Verification and Time Performance analysis of COMSOL v3.5a for solving the electromagnetic problem in a superconductor slab.

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Abstract: Numerical analysis based on finite element method (FEM) represents a powerful approach to solve electromagnetic problems. For instance, FEM methods have been broadly used to calculate the critical state current distribution in high temperature superconductors of various geometries. In the near future, we intend to develop a tool in COMSOL v3.5a for the analysis of power applications, such as motors and generators. To get an insight into the problem, we have first used COMSOL to obtain the electromagnetic solution for a simple superconducting geometry, a rectangular infinite slab, for which analytical and numerical solutions are documented. We discuss the verification of the method, and computing times obtained using different mesh densities and element polynomial shape functions.

Keywords: electromagnetic finite element method analysis, superconductivity, shape functions, mesh density, Partial Differential Equations (PDE).

1. Introduction

High temperature superconductors (HTS) can be used in many different applications, such as motors, generators, cables etc. This has motivated the study of the electromagnetic (EM) behavior of these materials, shaped in the different forms required for integrating them in devices. The critical-state model [3] has been used to solve this kind of problems for a handful of 2D or axi-symmetric geometries. An alternative and powerful approach is to use Finite Element Methods (FEM) based upon Maxwell equations to tackle EM problem, obtaining the distribution of currents, flux penetration and magnetization cycle in a sample when an external field is applied, among the possibility to solve thermal and mechanical items exploiting the multi-physics capability nowadays extended in some commercial FEM packages. COMSOL v3.5a is versatile, commercially available FEM software, which includes an EM package for solving Maxwell Partial Differential Equations (PDE). In our group, we intend to apply COMSOL v3.5a to simulate certain designs of superconducting motors. The aim of this work was to evaluate the results and time performance of COMSOL v3.5a by applying it first to a simple geometry for which the solution is approximate by using the solution found by Brandt for the critical state for HTS slabs and strips [2], integrating the 2D time dependent Maxwell equations at low frequency with the nonlinear constitutive relation between current density and electric field (Eq.3) for a HTS and by solving numerically the critical state [7]. We have considered a HTS slab of rectangular cross section $2a$x$2b$ and infinite length, under a sinusoidal magnetic field, applied perpendicular to it (Fig.1). The EM problem is then bidimensional, non-linear and time dependent. We have analyzed the verification and required computing time as a function of the mesh density and type of polynomial shape functions used.

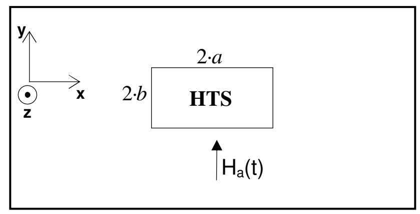

Figure 1. Rectangular HTS slab with infinite length (HTS) applying a magnetic source field perpendicular to the sample

2. Simulation procedures

2.1 PDE equations

The electromagnetic PDE considered are:

$$\nabla \times \vec{E} = -\mu_0 \cdot \mu \cdot \frac{\partial \vec{H}}{\partial t} = \begin{bmatrix} \frac{\partial E_z}{dy} \\ \frac{-\partial E_z}{dx} \end{bmatrix} = \quad \text{(Eq.1)}$$

$$= -\mu_0 \cdot \mu \begin{bmatrix} \frac{\partial H_x}{dt} \\ \frac{\partial H_y}{dt} \end{bmatrix}$$

$$J_z = \nabla \times \vec{H} = \frac{\partial H_y}{dx} - \frac{\partial H_x}{dy} \quad \text{(Eq.2)}$$

where μ_0 is the vacuum permeability; ***E*** and ***H*** are the electrical and magnetic fields, respectively, and J_z is the current density in z-direction, being XY in the plane of the slab and Z the out-of-plane coordinate as fig. 1 shows.

In addition, we assumed a constitutive power-law relation between the electrical field and current density [1, 4, 5] for HTS domain

$$E_z = E_0 \cdot \left(\frac{J_{z\_SC}}{J_c} \right)^n \quad \text{(Eq.3)}$$

with $E_0=10^{-4}$V/m, and typical exponent values of n=21, and the critical current density J_c=8x10^{-7} A/m^2.

