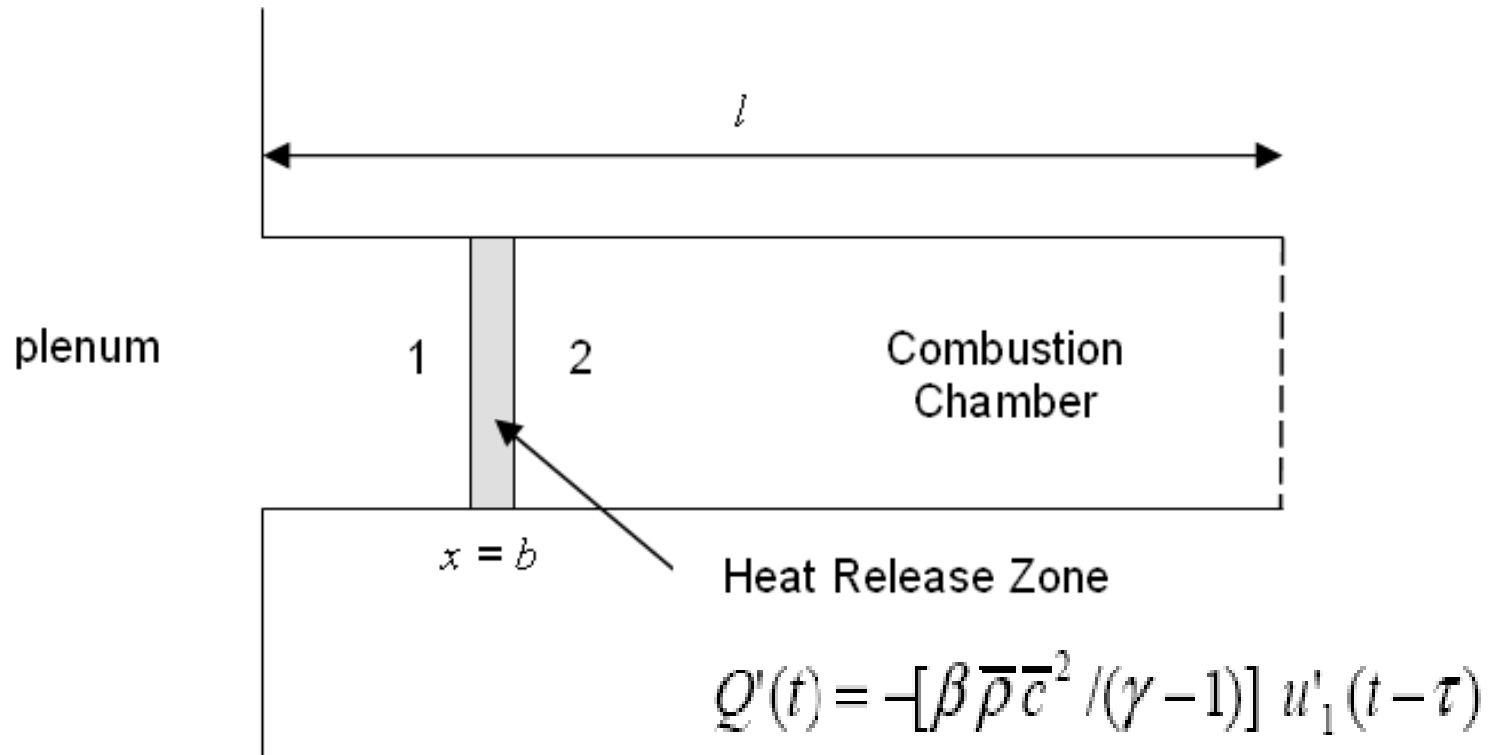
Inlet & Outlet wall are modelled with one of the following conditions:

- *Sound Hard Boundary*;
- *Sound Soft Boundary*;
- *Normal Acceleration*.

The right condition is defined in agreement with the examined test.



