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Numerical Sensitivity Analysis of a Complex Glass Forming Process by Means of Local Perturbations

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Abstract: Many industrial processes are characterized by a complex spatio-temporal and nonlinear dynamic behavior. Examples are rheological forming processes (e.g. glass, steel, plastic). For process optimization it is very important to know the impact of possible perturbations to the relevant process variables and the process "output" parameters (e.g. product properties). In this paper a glass forming process is investigated. We are concerned the effect of local perturbations to the global variable fields (e.g. temperature, velocity and geometric profile of the glass). The sensitivity of the process variables to the perturbations is calculated numerically. In detail perturbations of the oven temperature profile (stationary) and material inhomogeneities (time dependent) are investigated. The results of the sensitivity analysis are actually used for the development of optimal control strategies.

Keywords: Finite Element Model, Glass Forming Process, Sensitivity Analysis, Nonlinear PDEs, Process Optimization

1. Introduction

Over the last few years, numerical simulation models and especially Finite Element Models have become increasingly important for the design of process control strategies or process optimization. Many industrial processes are characterized by a complex spatio-temporal and nonlinear dynamic behavior. Examples are rheological forming processes (e.g. glass, steel, plastic). For process optimization it is very important to know the impact of possible perturbations to the relevant process variables and the process "output" parameters (e.g. product properties).

This *sensitivity* can be calculated straightforward for processes which can be modeled with lumped parameters, i.e. the model consists by a set of algebraic equations or ordinary differential equations. In this case a common definition of the local sensitivity S of an system response (respectively output variable) y to a parameter p is defined by the derivative of y with respect to a parameter p as equation (1) [3]

$$S = \frac{\Delta y}{\Delta p} \rightarrow \frac{\partial y}{\partial p} \qquad (1)$$

where Δp is the perturbation of p and Δy the change of y caused by Δp. By normalizing this sensitivity, the term sensitivity index is defined as equation (2) [7]

$$SI = \frac{\Delta y/y}{\Delta p/p} \rightarrow \frac{p}{y}\frac{\partial y}{\partial p} \qquad (2)$$

In other words, SI is the relation of relative change of the output variable $\Delta y/y$ to the relative parameter change $\Delta p/p$. For an introduction in the theme of sensitivity analysis see [3, 4, 7] and in the theme of perturbation theory see [5].

If the spatial distribution of the process cannot be neglected, the model consists by one or more partial differential equations (PDEs). In many cases also the perturbations of the process are spatially distributed. The derivative has to be generalized by using the definition of Gâteaux variation [3, 4]. The definition of local sensitivity of a system respect to a spatial distributed perturbation $h(z)$ is :

$$\delta y = \lim_{\varepsilon \to 0} \frac{y(p^0 + \varepsilon h) - y(p^0)}{\varepsilon} \qquad (3)$$

In Eq.(3) $p^0(z)$ denotes the nominal, unperturbed parameter distribution. According (3) the local sensitivity has to be calculated numerically.

In this paper a glass forming process is investigated [1, 2]. We are concerned the impact of local perturbations to the global variable fields (e.g. temperature, velocity and geometric profile of the glass). In detail perturbations of the oven temperature profile (stationary simulations) and material inhomogeneity (time dependent simulations) have been investigated. The results of the sensitivity analysis are actually used for the development of optimal control strategies.

The paper is structured as follows. In section 2 the nonlinear model of the glass forming process and its disturbance model are introduced. Section 3 provides information about the use of COMSOL. In section 4 the results of the sensitivity analysis are discussed.

2. Governing Equation

2.1 Model of the Glass Forming Process

The industrial process that is considered in this paper is a complex rheological forming process producing glass tubes and accordingly rods which are pre-products for optical fibers. The geometric properties of the produced rods resp. tubes have only a very small tolerance band.

The main physical phenomena arise from radiation, heat convection, and fluid dynamics. The process is strongly nonlinear in particular due to the impact of radiation and nonlinear material parameter laws (temperature dependence of specific heat, effective heat transfer coefficient and viscosity). In addition, the forming process involves a wide temperature range and is characterized by large deformations. The process setup is visualized in Figure 1. The cylinder is fed with slow velocity v_f in an oven where it is heated up to its forming temperature. Below the oven the tube is pulled with a higher velocity v_p resulting in thin glass rods (resp. tubes). The individual components of the model and their coupling terms and nonlinearities are illustrated in Figure 2.