The applied magnetic field changes monotonically at a very low frequency, 10^{-4} Hz, so as to obtain an EM solution close to that of the critical state. Quantification of the effect of that frequency over the results and how can it be linked to the exponent n will be reported.

2.2 Mesh and solver parameters

For the mesh design, we used triangular advance front elements with a minimum quality of 0.6 [9]. We used also Curl Elements allowing continuity of the tangential component of the field at each element boundary. This condition is obtained just in the boundary of the element but not inside it, allowing the propagation of the field in tangential and normal components [6]. The software supports Curl elements with linear, quadratic or cubic polynomial functions.

The problem was solved with Unsymmetrical MultiFrontal direct solver (UMFPACK) [10] for sparse matrices, with a relative tolerance and absolute tolerance of 0.01 and 0.0001 respectively [8]. Time dependent analysis uses Backward Differentiation Formula (BDF) [11]

Simulations were done in an Intel ® Core ™ i7 computer at 2.67GHz, with 16GB RAM and S.O. Windows 7, 64bits.

3. Simulation results

3.1 COMSOL solution versus critical-state based methods.

We simulated with COMSOL the current

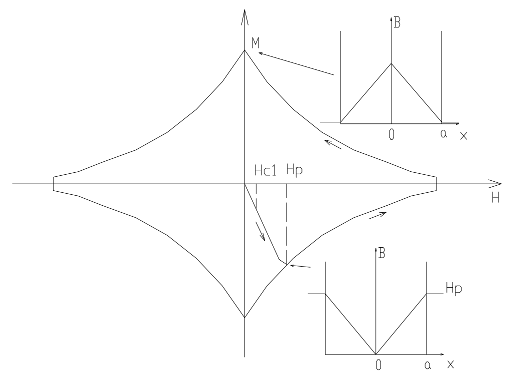

Figure 2. Typical magnetization curve for a HTS [8]

distribution and flux penetration as the applied magnetic field was cycled, as well as the penetration field H_p for slabs of different aspect ratio b/a. H_p is defined as the minimum applied field required to magnetize the center of the sample (see schematics in Fig. 2). These simulations were done using a constant mesh

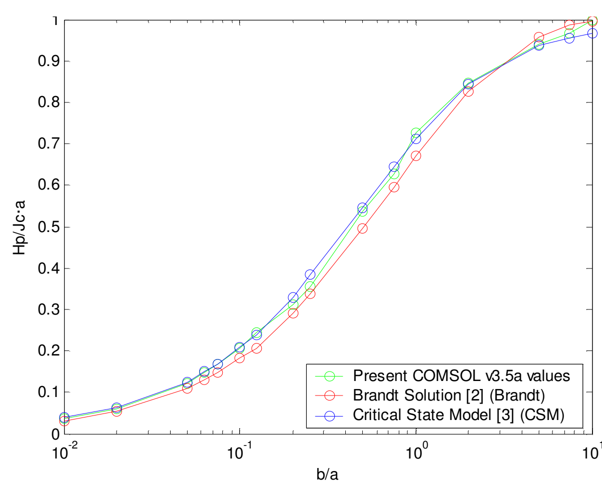

Figure 3. Normalized field penetration of a rectangular bar cross section 2ax2b in a perpendicular magnetic field using current-voltage power law represented for aspects ratio, b/a, of 0.01, 0.02, 0.05, 0.0625, 0.075, 0.1, 0.125, 0.2, 0.25, 0.5, 0.75, 1, 2, 5, 7.5, 10; obtained with COMSOL v3.5a (green line) Brandt's method (red line) and CSM (blue line).

density of 4 triangular (advance front) Curl elements/mm^2 and a second degree polynomial shape function

Fig. 3 plots the normalized penetration field $H_p/J_c{\cdot}a$ as a function of the aspect ratio *b/a* obtained with COMSOL v3.5a, compared to the curves predicted in the critical-state model context by (CSM) [7], and (Brandt) [2].

