Problem: Find the function u and a domain $\Omega = \{x : u(x) > 0\}$ such that

$$
\text{(P)} \qquad \begin{cases} \Delta u = \chi_\Omega - \mu, & \text{in} \quad \mathbb{R}^N, \\ u \geq 0, & \text{in} \quad \mathbb{R}^N, \\ u = 0, & \text{in} \quad \mathbb{R}^N \setminus \Omega, \end{cases}
$$

where μ is a given positive measure with compact support in Ω. The χ_Ω denotes characteristic function of Ω.

The domain Ω is called quadrature domain for μ.

Now as a special case suppose that $\mu = t(\chi_{B_1} + 2\chi_{B_2})$ is uniformly distributed on two circles $B_1(x_1, 1), B_2(x_2, 1)$ where $x_1 = (-2, 0), x_2 = (\sqrt{8}, 0)$. We can prove analytically that if $t = 4$ then the corresponding quadrature domain is consist of two balls touch each other at origin tangentially as you see in the Comsol file. Now for the next step we solve the following problem for $t = 5$

$$
(1) \qquad \begin{cases} \Delta u = 1 - t(\chi_{B_1} + 2\chi_{B_2}), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}
$$

to get the corresponding quadrature domain. Then we would like to move the boundaries with the velocity field $(ux/2, uy/2)$ to get the new geometry and do the similar steps on the new geometry. We continue this process to get the other boundary condition $\nabla u = 0$ on $\partial\Omega$.