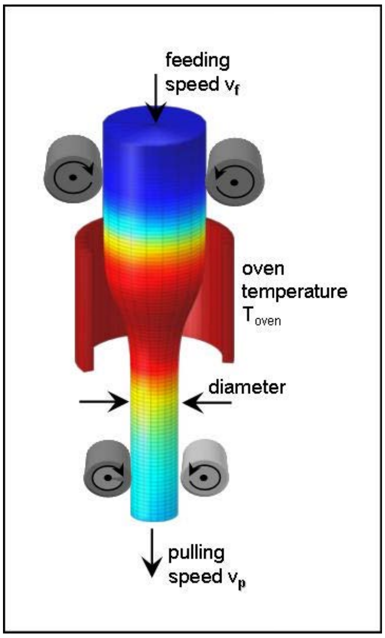

Figure 1. Industrial glass forming process

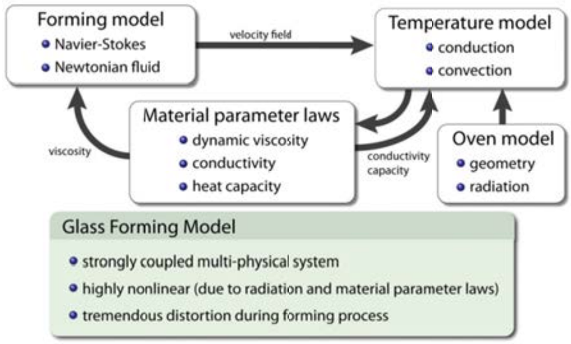

Figure 2. Parts and nonlinearities of the model

The forming is regarded as a Newtonian fluid with free surfaces. Basically, the model consists of two main parts describing (a) the glass flow and (b) the heat transfer in the glass and from the oven to the glass. Hence for a 3D simulation of the glass forming process the Navier-Stokes equations (momentum and mass balance) and the heat transfer PDE have to be solved. The state variables of the system are the velocity field $v(\vec{r},t)$, the pressure in the glass flow $p(\vec{r},t)$ and the temperature distribution of the glass $T(\vec{r},t)$. The geometry of the glass is obtained

The model can be reduced to a 2D model assuming axisymmetric. However, the calculation of 3D and even 2D scenarios is time consuming. If only mean values in radial direction are considered, 1D model can be obtained (Trouton model [6]):

$$\frac{\partial A}{\partial t} + \frac{\partial wA}{\partial z} = 0 \tag{4}$$

$$\frac{\partial}{\partial z}\left(3\mu(T)A\frac{\partial w}{\partial z}\right) = -\rho g A \tag{5}$$

$$A\rho\left(\frac{\partial C_p(T)T}{\partial t} + w\frac{\partial C_pT}{\partial z}\right) - \frac{\partial}{\partial z}\left(Ak(T)\frac{\partial T}{\partial z}\right) = 2\pi R S_r + Q_{dis}(z,t) \tag{6}$$

$$S_r = \epsilon\sigma_B\,(T_{oven}^4(z) - T^4(z))$$

In eq. (4) – (6) it denotes $A(z,t)$: cross section area of the glass rod, $w(z,t)$: velocity in z-direction, T(z,t): temperature of the glass, $\mu(T)$: dynamic viscosity, $c_p(T)$: specific heat, $k(T)$: effective heat transfer coefficient (which considers radiative heat transfer in a simple way), $T_{oven}(z)$: oven temperature, ρ: density of the glass, g: gravitational acceleration, ϵ: emissivity of the glass, σ_B: Stefan-Boltzmann constant. Eq. (4) is the mass balance, eq. (5) is the momentum balance (note that the viscous force in the Trouton model is $3\mu(T)A\frac{\partial w}{\partial z}$) and eq. (6) the 1D heat transfer equation. The results in this paper are based on the Trouten model (eq. (4) – (6)).

2.2 Disturbance model in glass drawing process

From Eq.3 the spatial local distributed perturbation $h(z)$ is modeled as Gaussian distribution as:

$$h(z) = \left(\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{1}{2}\left(\frac{z-z_0}{\sigma}\right)^2)\right) \quad (7)$$

where z_0 locate the position and σ determine the width of the perturbation. Two descriptions of spatial coordinates are considers: (a) coordinate z is fixed in space (Eulerian) and (b) the coordinate z is fixed in material (Lagrangian). In this paper two disturbances are investigated. Firstly the perturbation of oven temperature profile (ΔT_{oven}) can be determined as the disturbance fixed in space. The welding joint of glass is fixed in material, therefore it has to be considered as the heat source fixed in material. Because our equation are describe by space fixed coordinate, the disturb heat source fixed in material is convert to disturb moving heat source $Q_{dis}(z,t)$ in space fixed coordinate.

