Finite element modelling of three-phase non-isothermal flow in heavy oil reservoirs- case study SAGD

Taofik H. Nassan[1], Mohd Amro[1]
1. Institute of Drilling and Fluid Mining Engineering, Freiberg University of Technology, 09599-Freiberg, SN, Germany

Abstract

The application of FEM in reservoir engineering has been introduced in the early 1970s but has not gained much popularity due to high computational expense.
With faster solvers in COMSOL Multiphysics 5.5, we try to reformulate the problem of non-isothermal three-phase flow during steam assisted gravity drainage (SAGD). The problem has been solved previously by Comsol along with Matlab code [1]. In this short communication we formulate the governing equations using total flux concept which end up in one pressure equation, two saturation equations, and an energy equation. This set of PDE’s can be solved easily in Comsol using equation based module.
The results show that using an alternative formulation of the multiphase flow equations, which is convenient for the FEM discretization in COMSOL, the physical behavior of the steam injection process can be modelled in a clear and user-accessible form.

Keywords: multiphase fluid flow, non-isothermal flow, thermal enhanced oil recovery, steam injection, heavy oil

1. Introduction

Thermal enhanced oil recovery (TEOR) methods form the majority of the enhanced oil recovery projects worldwide. Among the thermal methods, steam injection is the most effective and most applied technique in petroleum industry. The main mechanism of the steam injection method is the reduction of heavy oil viscosity by introducing heat into the reservoir. The mathematical description of the steam injection process involves multiphase fluid flow and heat transfer in porous media. Its numerical solution enables engineers to analyze incremental oil recovery and field performance.

For commercial reservoir simulators, the finite-difference method is the dominant numerical technique to solve the governing equations. However, in recent decades, finite-element method (FEM) has been adopted favorably in many fields due to its accurate solution and solid fundamental theory.
In this short communication we formulate the governing equations of non-isothermal flow in porous media using total flux concept which end up in one pressure equation, two saturation equations, and an energy equation. This set of PDE’s can be solved easily in Comsol using equation based module. The resulted model is a general form for steam injection in porous media and to illustrate the feasibility of the model we implement it for SAGD configuration as an example. The problem has been solved previously by Comsol along with Matlab code [1]. However, the proposed formulation and solution routine eliminates the need for external code.
The rest of the article is organized as the following; the mathematical model is introduced in the next chapter then model implementation in Comsol followed by the results, and finally the conclusion.

2. Mathematical model

The fluids that coexist in the formation during steam injection in an oil formation are steam (s), water (w), and oil (o). Therefore, the problem is considered as three phase (s, w, o) and two components (H_2O, oil). The mass balance equations can be formulated allowing the mass transfer between the steam and water. Oil is considered non-volatile in favor of fast solution and less complexity of the mathematical model.

2.1 Fluid flow model

Continuity equation for oil, water, and steam is defined as the following [2, 3, 4, 5, 6]:

$$\frac{\partial(\varphi\rho_o S_o)}{\partial t} + \nabla.\ (\rho_o u_o) = -q_{o,pro} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

$$\frac{\partial(\varphi\rho_w S_w)}{\partial t} + \nabla.\ (\rho_w u_w) = -q_{w,pro} + q_{w,inj} + q_{cond,w} \ldots\ldots\ldots (1b)$$

$$\frac{\partial(\varphi\rho_s S_s)}{\partial t} + \nabla.\ (\rho_s u_s) = -q_{s,pro} + q_{s,inj} - q_{cond,s} \ldots\ldots\ldots (1c)$$

Where:
Subscripts *o*, *w*, and *s* stand for oil, water, and steam respectively.
Equations (1b) and (1c) can be added to eliminate the condensation terms of steam and water (the last term in both equations) which yields:

$$\varphi\frac{\partial}{\partial t}(\rho_w S_w + \rho_s S_s) + \nabla.\ (\rho_w u_w + \rho_s u_s) = -q_{w,pro} + q_{w,inj} - q_{s,pro} + q_{s,inj} \ldots (2)$$

