Coupled Magnetic-Structural Finite Element Analysis

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Abstract: In general, there is a wide range of applications requiring coupled electromagnetic-structural analysis. In this paper, two of such coupled problems have been analyzed. The first one is a transient analysis of Electromagnetic Forming process and the other problem is a deformation analysis of a coil in a static electromagnetic field. These problems are simulated in COMSOL using AC/DC module and Structural-Mechanics module. Simulation results obtained in the first problem show good matching with experimental results. The results in the available literature are used to validate the latter problem.

Keywords: Electromagnetic Forming, coupled magnetic-structural analysis, COMSOL, Finite Element Method

1. Introduction

For some complex problems, it is difficult to get analytical or closed form solution. Such problems can be easily solved using numerical techniques. Numerical methods have come into prominence and have become more attractive with the advent of fast digital computers. The Finite Element Method (FEM) is a powerful numerical technique that has been used to solve a variety of problems in thermal, electromagnetic, fluid and structural mechanics domains.

Coupled magnetic-structural analysis finds many applications like electromagnetic forming, MEMS, etc. Noise and vibration analyses of electrical machines may also fall under the gamut of the coupled analysis.

Most of the coupling models treat the coupled problem in two steps: first the electromagnetic field equations are solved to get forces, and then the structural equations are solved to get deformations. In these models the time step should be small to get accurate results. This is generally known as sequential coupling or weak coupling. The second approach involves solving the magnetic and structural equations simultaneously, which is a strong coupling approach. In this paper, two problems have been analyzed using a strong magnetic-structural coupling option available in COMSOL. The two problems analyzed are viz. the simulation of electromagnetic forming process and the deformation analysis of coil in a static electromagnetic field.

2. Coupled Magnetic-Structural Formulation

The time-dependent magnetic diffusion equation, governing the electromagnetic field in terms of magnetic vector potential (**A**) is

$$\nabla\times\frac{1}{\mu}(\nabla\times\mathbf{A})=\mathbf{J}_0-\sigma\frac{\partial\mathbf{A}}{\partial t}+\sigma(v\times\nabla\times\mathbf{A})\text{.........}(1)$$

where σ, μ, v and $\mathbf{J_0}$ are conductivity of the material, permeability of the material, velocity and current density respectively.

Since the problem is axi-symmetric with the magnetic vector potential (A_Φ) in Φ-direction, Eq. (1) becomes

$$\frac{\partial}{\partial r}\left(\frac{1}{\mu r}\frac{\partial}{\partial r}(rA_\phi)\right)+\frac{\partial}{\partial z}\left(\frac{1}{\mu r}\frac{\partial}{\partial z}(rA_\phi)\right)+J_\phi$$
$$-\sigma\frac{\partial A_\phi}{\partial t}+\sigma\left(\frac{\partial x}{\partial t}\times B\right)_\phi=0\text{............}(2)$$

where x and B are the displacement and flux density respectively. With the application of Galerkin finite element method to Eq. (2), the global system of equations can be obtained as

$$[T]\left\{\frac{\partial A_\phi}{\partial t}\right\}-[S]\{A_\phi\}=\{J\}\text{.....................}(3)$$

where

$$[T]=\iint\left(\sigma r[N]^T[N]\right)drdz$$

$$[S]=\iint\frac{1}{\mu r}\left(\frac{\partial}{\partial r}r[N]^T\frac{\partial}{\partial r}(r[N])+\frac{\partial}{\partial z}r[N]^T\frac{\partial}{\partial z}(r[N])\right)drdz$$

$$\{J\} = \iint r[N]^T \left(\sigma \left(\frac{\partial x}{\partial t} \times B \right)_\phi + J_\phi \right) drdz$$

and N is the element shape function.

The finite element equation of motion of a body is obtained by applying the principle of virtual work along with D'Alembert principle [1]:

$$[M]\frac{\partial^2}{\partial t^2}\{x\} + [K]\{x\} = \{F\} \qquad (4)$$

where

$$[M] = \iint 2\pi r [N]^T \rho [N] drdz$$

$$[K] = \iint 2\pi r [C]^T [D][C] drdz$$

$$\{F\} = \iint 2\pi r \left\{ [N]^T (\mathbf{J} \times \mathbf{B}) \right\} drdz$$

in which $[M]$ is the mass matrix, $[K]$ is the structural stiffness matrix, $\{F\}$ is the vector of nodal Lorenz force, ρ is the density of the conductor, $[C]$ is the displacement-strain matrix, and $[D]$ is the elasticity matrix. Eq. (3) and Eq. (4) are solved simultaneously to determine magnetic vector potential and mechanical deformation with time.