3. Use of COMSOL Multiphysics

Eq. (4) – (6) have been implemented in COMSOL (version 4.2). The equations are solved for the variables $A(z,t)$, $w(z,t)$ and $T(z,t)$. A length of 2 m of the total glass rod is simulated. The feeding and pulling speed (v_f, v_p) have been set as Dirichlet boundary conditions (BC). Cross section area of the cylinder which is fed in the oven is also defined as Dirichlet BC, while the resulting cross section area below the oven is calculated. The BCs for the heat transfer equation (6) are chosen as Neumann type. The model has been initialized with pre-calculated solutions $A_0(z)$, $w_0(z)$ and $T_0(z)$ of the unperturbed system. For the perturb systems the term $h(z)$ is added in order to calculate the term δy as Eq.(3). The perturb system is simulated by using the parametric sweep module in COMSOL by varying the parameters z_0 , σ and ε. The Matlab interface has been used for analyzing the results and the calculation of the sensitivity

$$\Delta y = \frac{y(p^0+\varepsilon h) - y(p^0)}{\varepsilon} \quad (8)$$

4. Sensitivity Analysis

Two different perturbation scenarios have been investigated, namely perturbations of the oven temperature profile (stationary simulations) and material inhomogeneity (time dependent simulations). The scenarios and results are discussed in the following subsections.

4.1 Stationary Perturbations of the Oven Temperature Profile

In Figure 3 an exemplary result with perturbations of the oven temperature profile is shown. Some perturbation scenarios are shown in subplot 4 of Fig. 4. The perturbations are modeled as a Gaussian profile as Eq.(7). The corresponding sensitivity index of glass temperature, velocities and cross section area are shown in subplots 1 – 3. It can be seen that the maximal sensitivity with respect to temperature is reached near the upper end of the oven (z ~ -0.2 m), while for velocity and cross section the maximum of the sensitivity is at z ~ 0 m which is near to the lower end of the oven. As due to conservation of mass the Volume flow $V = A{\cdot}w$ is unchanged, it holds

$$\frac{\Delta V}{V} = \frac{\Delta A}{A} + \frac{\Delta w}{w} = 0 \rightarrow \frac{\Delta A}{A} = -\frac{\Delta w}{w} \quad (9)$$

Hence the relative changes of cross section area are equal to the relative changes of the cross section area. As a consequence, also the maximal sensitivity is located at the same position (z ~ 0 m).

From Figure 3 it is obvious that the relative maximum of sensitivity index of temperature is moving with the center of the perturbations, while the center of the sensitivity index of velocity and cross section area is nearly unchanged at z ~ 0 m. It is interesting that the sign of sensitivity index changes when the center of oven temperature perturbations moves along the z-axis.