Momentum equation is represented by Darcy’s law for the three-phase flow and reads:

$$u_o = -\frac{k_o k}{\mu_o}(\nabla p_o - \rho_o g \nabla z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

$$u_w = -\frac{k_w k}{\mu_w}(\nabla p_w - \rho_w g \nabla z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

$$u_s = -\frac{k_s k}{\mu_s}(\nabla p_s - \rho_s g \nabla z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3c)$$

2.2 Heat transfer model

Energy equation can be written for each phase including an equation for the matrix as:

$$\frac{\partial}{\partial t}(\varphi\rho_o S_o U_o)+\nabla.(\rho_o u_o h_o)-\nabla.(k_o\nabla T)-Q_{O,pro}=0\ldots\ldots\ldots\ldots(4a)$$

$$\frac{\partial}{\partial t}(\varphi\rho_w S_w U_w)+\nabla.(\rho_w u_w h_w)-\nabla.(k_w\nabla T)+Q_{w,inj}-Q_{w,pro}=0\ldots\ldots\ldots\ldots(4b)$$

$$\frac{\partial}{\partial t}(\varphi\rho_s S_s U_s)+\nabla.(\rho_s u_s h_s)-\nabla.(k_s\nabla T)+Q_{s,inj}-Q_{s,pro}=0\ldots\ldots\ldots\ldots(4c)$$

$$\frac{\partial}{\partial t}((1-\varphi)\rho_m U_m)-\nabla.(k_m\nabla T)=0\ldots\ldots\ldots\ldots(4d)$$

Where:
m subscript refers to matrix
Specific internal energy of phase α $\quad U_\alpha=C_{v\alpha}T$
Enthalpy of phase α $\quad h_\alpha=C_{P\alpha}T$
Considering local thermal equilibrium condition in the reservoir [8] and by summing the above four equations (4a, 4b, 4c, and 4d):

$$\begin{aligned}\frac{\partial}{\partial t}(\varphi\rho_o S_o U_o+\varphi\rho_w S_w U_w+\varphi\rho_s S_s U_s+(1-\varphi)\rho_m U_m)&\\ +\nabla.(\rho_o u_o h_o+\rho_w u_w h_w+\rho_s u_s h_s)&\\ -\nabla.(k_T\,\nabla T)-Q_{O,pro}+Q_{w,inj}-Q_{w,pro}&\\ +Q_{s,inj}-Q_{s,pro}=0\ldots\ldots\ldots\ldots&(5)\end{aligned}$$

Where

$$k_T=k_o\varphi\,S_o+k_s\varphi\,S_s+\;k_w\varphi\,S_w+(1-\varphi)k_m$$

2.3 Mathematical formulation of the model

Total velocity is defined as the sum of the fluid velocity of all fluids in the pore space [3, 4]:

$$\vec{u}=\overrightarrow{u_o}+\overrightarrow{u_w}+\overrightarrow{u_s}\ldots\ldots\ldots\ldots(6\,)$$

Summing equations (1) yields:

Pressure equation

$$\nabla.\vec{u}=-\varphi\left[\frac{S_w}{\rho_w}\frac{\partial\rho_w}{\partial t}+\frac{S_s}{\rho_s}\frac{\partial\rho_s}{\partial t}+\frac{S_o}{\rho_o}\frac{\partial\rho_o}{\partial t}\right]-\frac{\overrightarrow{u_w}}{\rho_w}\,\nabla\rho_w-\frac{\overrightarrow{u_s}}{\rho_s}\,\nabla\rho_s-\frac{\overrightarrow{u_o}}{\rho_o}\,\nabla\rho_o\ldots\ldots\ldots\ldots(7)$$

