On The Modelling of Electrowetting in COMSOL MultiPhysics

J.F. Dannenberg[*,1], J. Brinkert[1] and E.A.D. Lamers[1]
[1]Reden B.V., Twekkelerweg 263, 7553 LZ Hengelo (Ov.), The Netherlands
*Corresponding author: e.dannenberg@reden.nl

Abstract: Electrowetting is a popular tool for manipulating small amounts of liquid. In this paper, we take a closer look at the 'mechanism' of electrowetting displays. We describe the physics which drive the phenomenon (Section 2). The equations are rewritten such that they can be implemented in COMSOL Multiphysics (Section 3). And in the last section, results of the implementation using COMSOL's two phase flow with level set and the electrostatic module are shown (both for 2D and for 3D). A comparison of the 'break up' of the oil layer in 2D and an analytical calculation of the forces are compared.

Keywords: Electrowetting, COMSOL Multi-physics, level set, FE modelling.

1. Introduction

Electrowetting is becoming more and more popular and is probably the most flexible way of manipulating small amounts of liquids. Applying an electric field in a liquid, contact angles are changed and/ or surface pressures introduced. This causes the liquids to move to a new equilibrium position. As an example, consider electrowetting for displays [1].

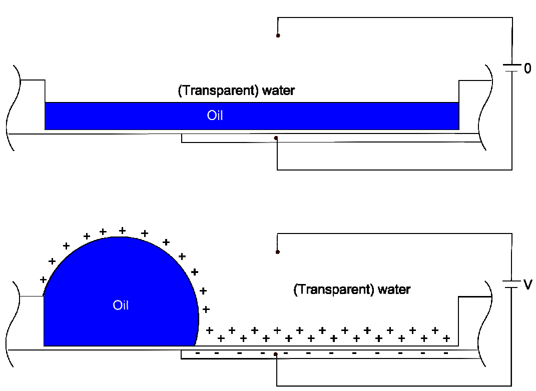

Figure 1. Principle of electrowetting of displays. The oil layer covers the substrate without any applied voltage (above), whereas applying a voltage leads to the retraction of the oil (below).

Consider a small cell containing (a non conducting) oil and (conducting) water. When a voltage is applied at the water, the oil retracts to the side and the water will cover the substrate, whereas without a voltage, the oil would cover the substrate (see also Figure 1). Using a coloured or light absorbing oil, the light transmittance of the display can be manipulated. When the voltage is removed, surface tension restores the original situation, with oil covering the (hydrophobic) surface. The process takes just milliseconds, fast enough to achieve video rate image change, solely based on the hydrodynamics.

2. Physics

The main physical mechanism is the movement of the different liquids due to the electric field applied and the surface tension. To gain insight of these physics, a simplified model will be analysed first: the plate capacitor.

2.1 The plate capacitor

When looking at a cell as described above, in which the water is assumed to be a perfect conductor, the cell can be seen as a plate capacitor:

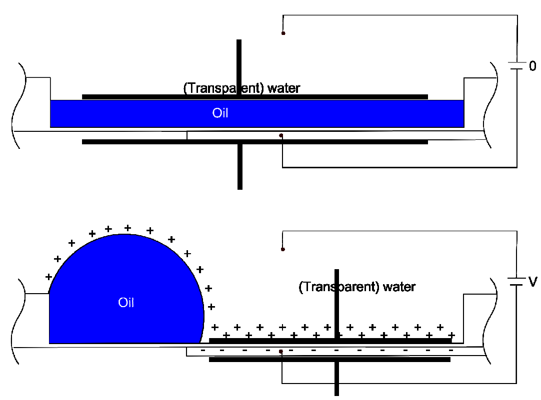

Figure 2. Plate capacitor approach; two situations can be distinguished, a) the plate capacitor over the oil and insulator and b) the plate capacitor only over the insulator.

When looking at a plate capacitor, the capacity of the capacitor can be written as [2]:

$$C = \frac{\varepsilon A}{d}, \quad \varepsilon = \varepsilon_0 \varepsilon_r \tag{1}$$

In which ε_0 is the permittivity of vacuum, ε_r is the relative permittivity of the insulator/oil, *A* the surface and *d* the height of the insulator/oil. The relative permittivity of the oil and the insulator is taken to be equal for simplicity ($\varepsilon_{r,oil} = \varepsilon_{r,\ insulator} = \varepsilon_r$). Note that the equation above is only valid if the dielectric and the oil have the same permittivity.