3. Case Studies

3.1 Electromagnetic Forming

Electromagnetic forming offers a number of advantages over conventional forming methods and finds applications in a number of areas. It is used to join and shape metals with precision and rapidity, and that to without the heat effect and tool marks associated with other methods. The principle is based on the plastic deformation of high conductive materials under electromagnetic pressure created by a fast changing magnetic field. This field is obtained by discharging energy stored in capacitor bank to forming coil with the help of a spark gap [2]. The schematic of the forming process is shown in Fig. 1.

Closing of the spark-gap by the application of trigger pulses creates a damped sinusoidal current in the coil for few microseconds due to discharge of the energy stored in L-C circuit. The corresponding induced currents in the work-piece interact with flux density to produce Lorentz forces leading to deformation. The axi-symmetric model of the electromagnetic system used in the analysis is shown in Fig. 2. The final deformation of the work piece obtained experimentally is shown in Fig. 3.

Specifications of coil and work piece are given in Table 1. 2-D strongly coupled magnetic-structural finite element analysis is carried out using AC/DC module and Structural-Mechanics module in COMSOL. The applied exponentially decaying current is [3]

$$i(t) = 116260e^{(-210t)} \sin(2\pi \times 8761.2 \times t) \qquad (5)$$

To analyze the deformation of the work piece, a transient analysis is carried out for 500 µs.

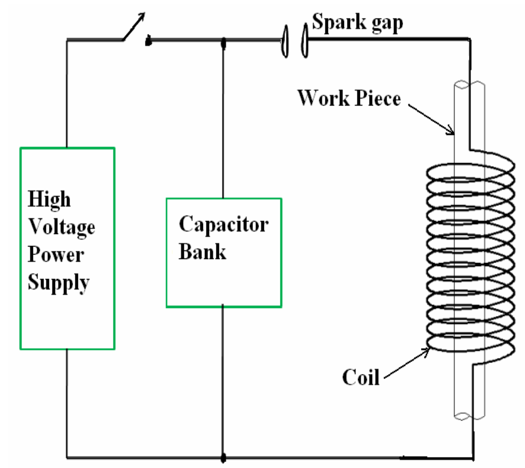

Figure 1. Schematic of Electromagnetic Forming equipment

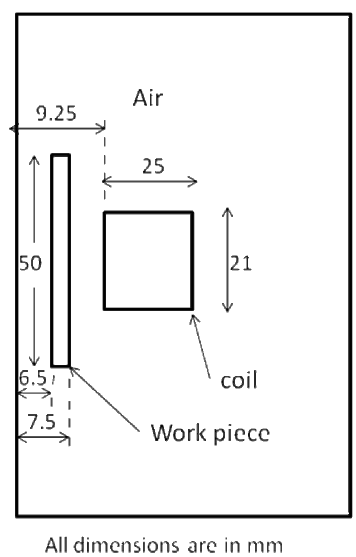

Figure 2. Geometry of the forming system

Table 1: Specifications of coil and work piece

Item	**Work-piece**	**Coil**
Inner radius	6.5 mm	9.25 mm
Outer radius	7.5 mm	34.25 mm
Length	50 mm	21 mm
Material	Aluminum	Copper

The 2-D surface plot of deformation at 500 μs is shown in Fig. 4. The comparison of deformations obtained from simulation and experiment is given in Table 2; the two values match closely.