Water saturation equation can be obtained from (2)

$$\begin{aligned}\varphi\,\frac{\partial S_w}{\partial t}+\nabla.\overrightarrow{u_w}&\\ =-\frac{1}{\rho_w}\overrightarrow{u_w}\,\nabla\rho_w-\varphi\rho_w S_w\,\frac{\partial\rho_w}{\partial t}+\frac{\rho_s}{\rho_w}\left(\varphi\frac{\partial S_s}{\partial t}-\nabla.\overrightarrow{u_s}\right)&\\ -\frac{1}{\rho_w}\left(\varphi S_s\,\frac{\partial\rho_s}{\partial t}+\overrightarrow{u_s}\,\nabla\rho_s\right)\ldots\ldots\ldots\ldots&(8)\end{aligned}$$

Oil saturation equation for (1a)

$$\varphi\,\frac{\partial S_o}{\partial t}+\nabla.\overrightarrow{u_o}=-\frac{1}{\rho_o}\left[\varphi\,\frac{\partial(\rho_o S_o)}{\partial t}+\overrightarrow{u_o}\,\nabla\rho_o\right]\ldots\ldots\ldots(9)$$

Energy equation can be arranged after taking latent heat of steam into consideration as

$$\begin{aligned}\left(\varphi\rho_o S_o C_{po}+\varphi\rho_w S_w C_{pw}+\varphi\rho_s S_s C_{ps}+(1-\varphi)\rho_m C_{pm}\right)\frac{\partial T}{\partial t}&\\ +\left(\rho_o u_o C_{po}+\rho_w u_w C_{pw}+\rho_s u_s C_{ps}\right)\nabla.T&\\ -\nabla.(k\,\nabla T)+L_v(p)\left[\varphi\,\frac{\partial(\rho_s S_s)}{\partial t}+\overrightarrow{u_s}\,\nabla\rho_s\right]&\\ -Q_{O,pro}+Q_{w,inj}-Q_{w,pro}+Q_{s,inj}&\\ -Q_{s,pro}=0\ldots\ldots\ldots\ldots&(10)\end{aligned}$$

Where L_v is latent heat of steam

2.4 Rock and fluid properties

Three-phase Relative permeability is calculated from Stone II [7]

$$k_{ro}(S_w,S_g)=(k_{row}+k_{rw})(k_{rog}+k_{rg})-(k_{rw}+k_{rg})\ldots\ldots\ldots\ldots(11)$$

Steam water equilibrium temperature depends on pressure:

$$T=T_s(p)\ldots\ldots\ldots\ldots(12)$$

The relation among phases' saturations:

$$\sum S_i=S_o+S_w+S_s=1\ldots\ldots\ldots\ldots(13)$$

Capillary pressure relations between every two phases reads:

$$p_{cow}(S_w,T)=p_o-p_w\ldots\ldots\ldots\ldots(14)$$
$$p_{cso}(S_s,T)=p_s-p_o\ldots\ldots\ldots\ldots(15)$$

Assuming that the fractional flow is only dependent on saturations, capillary pressure can be written as:

$$\nabla p_c=f_w\nabla p_{cow}-f_s\nabla p_{cso}\ldots\ldots\ldots\ldots(16)$$

Defining the global pressure by:

$$\nabla p=\nabla p_o+\nabla p_c\ldots\ldots\ldots\ldots(17)$$

For further simplification, the capillary pressure will be neglected which means that the global pressure is the oil pressure:

$$\nabla p=\nabla p_o\ldots\ldots\ldots\ldots(18)$$

3. Model implementation for SAGD configuration

Relatively few authors demonstrated the modelling of heat transfer in steam-injection process with COMSOL Multiphysics. Bogdanov et al [1] simulated SAGD process in 2D with Comsol along with Matlab Code. Nassan & Amro [9] simulated water flooding in 3D by Comsol and proved to be relatively fast simulator. Nassan [10] solved the steam flooding process with the energy equation in dimensionless form to investigate the effect of the convection and conduction terms on the temperature propagation through the reservoir. He noted that at higher fluid velocity, heat transfer within the reservoir is dominated by the convection mechanism, while the conduction mechanism dominates in steam injection process with relatively low velocity. He also concluded that the most important factors to rise the reservoir temperature, are the injection time and injected steam volume.
In this section, a steam injection model and its simulation results are presented. The operational and reservoir parameters in the model are based on the input data in [1]. Water and steam thermodynamic properties are adapted from MINI-REFPROP open source program as a function of temperature. The relative permeability data are adapted from [11] and the relative permeability of oil is calculated by Stone II model for three-phase flow [7].
A reservoir model is built in Comsol 5.5” in 2 dimensional plane with 36 m length and 15 m height. Injection well is placed at 8 m offset from the top and the production well is placed at 2 m from the bottom of the reservoir as illustrated in figure (1). The distance between the wells is 5 m which is typical SAGD configuration. The initial pressure distribution in the reservoir is at the hydrostatic condition. The 100% quality steam is injected with $u_s = 2 * 10^{-6}$ m/s velocity. The production well maintains the constant initial pressure, thus a small differential pressure between the two horizontal wells is resulted.

Tables (1) and (2) show the main parameters and initial and boundary conditions applied in the model.

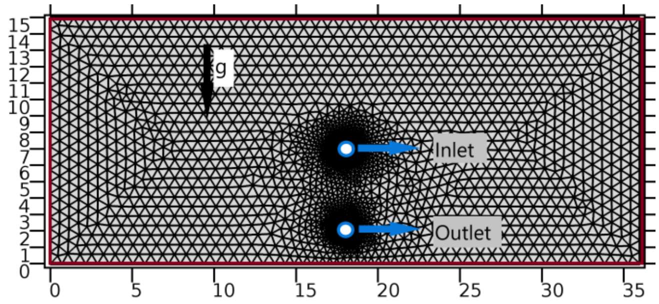


Figure (1) Geometry and mesh generation of SAGD configuration

Table (1) Main parameters in the model

Name	**Value**	**Description**
T_inj	450 K	Injection temperature
dPinj	45000 Pa	Injection pressure difference
phi	0.32	Porosity
k	9.8692E-13 m²	absolute permeability
L	15 m	Total thickness
rho_o_ini	900 kg/m³	Oil Density
mu_o_ini	0.1 Pa·s	Initial oil viscosity
p_ini	1E6 Pa	Reservoir initial pressure
Sw_ini	0.2	Initial water saturation
So_ini	0.8	Initial oil saturation
Cp_o	2093.4 J/(kg·K)	Heat capacity of oil
Cp_r	1055.1 J/(kg·K)	Heat capacity of rock
T_ini	298.15 K	Initial temperature
p_top	8.785E5 Pa	Pressure at top of reservoir
e	2E-6 m²/s	Numerical diffusivity
rho_r	2224 kg/m³	Rock density
Sor	0.15	Residual oil saturation
M	0.68	
k_r	2.6 W/(m·K)	Rock thermal conductivity
k_w	0.7 W/(m·K)	Water thermal conductivity
k_o	0.387 W/(m·K)	Oil thermal conductivity
k_g	0.0039 W/(m·K)	Steam thermal conductivity

Table (2) Initial and boundary conditions

	Initial	inlet	outlet	Outer boundary
p	p_{ini}	$u=u_{in}$	$p=p_{ini}$	$-n.\ u=0$
S_w	S_{w_ini}	$S_w= S_{wc}$	$S_w= S_{wc}$	$-n.(-c\ \nabla S_w+ u_w)=0$
S_o	S_{o_ini}	$S_o= S_{or}$	$-n.(-c\ \nabla S_o+ u_o)=-u_o$	$-n.(-c\ \nabla S_o+ u_o) =0$
T	$T=T_{inj}$	*Convective flux*	*No flux*	

Pressure equation (7) is applied in the general form PDE in Comsol which reads [12]:

$$\boldsymbol{e_a}\frac{\boldsymbol{\partial^2 u}}{\boldsymbol{\partial t^2}}+\boldsymbol{d_a}\frac{\boldsymbol{\partial u}}{\boldsymbol{\partial t}}+\boldsymbol{\nabla.\Gamma}=\boldsymbol{f} \ldots\ldots\ldots\ldots\ldots\ldots.\ldots..(19)$$