The plate capacitor will aim for a high capacity *C*, which means that it will try to decrease the thickness *d* or increase surface *A*.

In the first situation the oil covers the insulator completely, meaning that the capacitor cannot increase the surface, only decrease the thickness. The oil will be pushed away, so the water will move down, until it reaches the insulator. At this point, the capacitor can be defined over the insulator, without the oil. In this case, the thickness cannot become any smaller, but the surface can grow bigger, which means that the oil will be pushed aside.

2.2 The driving forces of electrowetting.

Let us start from the well-known Maxwell stress tensor T_{ik}, which says that the stress introduced by applying an electrostatic field is [3]:

$$\begin{aligned} \mathrm{T}_{ik} &= \varepsilon_0 \varepsilon_r \left(E_i E_k - \frac{1}{2} \delta_{ik} E^2 \right), \\ i &= 1,2,3 \qquad k = 1,2,3 \end{aligned} \tag{2}$$

in which is assumed that the electric field can be modelled as (quasi-)static. Therefore magnetism can been neglected (B=0).

The force per volume can then be found by taking the divergence of the stress tensor:

$$f = \nabla \cdot T_{ik} \ \text{[N/m}^3\text{]} \tag{3}$$

As in the example of the plate capacitor, the electric field in the x-direction is zero (E_x=0). This leads to that the electric field E = E_y. Note that in the situation of the plate capacitor the force only appears at the oil/water interface. When integrating over the y-direction, one obtains the pressure on this interface between water and oil:

$$P_{elec} = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 \ \text{[N/m}^2\text{]} \tag{4}$$

When looking near the Triple Contact Line (TCL), the pressure as described above will only be significant in a small region (d_2) around the TCL. When assumed that the water is a much better conductor than the dielectric, the gradient of the voltage V is only significant in the dielectric layer. This gives, with a thickness *d* of the dielectric layer:

$$E = \nabla V = \frac{\Delta V}{d} \tag{5}$$

where ΔV is the difference of the voltage over the dielectric. When integrating the pressure (equation (4)) over d_2, and combining with equation (5) this results in:

$$\begin{aligned} &\int \frac{1}{2} \varepsilon_0 \varepsilon_r \frac{(\Delta V)^2}{d^2} \partial d_2 = \\ &\frac{1}{2} \varepsilon_0 \varepsilon_r \frac{(\Delta V)^2 d_2}{d^2} = \frac{1}{2} \varepsilon_0 \varepsilon_r \frac{(\Delta V)^2}{d} \end{aligned} \tag{6}$$

in which d_2 is chosen to be length *d*. This is exactly the force per meter as prescribed by Lippmann [4] (note that V is 0 at the ground, leading to $\Delta V = V$).

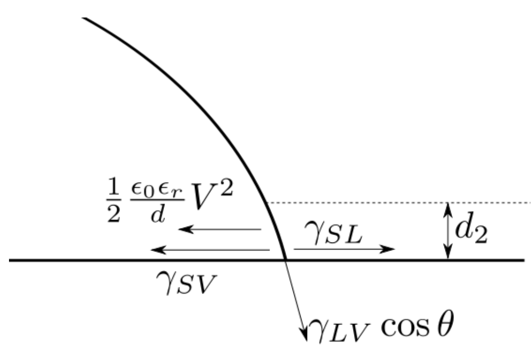

Figure 3. Forces in the triple contact line, in which γ represents the surface tension [N/m] of the oil/water (LV), the oil solid (SV) and the water solid (SL) and θ the contact angle.

When implementing this force in Young's equation, the well-known Young-Lippmann equation is found:

$$\cos(\theta) = \cos(\theta_0) + \frac{\varepsilon_0 \varepsilon_r}{2d\sigma} V^2 \qquad (7)$$

prescribing the contact angle depending on the applied voltage and the static contact angle (θ_0), see also Figure 3.

More information about the basics of electro-wetting can be found in Mugele [5].

3. Implementation in COMSOL Multi-physics

Besides the density and the viscosity, also the relative permittivity has to be prescribed in the fluid domain. This can be done in the same way as the density and viscosity, by using the level set function ϕ:

$$\varepsilon_r = \varepsilon_{r1} + (\varepsilon_{r2} - \varepsilon_{r1})\phi \qquad (8)$$

with ε_{r1} and ε_{r2} the relative permittivity of respectively fluid 1 and fluid 2.

The force on the volume can be described using equation (2) and (3). Note that the force only appears at the level set interface, since the derivative of ε_r and E are only non-zero in this domain.

