Boundary Arbitrary Lagrangian-Eulerian and Deformable Boundary Perturbation Method

J. Rivero-Rodriguez[1], B. Scheid[1]

[1]Université Libre de Bruxelles, Brussels, Belgium

Abstract

Partial differential equations on unknown domains is a mathematical problem that arises in many applications. Several examples can be easily found in the dynamics of free surfaces in fluid dynamics or in fluid-structure interactions. In many situations, large deformations are encountered and in many others, large deformation analysis can be avoided with the help of some analysis which rely on a small deformation of the boundaries of the domain. These are the sensitivity analysis of an objective function, small perturbation of a parameter of the problem or stability analysis among others. To tackle the former kind of problems in arbitrary geometries and a systematic manner, we propose a mathematical approach to track the surface, the Boundary Arbitrary Lagrangian-Eulerian method (BALE). To handle with the latter kind of problems, we propose an approach to perturbe the domain and write the partial differential equation and boundary conditions on the unperturbed geometry. We name this method as the Deformable Boundary Perturbation (DBP). We also provide an introduction on the exterior differential operator as the fundamental operator within the aforementioned methods regarding its geometrical interpretation. Finally, we apply the BALE and the DBP methods to the system of equation governing the capillary migration force that a bubble experience in a microchannel to obtain it sensitivity against its position. This relevant problem serves as a convenient benchmark to test both methods and validate its results by comparison between the two. The recent work by the authors (Rivero-Rodriguez, J., & Scheid, B. (2018). Journal of Fluid Mechanics, 842, 215-247) rely on both methods proposed in this abstract. To numerical solve the governing systems of equations, we use the finite element method and provide the weak formulation of the system, including a discussion on the weak formulation of the exterior differential operator.