Figure 3. Deformation obtained experimentally [3]

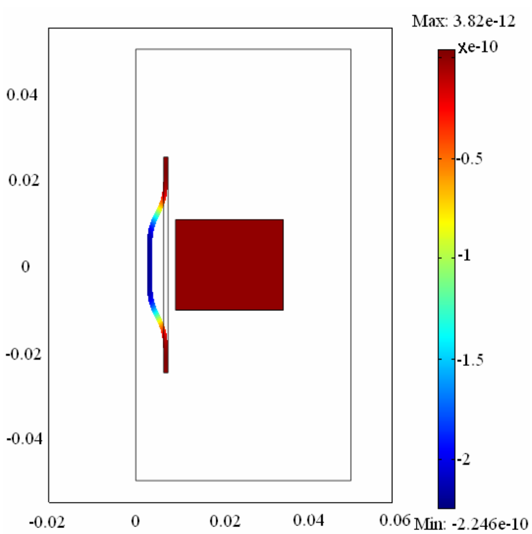

Figure 4. 2-D surface plot of deformation

Table 2: Comparision of results

	Maximum deformation
Simulation	3.85 mm
Experiment	3.80 mm

3.2 Deformation analysis of a coil

Noise and vibrations are the two aspects to be considered while designing electrical machines. The mechanical behaviour of coils under the action of magnetic field needs to be analyzed. Euxibie et al. performed a steady state analysis for deformation of a non-magnetic conductor under the electromagnetic forces using finite element method with strong coupling [4]. Hirsinger and Billardon conducted a similar analysis on ferromagnetic materials with magnetostriction effects [5].

Here, the deformation of a current carrying coil in a steady magnetic field is analyzed using coupled magnetic-structural formulation. The axi-symmetric model of the problem is shown in Fig. 5. The air region is represented by a rectangle with sides sufficiently larger than the radius of the coil. This ensures that the boundary conditions have negligible effect on the investigated field phenomenon.

A solenoid coil with uniform current density ($5e+6$ A/m^2) is considered in this analysis. In the structural analysis, the coil is modeled as a homogeneous, isotropic material with specific values of modulus of elasticity E and poissons ratio ν. The resultant deformation of the coil in the steady magnetic field is shown in Fig. 6. The comparision of results obtained from the simulation with that reported in [4] is given in Table 3.

Table 3: Comparision of results

Results	**Max. Flux density, (T)**	**Max. Force Density, (N/m^2)**	**Max. Deformation, (m)**
COMSOL	0.05	2.50e5	1.775e-9
[4]	0.04877	1.946e5	1.923e-9

4. Conclusion

In this paper, coupled electromagnetic-structural finite element formulation is discussed and two problems have been analysed using COMSOL as case studies. In the first problem, a

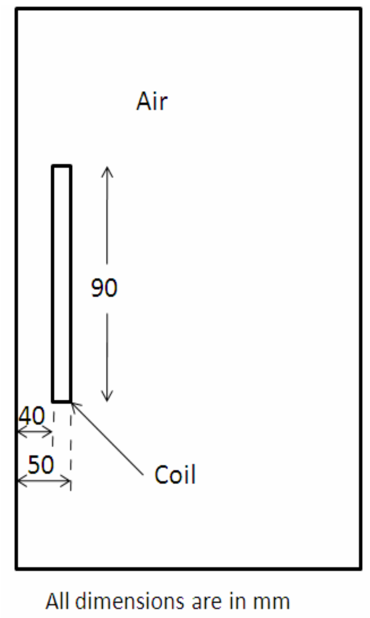

Figure 5. Problem model [4]

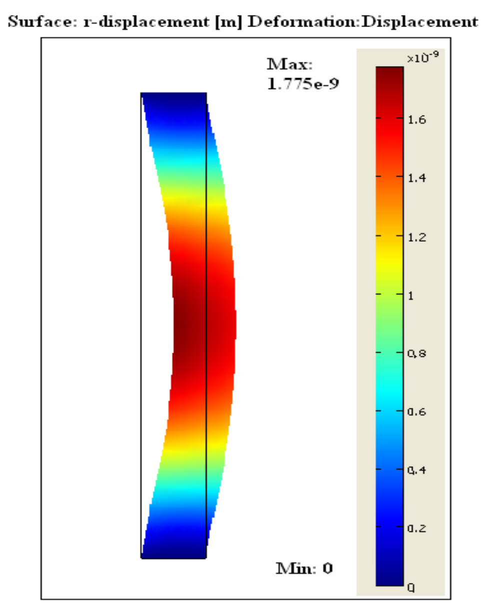

Figure 6. Deformation in coil

transient analysis of electromagnetic forming process is carried out to obtain deformation of the work piece. The deformation obtained from the simulation shows good agreement with experimental results.

In the second case, mechanical deformation of a current carrying coil under the influence of static magnetic field is analysed. The results obtained from the simulation are compared with those reported in the literature.

8. References

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