The assumption of a layer thickness *d*, which is thin compared to the system, leads to the prescription of the contact angle depending on the voltage difference over the dielectric layer. This is straight forward to implement in COMSOL since COMSOL already uses a prescribed contact angle. The contact angle prescription now only depends on the voltage, which is comparable with the implementation of Cahill et al [6].

4. Results

Numerical simulations and analytical calculation has been performed in 2D. A three dimensional simulation has been performed as well.

4.1 Two-dimensional

A two dimensional model is used to check the implementation of the forces and the coupling between Electrostatics, Navier-Stokes and Level Set.

Figure 4. Geometry of the two dimensional problem.

The geometry of the problem is shown in Figure 4. The oil layer height is $4x10^{-6}$ m and the insulator is $0.8x10^{-6}$ m. The cell width is $50x10^{-6}$ m and the cell height $10x10^{-6}$ m. The surface tension between the water and the oil is 0.054 N/m. The contact angle at the sides of the cell are set at 80 degrees for the oil. In the figure the ground is represented with blue, the water as transparent, the oil as red, the insulator as cyan and the electrode as yellow.

One can analytically calculate the necessary voltage needed to break the surface tension and thereby the oil layer by comparing the pressures of the surface tension and the electric force, using the plate capacitor approach as earlier described in section 2.1. When the oil/water interface is assumed to form a perfect circular arc due to the pressure above, the water to be a perfect conductor and the volume of the oil and water is kept equally in time; it follows that the needed voltage for 'break up' is approximately 50 V.

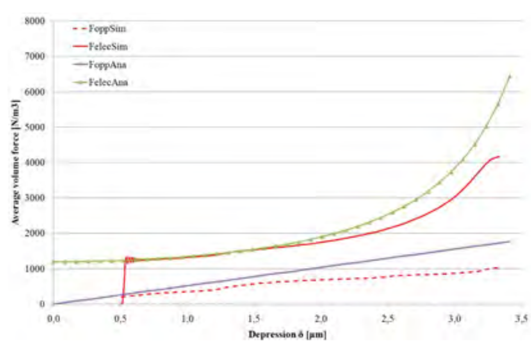

Figure 5. The analytically calculated forces vs. the forces in the 2D simulation.

In Figure 5 the average surface tension and average electric force on the surface are plotted.

The analytical values (green for the electric and purple for the surface tension) are compared with the simulation results (red solid for electric and red dashed for the surface tension). As can be seen in the figure, both the surface tensions are lower than the electric forces. This will result in the breakup of the oil layer.

In the simulation, the oil has a contact angle of 80 degrees at the sides. This causes the oil/water interface to be slightly curved in the initial situation. This can be seen in the figure; the red dashed line is nonzero at the beginning. The electric field is applied after a small amount of time; therefore the solid red line does not start at a deflection of zero, but by approximately $5x10^{-7}$ m. This deflection is caused by the initial curvature of the surface due to the contact angles at the sides. At this stage the surface tension and electric force are equal in the analytical and simulation calculation.

However, with a larger deflection the difference in the forces becomes larger. This is expected, since in the analytical calculation a perfect circular arc of the oil/water interface is assumed, whereas in the simulation this is not true, see Figure 6. This is caused by the prescribed contact angle at the sides of the cell. Also the water is assumed to be a perfect conductor, as in the simulation it is not. This causes an error, which increases when the amount of water between the ground and the electrode is larger (the voltage drop of the simulation just before breaking is lower than the voltage drop used analytical).

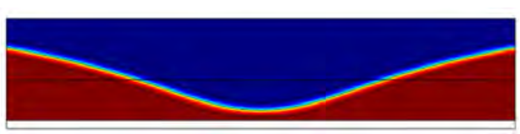

Figure 6. The oil/water interface just before breaking.

4.2 Three-dimensional

Also a three dimensional model is investigated, to see the real phenomenon of electrowetting. At this stage it is not possible to compute entire cells, so the model results have not yet been compared to experimental data.

The geometry of the three dimensional model is given in Figure 7, in which the cell width is $25x10^{-6}$ m and the cell height is $14x10^{-6}$ m. Again, the grounds are represented by blue, the oil by red, the water by transparent and the electrode by yellow. The oil layer has a height of $7x10^{-6}$ m. The ground at the bottom covers approximately 20% of the surface. The insulator between the ground and electrode is $5x10^{-6}$ m wide.

