COMSOL CONFERENCE

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The Microplane Model for Concrete in COMSOL

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Agenda

- Aims of the work
- The Microplane Model
  - Main theory aspects
- Implementation in COMSOL
  - How the elastic behaviour was developed
- Few simple examples
- Conclusions

Why a new model for concrete?

- Safety of large civil structures:
  - Often evaluated by means of numerical models based on the Finite Element Method (FEM)
    - Classical approaches are able to properly simulate *only a few specific* characteristic of concrete
    - The Microplane Model seems to be a promising alternative to represent the *overall* behaviour of concrete

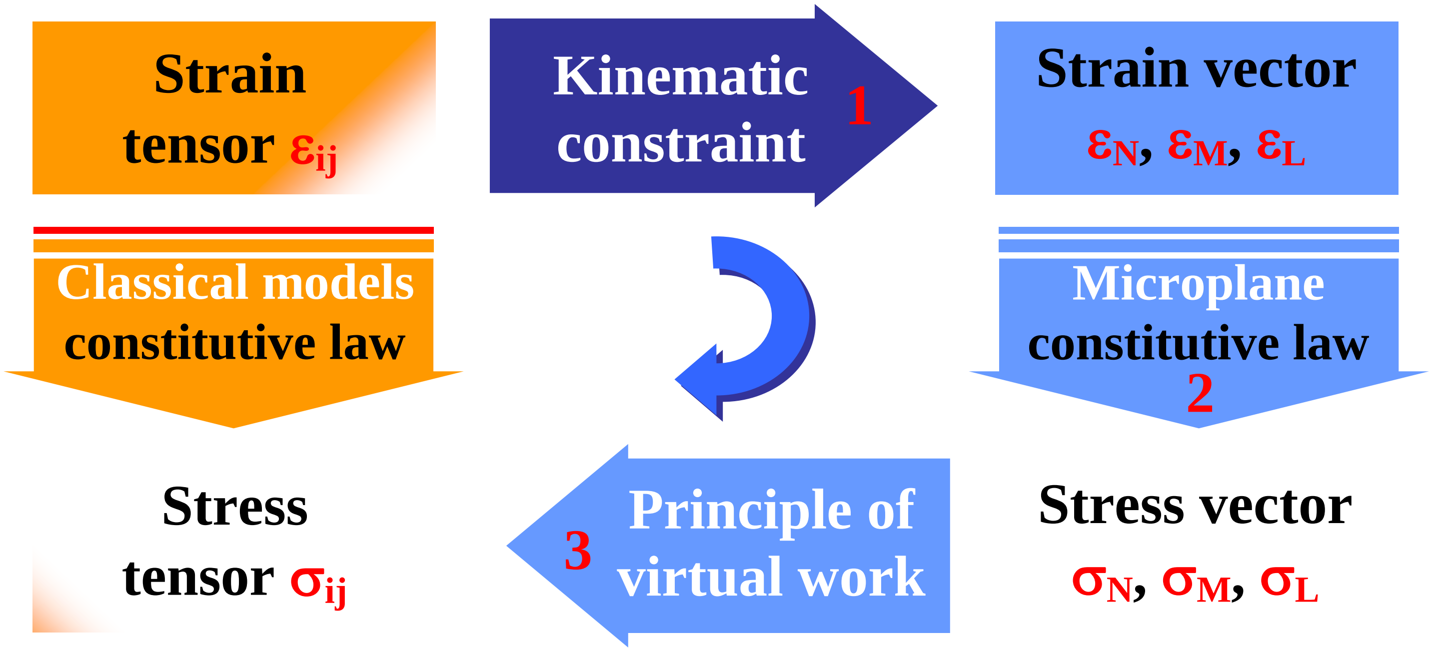
Microplane model: theory

- The major mechanical phenomena of concrete are always referable to a plane whose orientation depends on material microstructures, as well as loading and constraint conditions

- The microplane constitutive law is formulated by means of a relation between the strain and stress vectors acting on a plane whose orientation is arbitrary

Microplane model: theory

- The microplane logical scheme consists of three parts
  - The starting point and the final one are the same of classical approaches