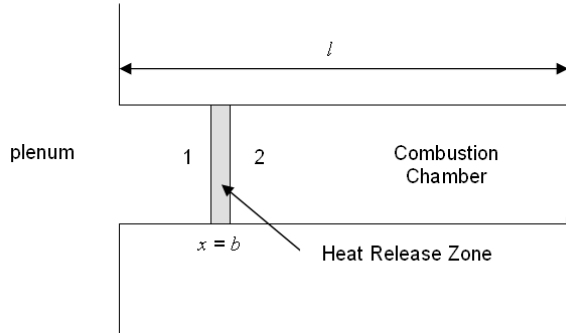
# Preliminary Tests on Linear Combustion Chamber





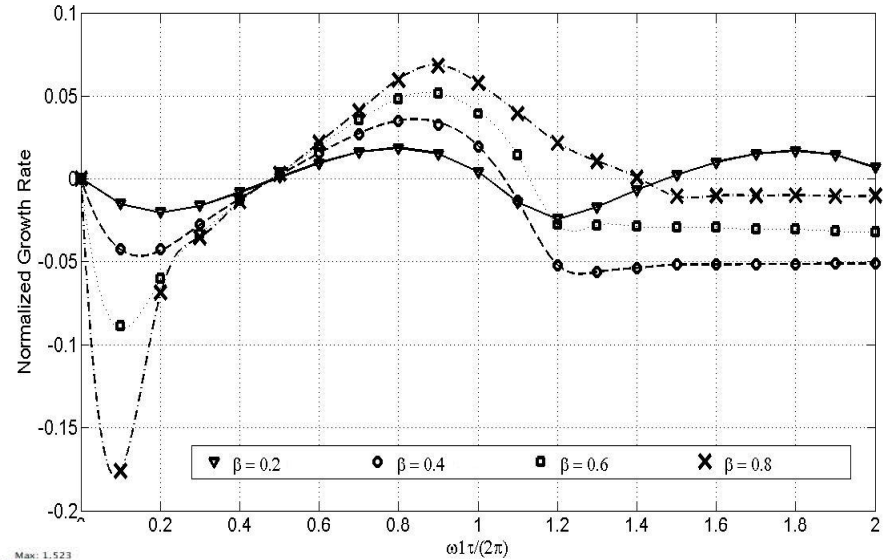


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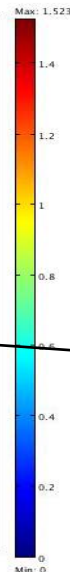
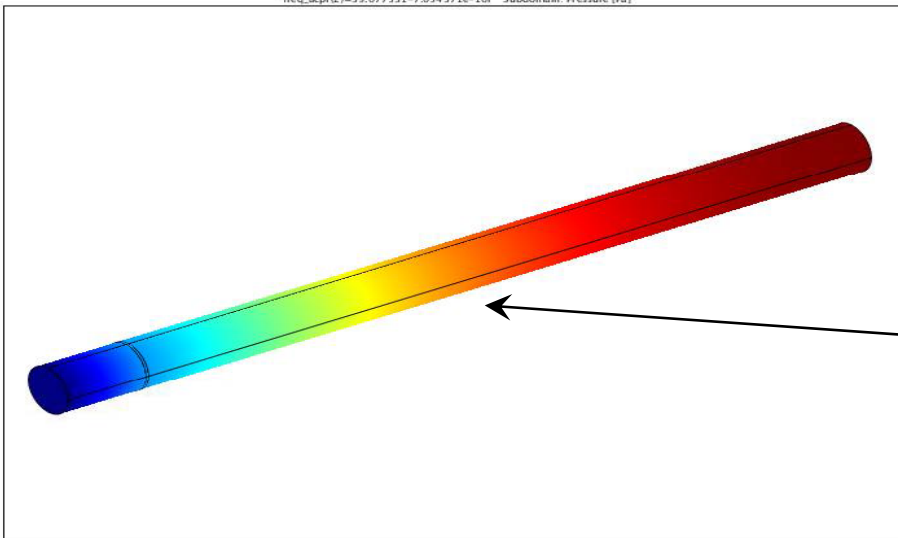


**Analytical solution is available:**

$$\tan\left(\frac{\omega}{c}b\right) \tan\left[\frac{\omega}{c}(l-b)\right] = 1 - \beta \exp(-i\omega\tau)$$