Where

$$\boldsymbol{\nabla}=\left(\frac{\boldsymbol{\partial}}{\boldsymbol{\partial x}},\frac{\boldsymbol{\partial}}{\boldsymbol{\partial y}}\right)$$

Γ Consists of the components of velocity u_x and u_y
f is the right hand side of equation (7)

Equations (8), (9), and (10) are applied in the coefficient form PDE in Comsol which is written in the form:

$$\boldsymbol{e_a}\frac{\boldsymbol{\partial^2 u}}{\boldsymbol{\partial t^2}}+\boldsymbol{d_a}\frac{\boldsymbol{\partial u}}{\boldsymbol{\partial t}}+\boldsymbol{\nabla.(-c\nabla u-\alpha u+\gamma)+\beta.\nabla u+a u} \\ =\boldsymbol{f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots.....(20)$$

Where f in equation (20) is the right hand side of equations (8) and (9), while f in equation (10) is the term:

$$L_v(p)\left[\varphi\ \frac{\partial(\rho_s S_s)}{\partial t}+\overrightarrow{u_s}\,\nabla\rho_s\right]$$

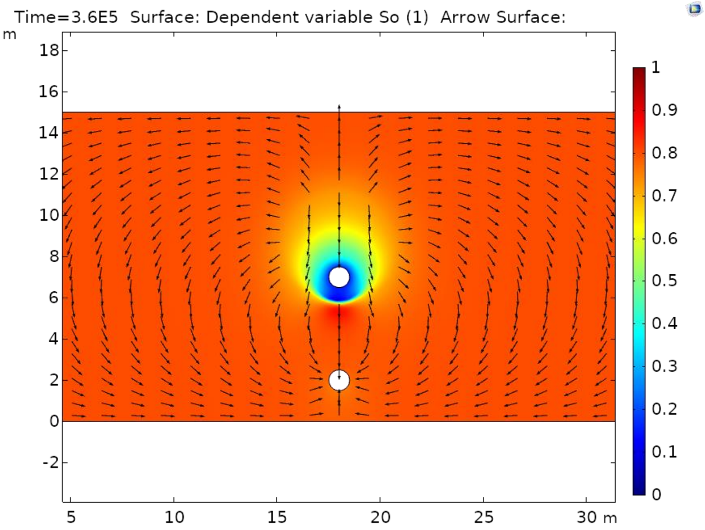
Mobility and velocity components of all phases are applied in the variables node under definitions. Relative permeability data and thermal dependent properties of oil, steam, and water are all applied as interpolation functions in definition node as well.

4. Results

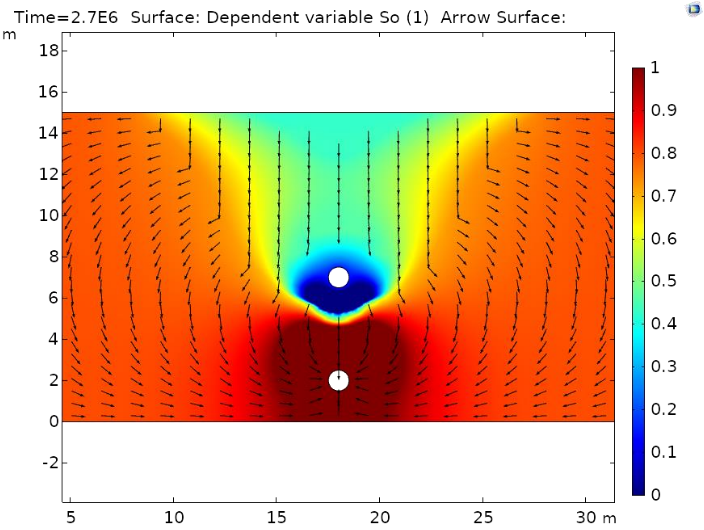
Figure (2) illustrates the change of oil saturation after different time steps for SAGD process. The figure show the growth of the steam chamber above and around the injection well. The computation time was 6 minutes where the system equations (7), (8), (9) and (10) is solved simultaneously. Figures (3) and (4) also show the steam saturation and temperature distribution after 75 days. The shape of the steam chamber can be recognized from these two parameters as well.