Microplane model: theory

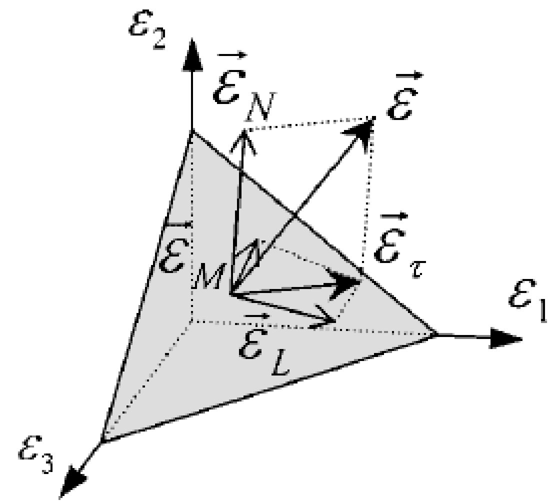
- **<u>Step 1</u>**: application of the kinematic constraint
  - The number and orientation of all microplanes related to a material point should be defined
  - In each material point the strain tensor ε_{ij} is projected on each microplane

$$\varepsilon_i^k = \varepsilon_{ij} n_j$$

$$\varepsilon_N = n_i \left(\varepsilon_{ij} n_j\right) = N_{ij}\, \varepsilon_{ij}$$

$$\varepsilon_M = m_i \left(\varepsilon_{ij} n_j\right) = M_{ij}\, \varepsilon_{ij}$$

$$\varepsilon_L = l_i \left(\varepsilon_{ij} n_j\right) = L_{ij}\, \varepsilon_{ij}$$



Microplane model: theory

- **<u>Step 2</u>**: the microplane constitutive law
  - The normal strain and stress vectors are split into their volumetric and deviatoric parts
  - The volumetric strain and stress are equal for all microplanes
  - The elastic response is defined by means of the elastic incremental relations in the rate form:

$$\dot{\sigma}_V = E_V \dot{\varepsilon}_V \qquad \dot{\sigma}_D = E_D \dot{\varepsilon}_D \qquad \dot{\sigma}_M = E_M \dot{\varepsilon}_M \qquad \dot{\sigma}_L = E_L \dot{\varepsilon}_L$$

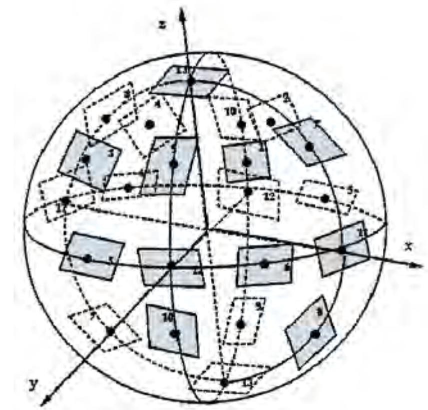
$$E_V = \frac{E}{1-2\upsilon} \qquad E_D = \frac{5E}{(2+3\mu)(1+\upsilon)} \qquad E_T = \mu E_D$$

Microplane model: theory

- **<u>Step 3</u>**: application of the principle of virtual work
  - The static equilibrium is written with reference to the surface **Ω** of a unit hemisphere whose centre is the material point

$$\sigma_{ij} = \frac{3}{2\pi}\int_{\Omega}\left(\sigma_N \cdot N_{ij} + \sigma_M \cdot M_{ij} + \sigma_L \cdot L_{ij}\right)d\Omega$$

Gaussian quadrature formulas of various degrees of approximation are used to solve the integral over the unit hemisphere