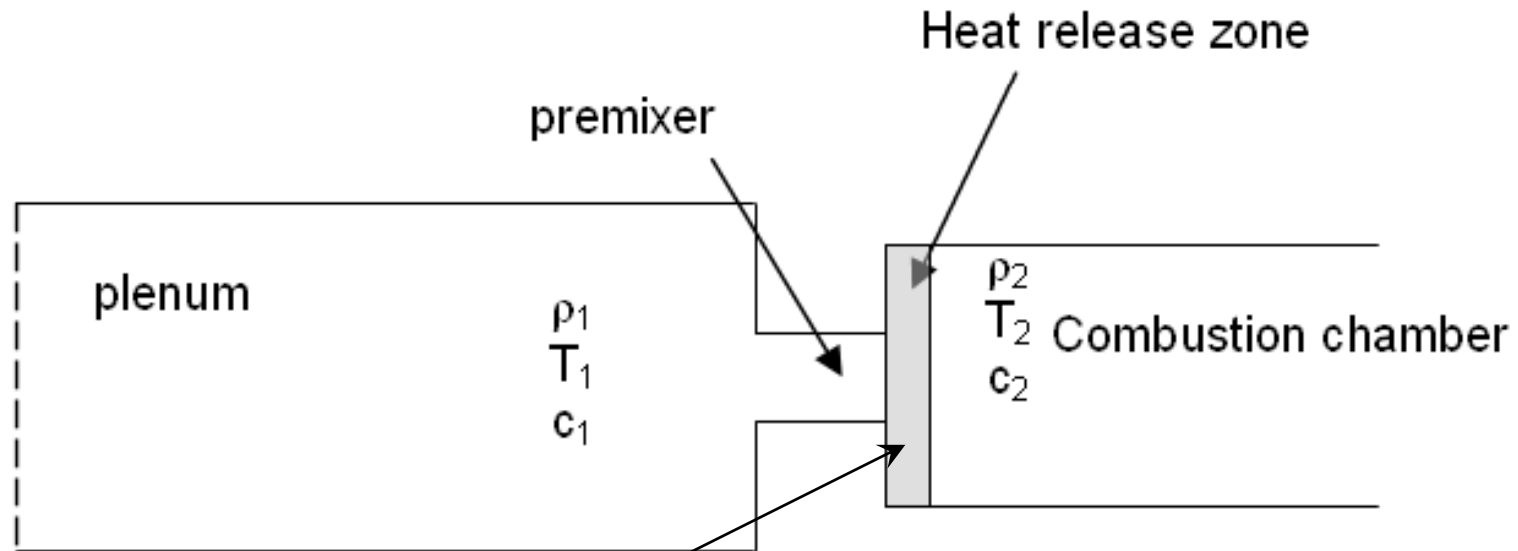
freq\_acpri(2)=39.077531-7.054371e-16i Subdomain: Pressure [Pa]