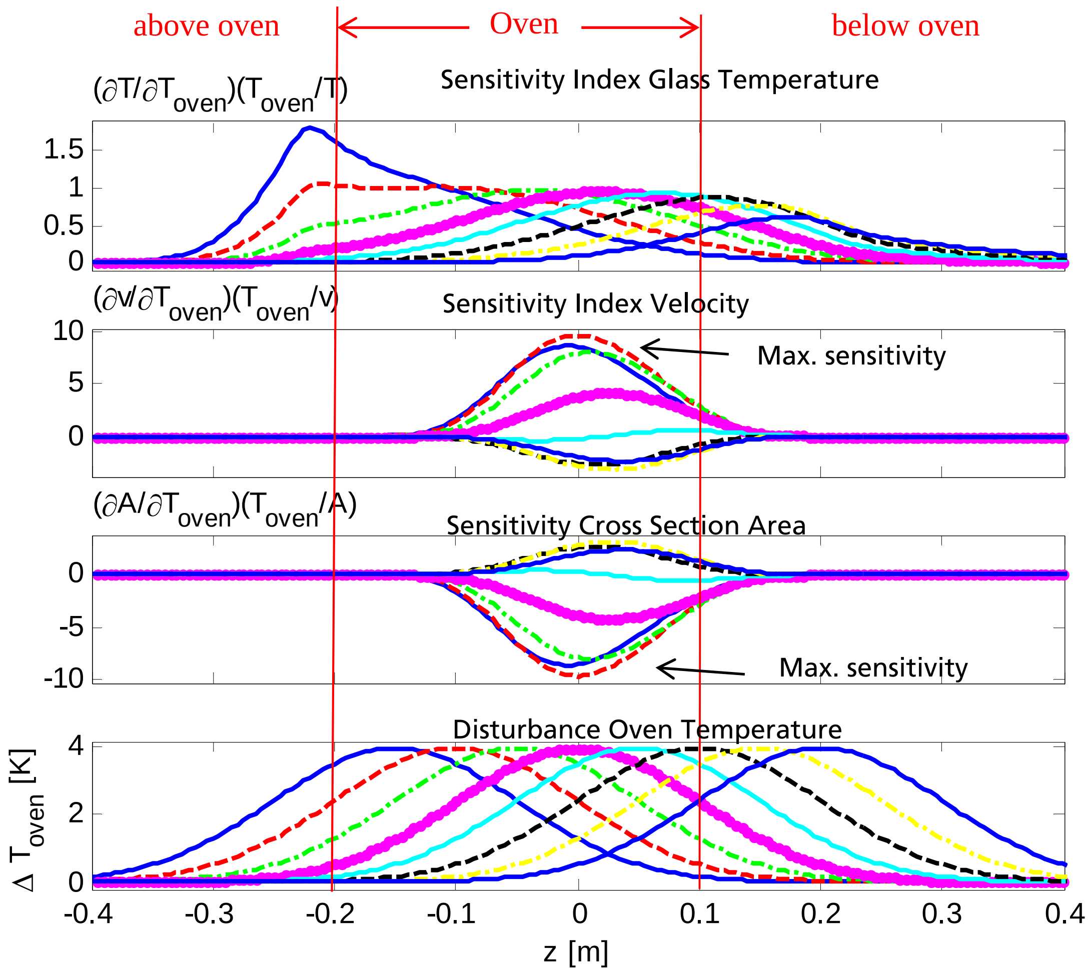

Figure 3. Results with stationary perturbations of the Oven Temperature Profile

While for z > 0 the relative changes of velocity (and hence the sensitivity index) are positive, it becomes negative for z < 0.1 m. If the center of oven perturbations is located n the range z ~ 0 … 0.1 m, the relative changes are partly positive and negative. In summary it can be stated that stationary oven temperature perturbations at the upper end of the oven have the strongest impact to glass temperature, velocity and cross section area.

4.2 Time Dependent Perturbations

Figure 4 shows results with time dependent perturbations, namely a moving heat source inside the glass. This heat source corresponds to the end of the batch process, where a transition to different glass material introduces additional radiative energy into the oven. The subplots at the bottom of Figure 4 show snapshots of the moving heat source. It is assumed that the amplitude of the heat source is increasing when it is moving into the oven. On the left side at the bottom the *volume specific heat source* q in [W/m^3] is shown, while on the right side the *length specific heat* source $q{\cdot}A$ [W/m] is plotted.

The subplots on the left side of Figure 4 show snapshots of the profile of the relevant process variables (glass temperature, velocity, cross section area, volume flow, viscosity (logarithmic scale), viscous force). In the subplots of the right side of Figure 4 snapshots of the *changes* of the profile with respect to the unperturbed profile are plotted.

The impact of the moving disturbance to temperature and velocity is similar to the stationary scenario of subsection 4.1. Regarding temperature, the maximum of the profile is near the maximum of the perturbation heat source. Regarding velocity, again the changes of the profile occur inside the oven in the range z ~ -0.1 … +0.1 m. Similar to the stationary scenario, the velocity changes are first positive (as long as the center of the perturbation is at position z < -0.1) and later on negative. The results of cross section area A are different compared to the stationary scenario. Inside the oven and slightly below the oven the relative changes of A correspond nearly to the negative relative changes of velocity. At position z > 0.3 m, which is about 0.2 m below the oven, cross section area is increasing. As the relative decrease of cross section area inside the oven is stronger than the relative increase of pulling speed, the volume flow inside the oven is temporarily decreasing, while below the oven it is greater than the nominal volume flow.

Summarizing, it can be stated that the time dependent perturbation scenarios lead to a transient perturbation of the glass flow. As consequence, a complex sequence of changes of the relevant process variables occurs. Due to the movement of the glass, an initial perturbation has a long range impact to the whole forming process.