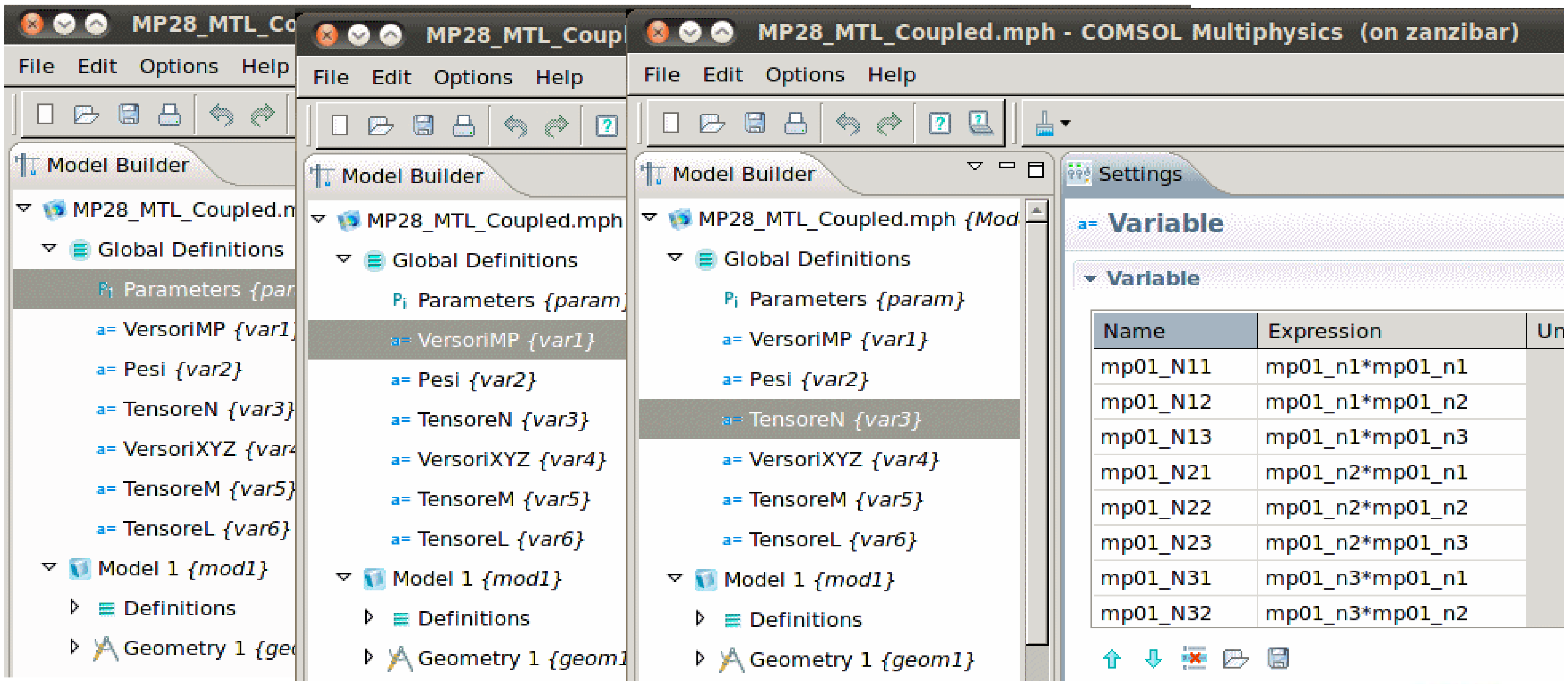
$$\sigma_{ij} \approx 6\sum_{k=1}^{N_{mp}} w_k\left(\sigma_N \cdot N_{ij} + \sigma_M \cdot M_{ij} + \sigma_L \cdot L_{ij}\right)^{(k)}$$

Implementation in COMSOL

- COMSOL does not necessarily require developing subroutines thanks to its user-friendly platform
- The implementation process was arranged in several phases, defining:
  - The global parameters and variables that are referred to all microplanes and they are common to all material points
  - The strain and stress vectors on each microplane
  - The constitutive law governing mechanical quantities at the microplane level