A low deviation between all the curves is observed. In order to better quantify the deviation of COMSOL v3.5a results from the other methods, we can define the relative deviation at a certain i=b/a ratio as:

b/a		**0.01**	**0.02**	**0.05**	**0.0625**
E_{r_i} (%)	Brandt	14.4	6.65	12.96	14.71
	CSM	7.93	6.4	0.46	1.31
b/a		**0.075**	**0.1**	**0.125**	**0.2**
E_{r_i} (%)	Brandt	15.42	13.92	18.64	6.99
	CSM	1	1.26	2.94	5.53
b/a		**0.25**	**0.5**	**0.75**	**1**
E_{r_i} (%)	Brandt	4.71	8.25	5.29	8.06
	CSM	7.68	1.47	2.72	1.7
b/a		**2**	**5**	**7.5**	**10**
E_{r_i} (%)	Brandt	2.43	1.88	2.03	0.4
	CSM	0.3	0.32	1.2	3.63

Table 1. Relative deviation between the result obtained by COMSOL v3.5a, and those of the CSM and reported by Brandt, at aspect ratios b/a=0.01, 0.02, 0.05, 0.0625, 0.075, 0.1, 0.125, 0.2, 0.25, 0.5, 0.75, 1, 2, 5, 7.5 and 10.

$$E_{r_i} = \frac{1}{A_i}\cdot\sqrt{(C_i - A_i)^2}\cdot 100, \qquad \text{(Eq.4)}$$

where C_i is the result obtained using COMSOL v3.5a, and A_i is the value obtained by either Brandt's solution or CSM. Then, the average relative deviation E_r over the whole curve can be calculated as:

$$E_r = \frac{1}{N}\cdot\sum_{1}^{N}\frac{1}{A_i}\cdot\sqrt{(C_i - A_i)^2}\cdot 100 \qquad \text{(Eq.5)}$$

being N the total number of aspect ratios considered.

The average deviation observed by COMSOL v3.5a relative to Brandt's results is 7.68%, and relative to the CSM is 3.8%. The maximum deviation between COMSOL v3.5a and Brandt (18.64%) occurs at *b/a*=0.125 and by CSM (7.93%) occurs at *b/a*=0.01.

Fig. 4 shows that the deviation between COMSOL v3.5a and Brandt's calculations

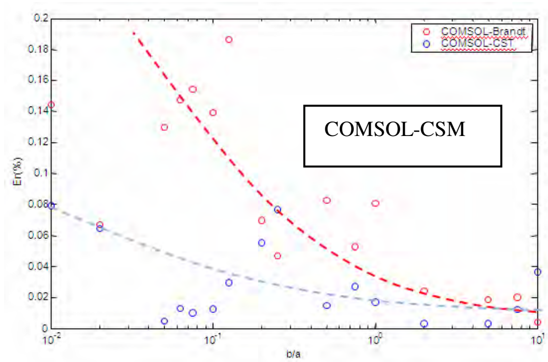

Figure 4. Deviation in the value of the field penetration simulated by COMSOL v3.5a for slabs of different aspect ratio b/a=0.01, 0.02, 0.05, 0.0625, 0.075, 0.1, 0.125, 0.2, 0.25, 0.5, 0.75, 1, 2, 5, 7.5 and 10, relatives to results calculated by Brandt (red points) and the CSM (blue points). Lines are guides-for-the-eye.

decreases when the aspect ratio increases. The deviation from the CSM is smaller, and decreases at a smaller rate with the aspect ratio.