5. Conclusions

The results of the investigated stationary and time dependent perturbation scenarios can be summarized as follows:

- The largest sensitivity of stationary perturbations of the oven temperature, velocity and cross section area profile is at the top of the oven
- For time dependent scenarios with a moving heat source it turned out that the results lead to a complex spatio-temporal pattern of changes of the relevant process variables. Hence disturbances have to be suppressed as early as possible.

Based on the results in the near future measures for the optimization of the industrial glass forming will be derived. e.g. based on the coupled forming and temperature model, optimal time dependent oven temperature decrease strategies could be calculated in order to minimize the impact of perturbations to the product quality.

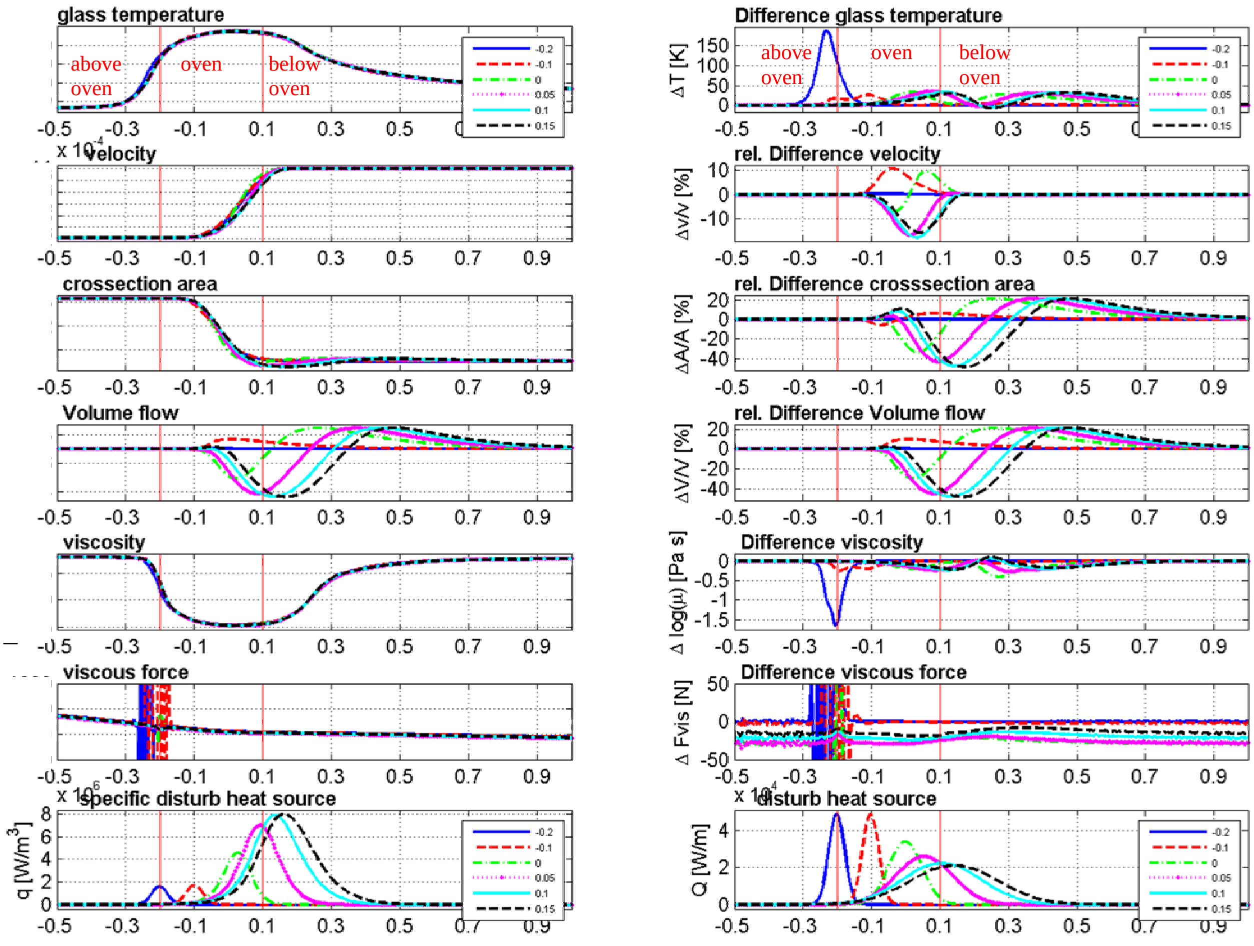

Figure 4. Results with time dependent perturbations (moving heat source)

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