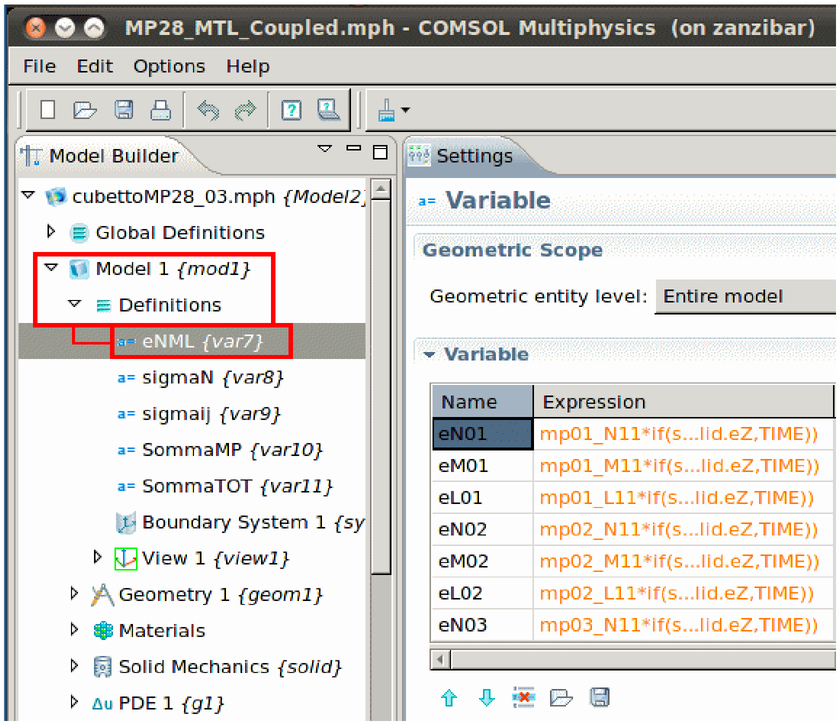
Implementation in COMSOL

- Global parameters and variables definition
  - In the Global Definition node of the Model Builder window



Implementation in COMSOL

- Strain vectors definition
  - In the Definition node that is a part of the Model node



$$\varepsilon_N = n_i \left(\varepsilon_{ij} n_j\right) = N_{ij}\, \varepsilon_{ij}$$

$$\varepsilon_M = m_i \left(\varepsilon_{ij} n_j\right) = M_{ij}\, \varepsilon_{ij}$$

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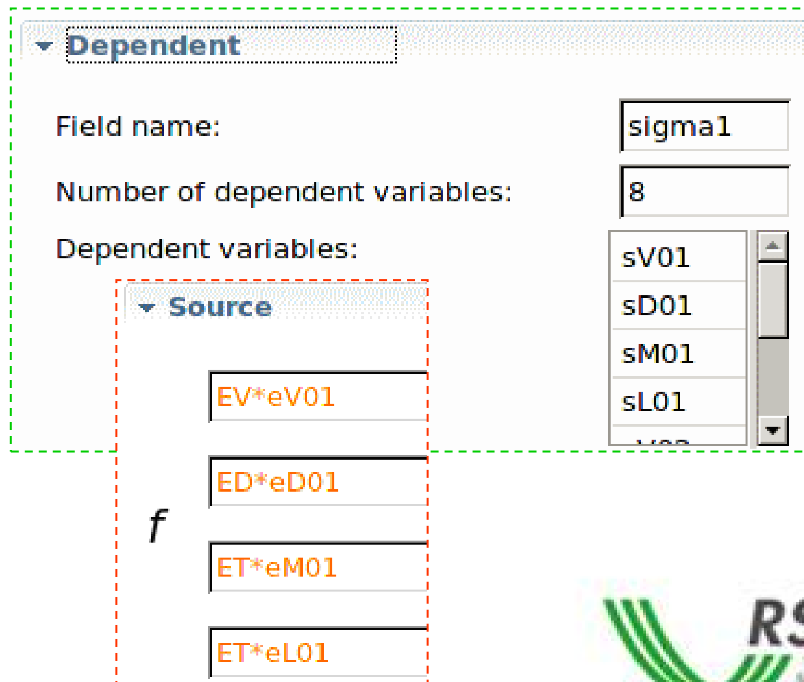
- All strain vector components are defined in the rate form

Implementation in COMSOL

- Constitutive law implemented by means of PDE modules
  - A total of 112 equations resulting from the 4 vector components per 28 microplanes in each material point
  - In the General Form PDE of the Setting window:

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \Gamma = f$$

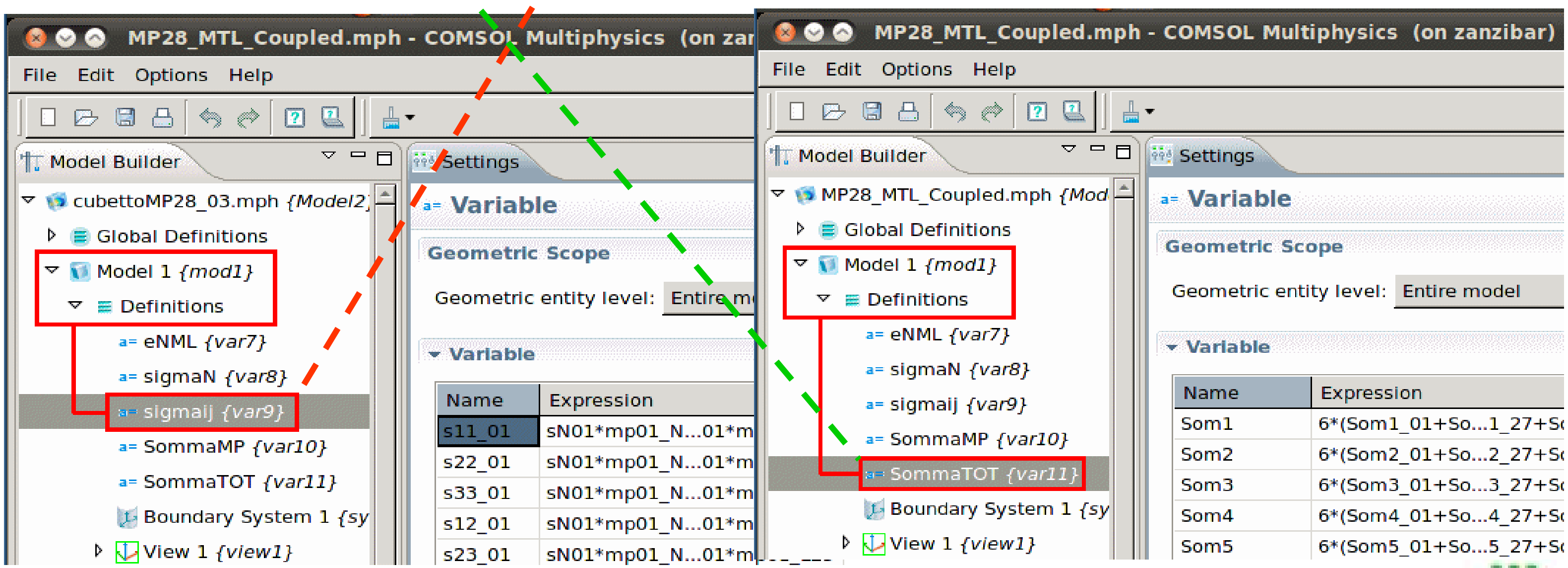
$$\dot{\sigma}_{V,D,M,L} = E_{V,D,T,T} \dot{\varepsilon}_{V,D,M,L}$$



Implementation in COMSOL

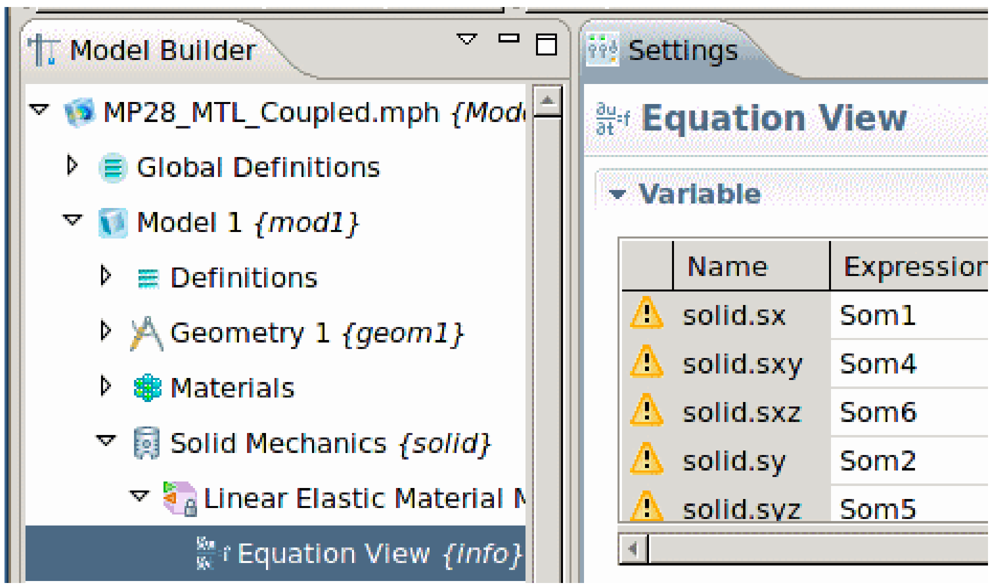
- Stress vectors definition
  - In the Definition node that is a part of the Model node