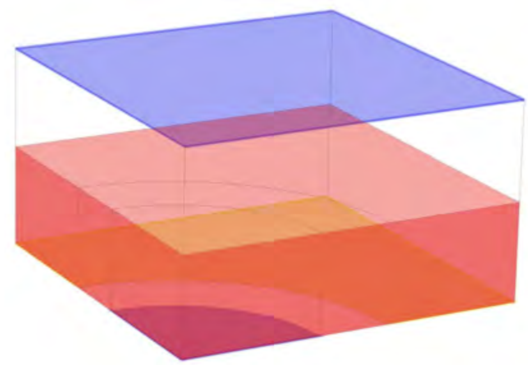

Figure 7. Geometry of the 3D problem.

Figure 9 shows the results of the three dimensional simulation. At the bottom of the geometry the voltage is plotted. On the side the level set function and in the middle the oil/water interface (level set = 0.5). The arrows illustrate the electric field. First the layer goes down, where after the layer 'breaks'. After the breaking the 'capacitor' will expand its surface, and pushes the oil away and off the insulator above the electrode.

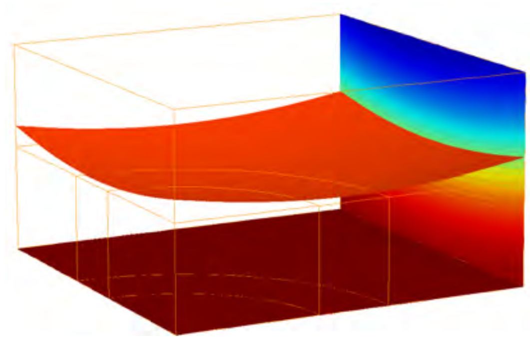

Figure 8. Results of the 3D simulation. The initial situation (top)

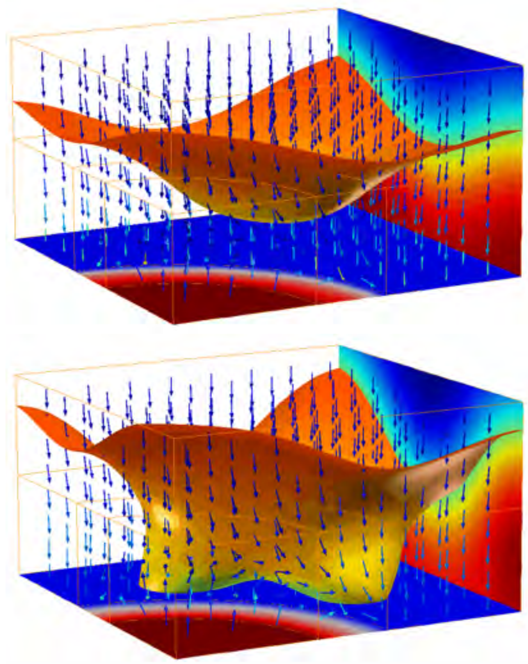

Figure 9. Results of the 3D simulation. The water moving down (top) and the water reached the substrate, and the oil is pushed away (bottom).

5. Conclusions

A coupling between the applied electric field and volume forces in the fluid due to this field is applied in COMSOL Multiphysics. Comparison between analytical and two-dimensional calculations resulted in only a small difference. This is probably due to the assumptions of the water to be a perfect conductor and the surface to be a perfect circular arc. Due to the lack of time, no three dimensional validation with experimental results have been made. However, the three dimensional simulation done in this research is promising.

Further research to the implementation of electrowetting (displays) should be investigated (experimental validation with the three dimensional model). The biggest issue with these problems is the large amount of degrees of freedom and the thereby large simulation times to find an accurate solution of the behaviour of the liquids due to the applied electric field.

6. References

1. Johan Feenstra and Rob Hayes, Electrowetting Displays, *Liquavista* (2009).

2. R.P. Feynman, R.B. Leighton and M. Sands, *Lectures on Physics*, California Institute of technology, **II**, 8-3 (1977).

3. J.A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company Inc, New York (1941).

4. G. Lippmann, Relation entre les phénomènes électriques et capillaries, *Annales de Chimie et de Physique*, **5**, 494 (1875).

5. F. Mugele and J.-C Baret, Electrowetting: from basics to applications, *Journal of Physics:Condensed matter,* R705 (2005).

6. Brian P. Cahill, Athanasios T. Giannitsis, Gunter Gastrock, Mart Min, Dieter Beckmann, “A Dynamic Electrowetting Simulation using the Level-Set Method”, *COMSOL Conference,* (2008).