We turn now our attention to the current density profiles obtained as an increasing applied magnetic field is applied to the sample. Fig. 5 shows the profiles simulated by COMSOL v3.5a for different aspect ratios. We have again compared the field profiles obtained by COMSOL v3.5a to those calculated by Brandt solution. Table 2 summarizes the maximum relative deviation observed between a COMSOL v3.5a and a Brandt's profile line, and the average

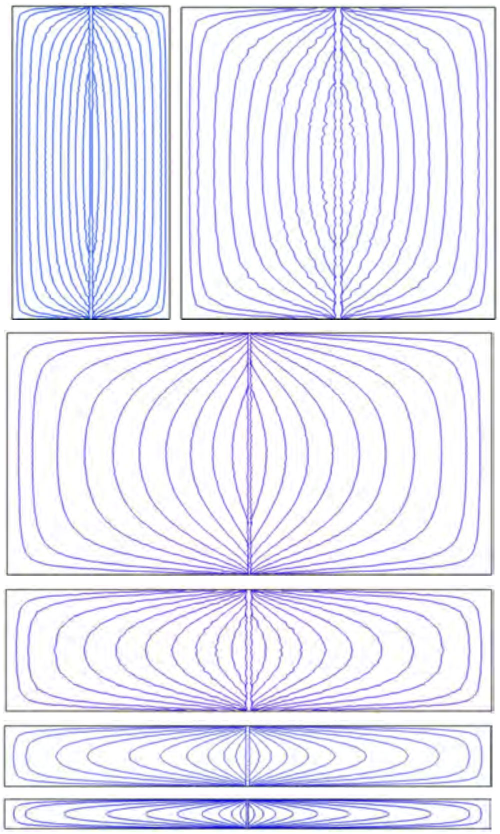

Figure 5. Current density fronts during penetration of a gradually increased field H_a (vertically oriented) into rectangular cross section infinite length bars of aspect ratios b/a=2, 1, 0.5, 0.25, 0.125 and 0.0625 (from top left to bottom). The contours of $J_z(x,y)=J_c/2$ for the field values Ha/Hp=0.05, 0.1, 0.2…. 0.9, 1 obtained with COMSOL v3.5a are shown.

deviation considering all profile lines, for each aspect ratio.

As shown in table 2, the average deviations observed in the current density profiles are lesser than 4 % compared with Brandt's calculations.

In conclusion, we cannot observe a great difference in the results obtained by both procedures considering that the CSM and Brand approaches neglect the non equilibrium currents which are time dependent. The quasi-static FEM solution fits the equilibrium solutions reasonably well. Fitting quality as a function on the frequency and the *n* exponent is on the way.

AR/ E_r	$\max(E_{r_i})$	E_r
2	0.8	1.1
1	1.9	1.27
0.5	1.1	1.14
0.25	5.1	2.31
0.125	4.2	3.67
0,0625	4.8	3.98

Table 2. Maximum and average errors between the current distribution obtained by COMSOL v3.5a and Brandt's method, for aspect ratios of b/a= 2, 1, 0.5, 0.25, 0.125 and 0.0625.

3.2 Performance as a function of the mesh density and polynomial element shape function.

In order to show the performance of the calculation, in this section, we present the results obtained for a fixed aspect ratio, b/a=0.5, at different mesh densities and polynomial element shape function orders.

Table 3 summarizes the values of the penetration field H_p obtained with COMSOL v3.5a at different mesh densities (in elements / mm^2) using polynomial shape functions of degree k=1, 2 and 3.

[Element/ mm²]	**Polynomial shape function degree**		
	k=1	**k=2**	**k=3**
	Penetration field measures [A/m]		
1e-2	6.5057e5	6.506e5	7.965e5
4e-2	6.693e5	8.78e5	7.993e5
8.5e-2	7.825e5	8.6363e5	8.077e5
16e-2	7.9645e5	8.6363e5	8.188e5
64e-2	8.6735e5	8.36325e5	8.371e5
256e-2	8.5244e5	8.7912e5	8.492e5

Table 3. Penetration fields applying mesh densities of 0.01, 0.04, 0.085, 0.16, 0.64 and 2.56 elements/mm^2 and for element polynomial shape functions of order 1,2 and 3 obtained with COMSOL v3.5a.

For comparison, the analytical value of Hp for a b/a=0.5 bar is Hp=8.72377e5 A/m [2].

As shown in Fig. 6, in order to make simulations with a relative deviation lesser than 5% it is possible to use a linear shape function with a minimum mesh density of 0.5 elements/mm^2; or a quadratic function with a mesh density of at least 0.05 elements/mm^2.