$$\sigma_{ij} \approx 6\sum_{k=1}^{N_{mp}} w_k \left(\sigma_N \cdot N_{ij} + \sigma_M \cdot M_{ij} + \sigma_L \cdot L_{ij}\right)^{(k)}$$



Implementation in COMSOL

- The Solid Mechanics module was added to study the mechanical behaviour of concrete structures
  - The dependent variables are the displacement field: *u*, *v* and *w*
  - This module is coupled with the PDE modules as follows:

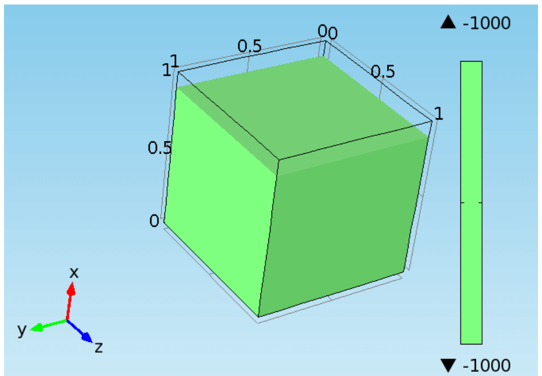


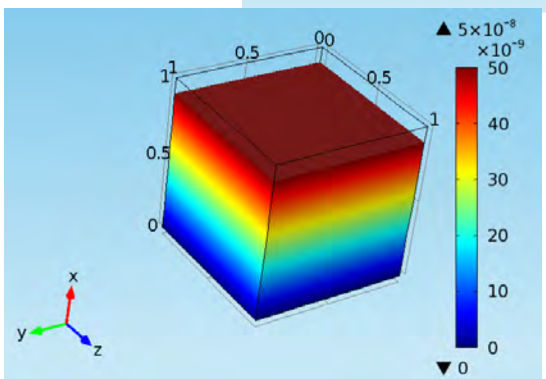
- $Som1 = \sigma_{xx}$
- $Som2 = \sigma_{yy}$
- $Som3 = \sigma_{zz}$
- $Som4 = \sigma_{xy}$
- $Som5 = \sigma_{xz}$
- $Som6 = \sigma_{yz}$