**First Axial Mode**  
**Pressure Wave**



# Tests on Linear Combustor with variation of section

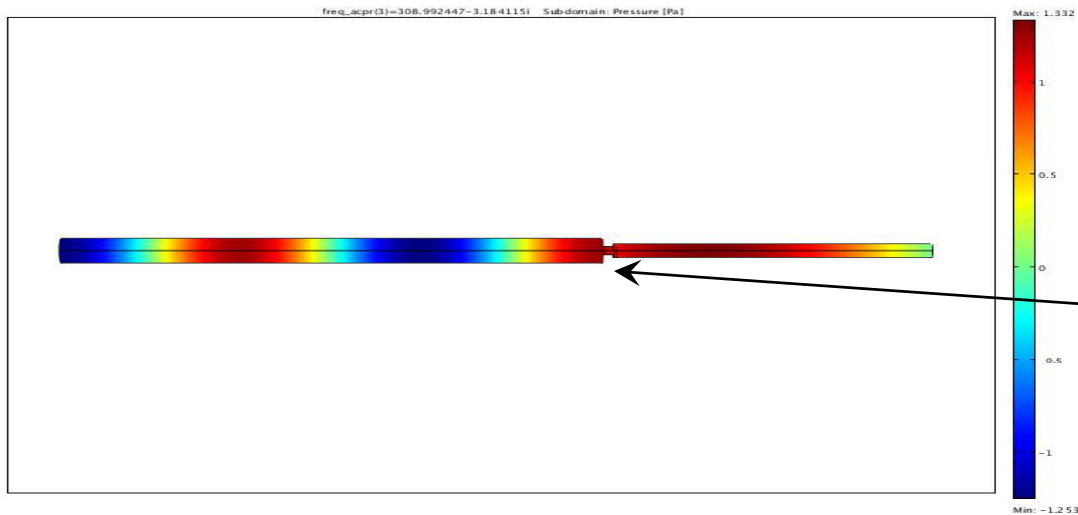
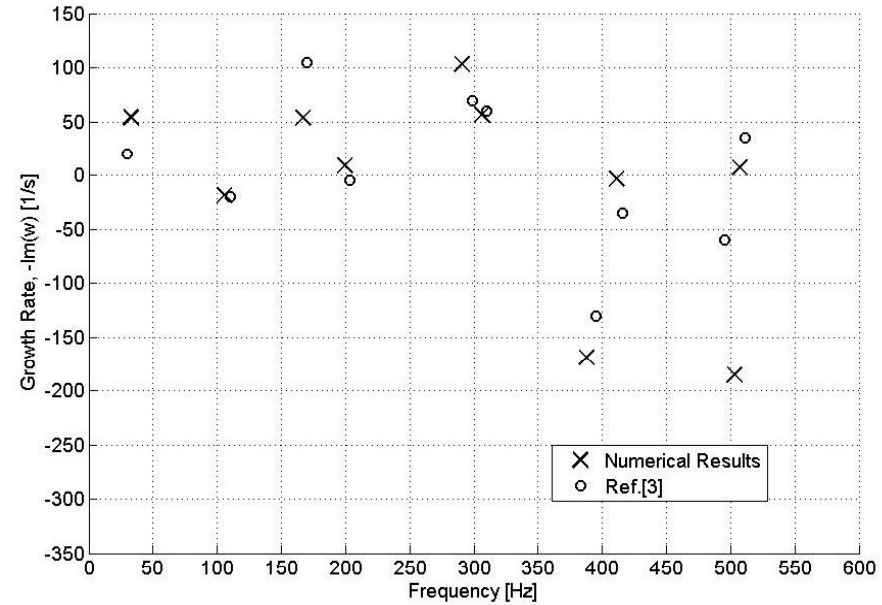
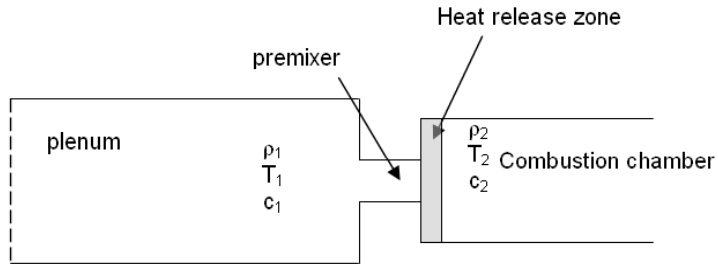