5. Conclusion and future work

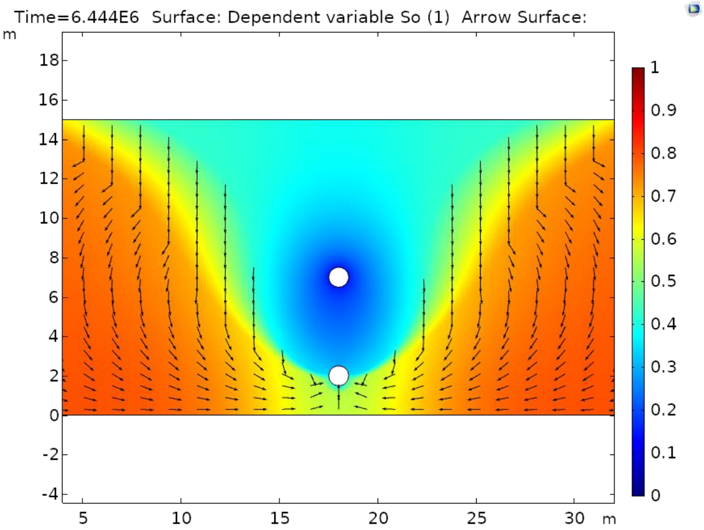
The results show that using an alternative formulation of the multiphase flow equations, which is convenient for the FEM discretization in COMSOL, the physical behavior of the steam injection process can be modelled in a clear and user-accessible form. The introduced model had many assumptions that simplified the problem considerably. The oil is considered as non-volatile fluid. The next step in this direction will be considering compositional model that will allow volatile hydrocarbon components to be present in gas phase along with steam.



(a) After 4 days



(b) After 31 days



(c) After 75 days

Figure (2) Oil saturation (So) distribution after different time steps of the kick off of SAGD process (a, b, c)

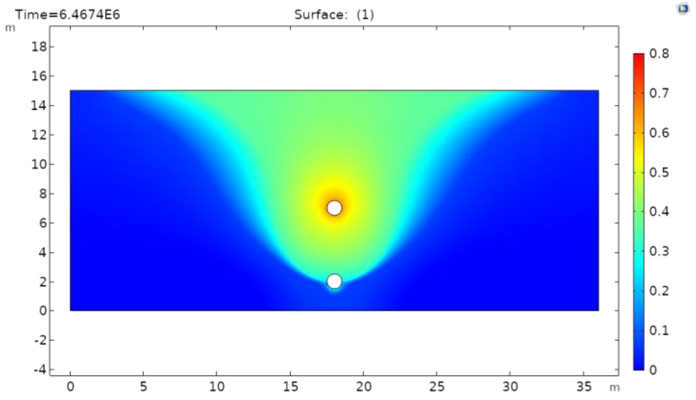


Figure (3) Steam saturation after 75 days

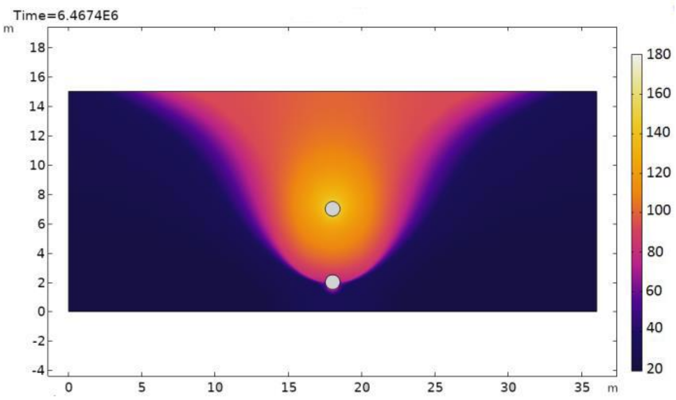


Figure (4) Temperature distribution (degC) after 75 days

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