Simple applications

- The elastic behaviour was verified on a concrete cube

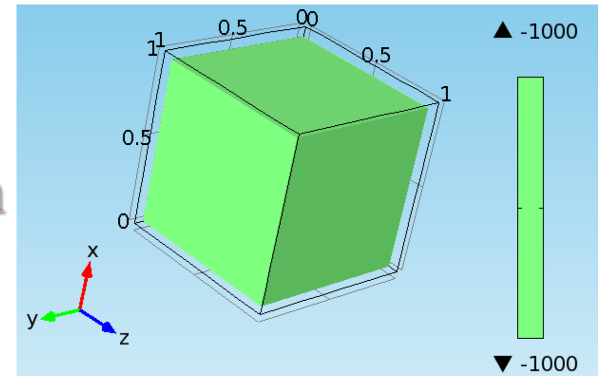
Uniaxial compression

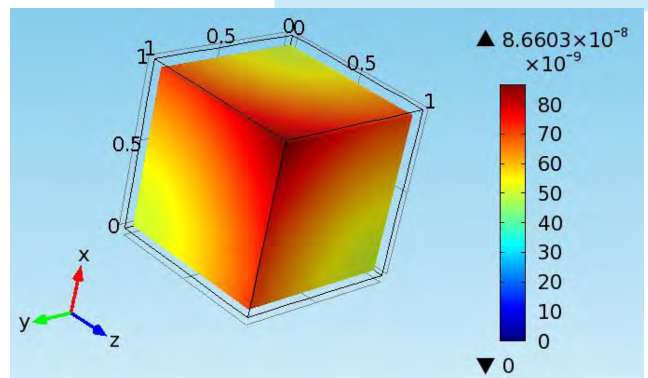




$E = 2.0e+10\ N/m^2$
$p = 1000\ N/m^2$

Triaxial compression

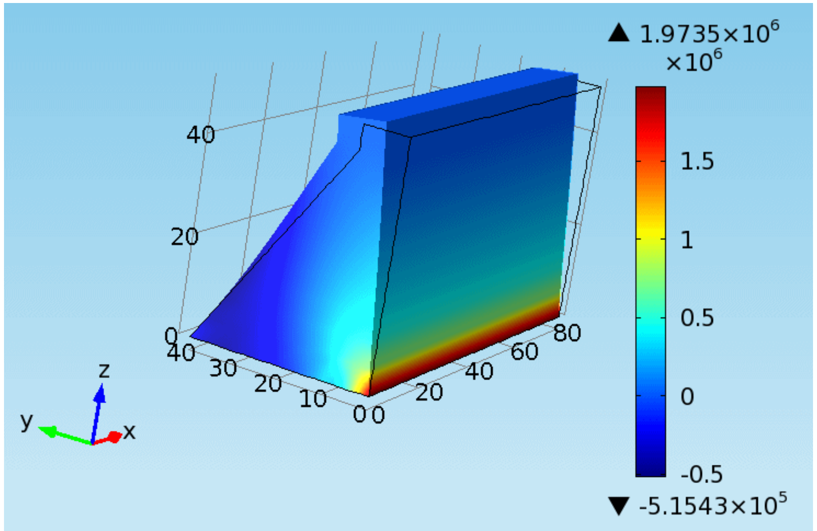




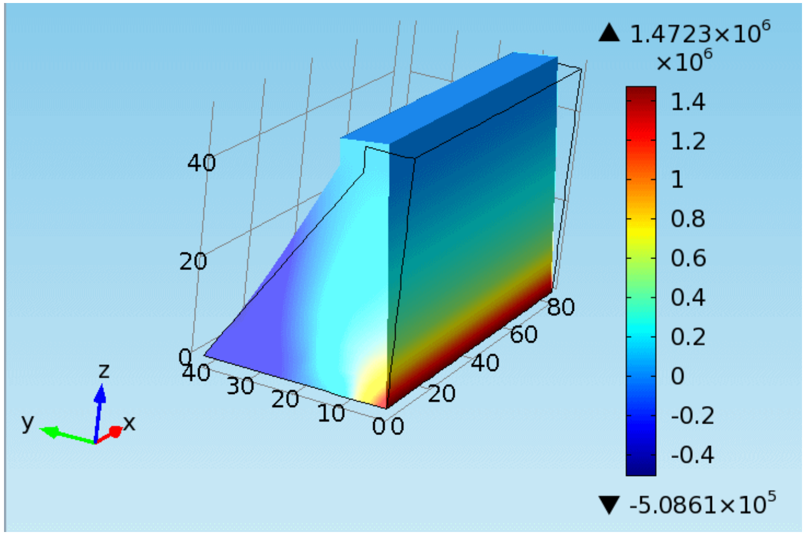
Simple applications

- The elastic behaviour was verified on a concrete gravity dam

Linear elastic model



Microplane model



Conclusions

- COMSOL has shown a good capacity to manage a complex implementation process
- 3D constitutive laws could be easily implemented without writing complex user subroutines
- The implementation of the non-linear behaviour of concrete is in progress
  - No external user subroutines are required
  - The implementation methodology is similar to that used for the linear elastic part

Thank you

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