$$\frac{\hat{Q}}{Q} = -k \frac{\hat{m}_i}{\bar{m}_i} \exp(-i\omega\tau)$$

Time delay



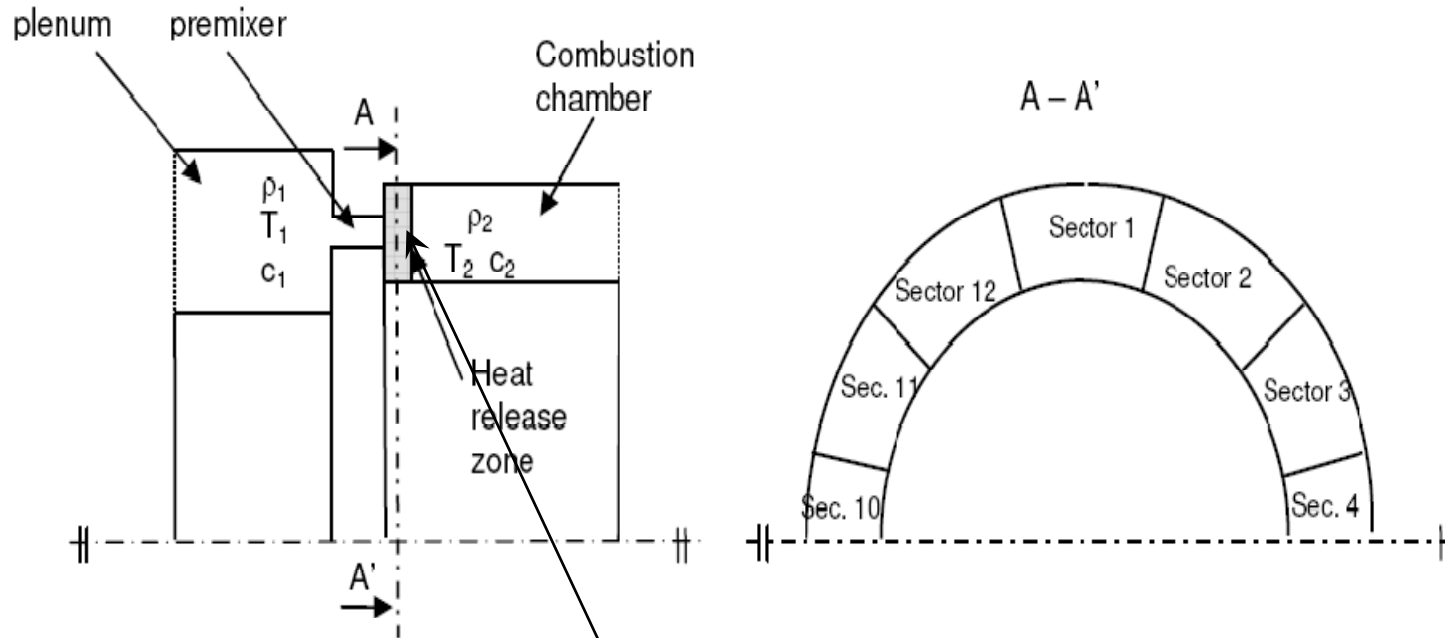
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**Axial Mode**  
**Pressure Wave**



