Numerical Modelling

The numerical model represents the skeleton of the project as it will allow the computer software to assess the behaviour of gas in the formation. Simply speaking, the numerical model is a mathematical expression relating the various reservoir parameters to the flow behaviour. Absorption, matrix shrinkage, and anisotropy are typically encountered in a coal seam reservoir and consequently the model will be designed to take them into account.

The dual porosity nature of the model require us to define two systems: one for the matrix and another for the fractures or cleats. Assuming single phase gas flow, the material balance equation for both systems looks as such:

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| $$\frac{∂}{∂t}\left(ϕ\_{f}ρ\_{f}\right)+∇\left(ρ\_{f}q\right)=Q\_{s}$$ | (1) |
| $$\frac{∂}{∂t}\left(ϕ\_{m}ρ\_{m}\right)+∇\left(ρ\_{m}q\right)=-Q\_{s}$$ | (2) |

Where $ϕ$ is the porosity of the system, $ρ$ is the density of the gas, $ϕρ$ represents the storage term, $q$ represents the gas flow rate, $k$ is the permeability of the system, $μ$ is the viscocity of the gas, $p$ is the pressure of the system, $Q\_{s}$ represents the exchange term between the two systems, and the subscripts f and m represent the fracture and matrix systems respectively.

The matrix system has the peculiarity of having a high storage capacity but almost negligible permeability. For this reason, any flow within the matrix is very small or even negligible (Barenblatt et al. 1960; Gilman et al. 2000; Kazemi et al. 1969). Ignoring the internal flow we can simplify equation 2 by eliminating the flow term as such

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| $$\frac{∂}{∂t}\left(∅\_{m}ρ\_{m}\right)=-Q\_{s}$$ | (3) |

Now, taking into consideration the Klinkernberg’s effect, the gas flow rate on equation (1) is given by Turgay et al on their study (1986) as:

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| $$q=-\frac{k\_{e}}{μ}∇p\_{f}$$ | (4) |
| $$k\_{e}=k\left(1+\frac{β\_{k}}{p\_{f}}\right)$$ | (5) |

Where $k$ is the absolute permeability, $β\_{k}$ is the Klinkenberg’s coefficient, and $p\_{f}$ represents the pressure in the fracture system. On a coal seam gas reservoir, however, the permeability $k$ is not constant. Cleat permeability may change due to gas desorption and as a response to decreasing gas pressures. Taking into account these two factors, Gilman et al. (2000) found that the permeability can be expressed as a function of fracture pressure as such:

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| $$k=k\_{0}exp\left(\frac{3ν}{1-ν}\frac{Δp\_{F}}{E\_{F}}\right)exp\left(-\frac{3αE}{1-ν}\frac{ΔS}{E\_{F}}\right)$$ | (6) |

Where $k\_{0}$ is the initial permeability, $ν$ is the Poisson’s ration of the formation, $E$ is the youngs modulus of the fracture, S is the change in absorbate mass, and $α$ is the volumetric swelling coefficient.

The exchange term between the two systems, $Q\_{s}$, is due predominantly to the gas release from the coal matrix into the fractures (Gilman et al. 2000). Gas desorption is described by a Knudsen diffusion profile (Rinker et al. 1979) and can be approximated to be (gilman et al. 2000)

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| $$Q\_{s}=\frac{M}{RT}\frac{D^{\*}}{λ}\frac{p\_{m}-p\_{f}}{λ}$$ | (7) |

Where $D^{\*}$ is the effective diffusion coefficient, $M$ is the molar mass of methane, R is the universal gas constant, T is the absolute temperature, $λ$ is the average distance between fractures, and $p\_{m}$ is the pressure in the matrix system.

As expressed by Gilman et al. (2000), the matrix storage term can be expressed through the dubinin-astakhov sorption model is

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| $$∅\_{m}ρ\_{m}=V\_{L}\frac{p\_{st}M}{RT\_{st}}exp\left[-\left(\frac{RT}{βE}ln\frac{p\_{s}}{p}\right)^{2}\right]$$ | (8) |

Where $V\_{L}$is the Langmuir volume, R is the universal gas constant, $T$ is the reservoir temperature, $β$ is the sorbate affinity coefficient, $E$ is the characteristic energy [psi-ft3/lb-mole], $p\_{s}$is the saturation vapour pressre [atm], $p\_{st}$ is the standard pressure, and $T\_{st}$is the standard temperature. Note that

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| $$p\_{s}=p\_{c}\left(\frac{T}{T\_{c}}\right)^{2}$$ | (9) |

Gas flow inside CSG formations sometimes occur at very high speeds. These speeds can deviate fluid behaviour from Darcy’s law. In their study, Wang et al. (2012) devised a mathematical equation describing the behaviour of such flow.

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| $$-\frac{k\_{e}}{μ}ρ\_{f}∇p\_{f}=-\frac{k\_{e}}{μ}\left(δρ\_{g}\right)∇p\_{f}$$ | (10) |

Where $δρ\_{g}$ is regarded as the correction of density. Taking equations 3-9 into equations 1-2 we end up with the two systems

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| $$ϕ\_{f}\frac{∂}{∂t}\left(p\_{f}\right)+∇\left(-\frac{k\_{e}}{μ}\left(δρ\_{g}\right)∇p\_{f}\right)=\frac{M}{RT}\frac{D^{\*}}{λ}\frac{p\_{m}-p\_{f}}{λ}$$ | (11) |
| $$\frac{∂}{∂t}\left(V\_{L}\frac{p\_{st}M}{RT\_{st}}exp\left[-\left(\frac{RT}{βE}ln\frac{p\_{s}}{p}\right)^{2}\right]\right)=-\frac{M}{RT}\frac{D^{\*}}{λ}\frac{p\_{m}-p\_{f}}{λ}$$ | (12) |

Comsol’s default equation is presented as such

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| $$δ\_{s}S\frac{∂p\_{i}}{∂t}+∇\left[-δ\_{k}\left(\frac{k\_{s}}{η}\right)\left(∇p\_{i}+ρ\_{f}g∇D\right)\right]=δ\_{Q}Q\_{s}$$ | (13) |

To fit equations 11 and 12 to equation 12 we need to simplify equation 12