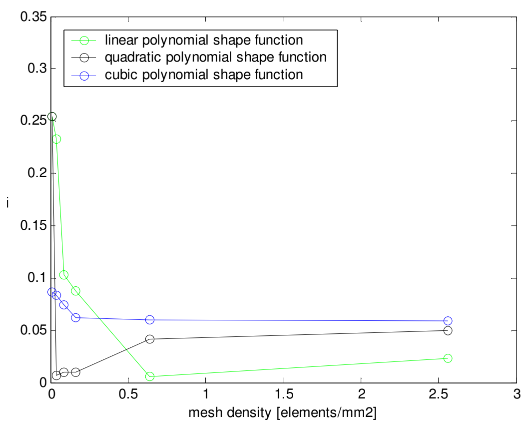

Figure 6. Relative deviations of field penetration observed for mesh densities 0.01, 0.04, 0.085, 0.16, 0.64 and 2.56 elements/mm^2 and for element polynomial shape functions of order k=1, 2 and 3 obtained with COMSOL v3.5a.

[Elements/mm^2]	Polynomial shape function degree		
	k=1	**k=2**	**k=3**
	Computing time [s]		
1e-2	4	48	1498
4e-2	5	160	13164
8.5e-2	62	3010	60285
16e-2	65	7200	117360
64e-2	372	10800	500000
256e-2	9043	60000	1500000

Table 4. Time consumed to obtain the penetration field applying mesh densities of 0.01, 0.04, 0.085, 0.16, 0.64 and 2.56 elements/mm^2 and for element polynomial shape functions of order k=1, 2 and 3 obtained with COMSOL v3.5a

It is also interesting to analyze the computing time consumed for each mesh density and shape function applied (table 4 and Figure 7). It is clear that the time consumed with a k=3 shape

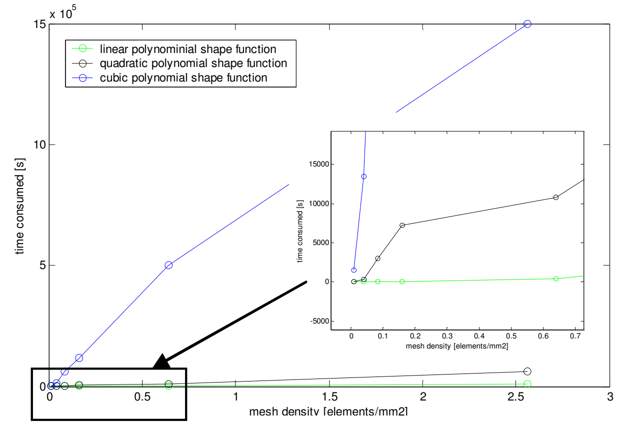

Figure 7. Time consumed by COMSOL v3.5a to obtain H_p for mesh densities 0.01, 0.04, 0.085, 0.16, 0.64 and 2.56 elements/mm^2 and for element polynomial shape functions of order k=1, 2 and 3.

function is very high compared with that consumed when using a k=2 or a k=1 with the same mesh density.

4. Conclusions

COMSOL v3.5a is a very attractive approach to solve at least EM problem in superconducting materials of various shapes. By analyzing a simple bidimensional problem of a rectangular in a magnetic perpendicular field, we have been able to conclude that COMSOL v3.5a is able to correctly describe the physical EM problem and achieves a high precision.

In addition, the software time performance has been analyzed as a function of two important FEM parameters, the mesh density and the degree of the shape function. We can conclude that second order shape functions can be used to obtain acceptable results at reduced mesh densities. More specifically, with a mesh density of 0.05 el./mm^2 and a k=2 shape function, the field penetration can be calculated with a relative deviation of 1.5% and the computer time is 160 s, thus these conditions are highly recommended for this kind of problem. When using a linear polynomial shape functions, k=1, a large mesh density is required to obtain a deviation smaller than 5%, but the computing time is still acceptable (300 s).

5. References

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