# Tests on Annular Combustion Chamber

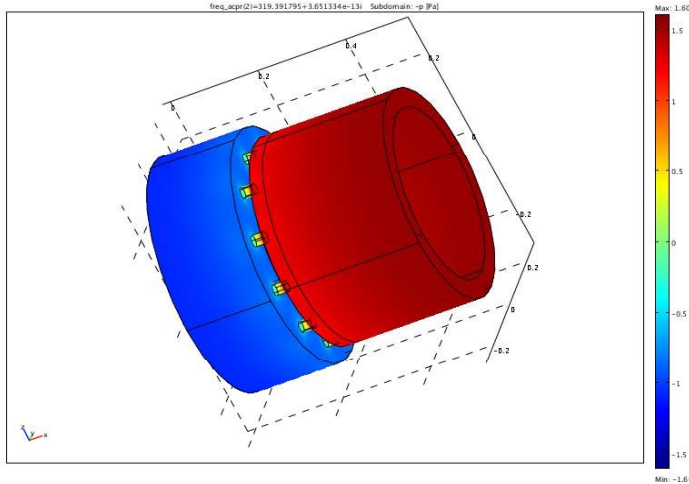


$$\frac{\hat{Q}}{Q} = -k \frac{\hat{m}_i}{m_i} \exp(-i\omega\tau)$$



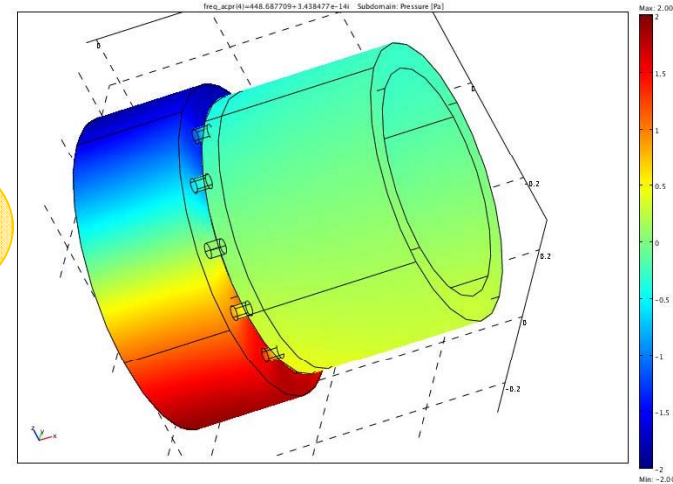
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(1,0,0)



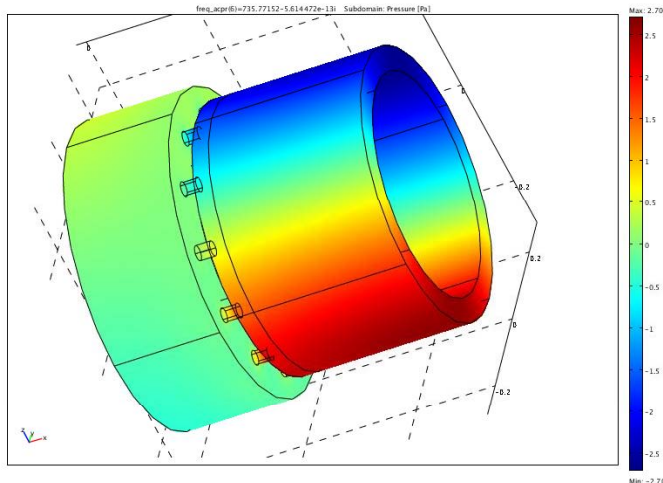
**Unstable for  $\tau > 0.002$  s**

(0,1,0)



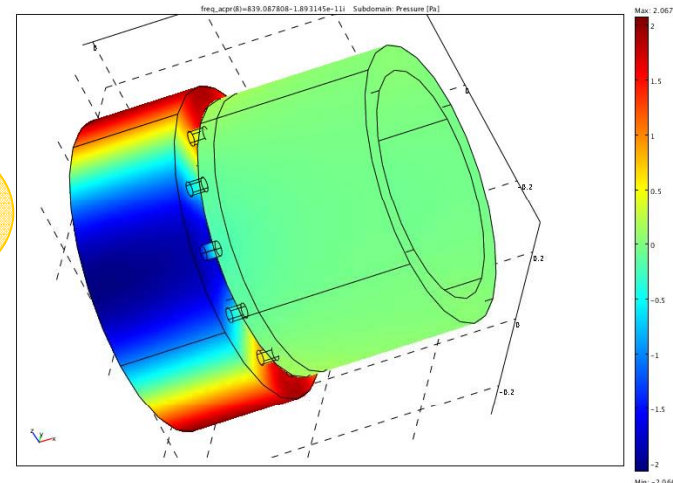
**Unstable for  $\tau = 0.001$  s  $0.003$  s  $0.005$  s**

(1,1,0)



**Unstable for  $\tau < 0.002$  s and  $\tau > 0.004$  s**

(0,2,0)



**Unstable for  $0.003$  s  $< \tau < 0.006$  s**



# Conclusions

- The mathematical model introduced has been successfully solved;
- COMSOL Multiphysics provides sufficient level of accuracy in the identification of stable and unstable eigenmodes;
- The FEM analysis has been successfully applied to different kinds of heat release law and different boundary conditions;
- The present approach is appropriate to treat complex geometry;
- COMSOL Multiphysics provides a commercially available tool that can analyze combustion instability problem.



# Further Applications

The method can be applied to analyze the effects of:

- Passive damping devices;
- Geometry of the system;
- Flame response functions;
- Transfer function matrices of the burners.



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Thank you  
for attention

Questions?