

# Eigenmodes of a Room

## *Introduction*

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Resonance can at times be a problem in everyday life. The low bass notes from the music system or home theater in the living room can shake the windows and make the floor vibrate. This happens only for certain frequencies—the eigenfrequencies of the room.

It is only in the low-frequency range that the eigenfrequencies are well separated. In the mid- and high-frequency ranges, the eigenfrequencies are packed so closely, with less than a half-tone between them, that the individual resonances are insignificant for music and other natural sounds. Nevertheless, the music experience is affected by the acoustics of the room.

When designing a concert hall, it is extremely important to take the resonances into account. For a clear and neutral sound, the eigenfrequencies should be evenly spaced. For the home theater or music system owner, who cannot change the shape of the living room, another question is more relevant: Where should the speakers be located for the best sound?

## *Model Definition*

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For example, take a room with the dimensions 5 by 4 by 2.6 meters equipped with a flat-screen TV, a sideboard, two speakers, and a couch. To illustrate the effects on the music, compute a few resonance frequencies in the vicinity of 90 Hz together with the corresponding eigenmodes. The eigenmode shows the sound intensity pattern for its associated eigenfrequency. From the characteristics of the eigenmodes, you can draw some conclusions as to where the speakers should be placed.

### **DOMAIN EQUATIONS**

Sound propagating in free air is described by the wave equation:

$$-\Delta p + \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

where  $p$  is the pressure, and  $c$  is the speed of sound. If the air is brought into motion by a harmonically oscillating source, for example, a loudspeaker, only one frequency  $f$

exists in the room. For that reason it makes sense to look for a time-harmonic solution on the form

$$p = \hat{p} e^{i\omega t}$$

The wave equation then simplifies to the Helmholtz equation for  $p$ , the amplitude of the acoustic disturbances:

$$\Delta \hat{p} + \frac{\omega^2}{c^2} \hat{p} = 0$$

### BOUNDARY CONDITIONS

This model assumes that all boundaries—walls, floor, ceiling, and furniture—are perfectly rigid (sound hard boundaries). This means that it returns no information of the damping properties of the room, but the distribution of the pressure should still be reasonably correct.

### ANALYTIC COMPARISON

It is possible to solve the simpler case of an empty room analytically. Each eigenfrequency corresponds to an integer triple  $(i, l, m)$ :

$$f_{i,l,m} = \frac{c}{2} \sqrt{\left(\frac{i}{L_x}\right)^2 + \left(\frac{l}{L_y}\right)^2 + \left(\frac{m}{L_z}\right)^2}$$

The eigenmodes can be divided into three distinct classes:

- Eigenfrequencies with only one index different from zero give rise to axial modes, that is, plane standing waves between two opposite walls.
- If one index is zero, the mode is tangential.
- If all indices are different from zero, the mode is oblique.

Theoretical resonance frequencies below 100 Hz for a room without furniture are found in the following table.

MODE INDEX	FREQUENCY	MODE INDEX	FREQUENCY
0,0,0	0	0,1,1	78.7
1,0,0	34.3	2,1,0	80.9
0,1,0	42.9	0,2,0	85.8
1,1,0	54.9	1,1,1	85.8
0,0,1	66.0	1,2,0	92.4

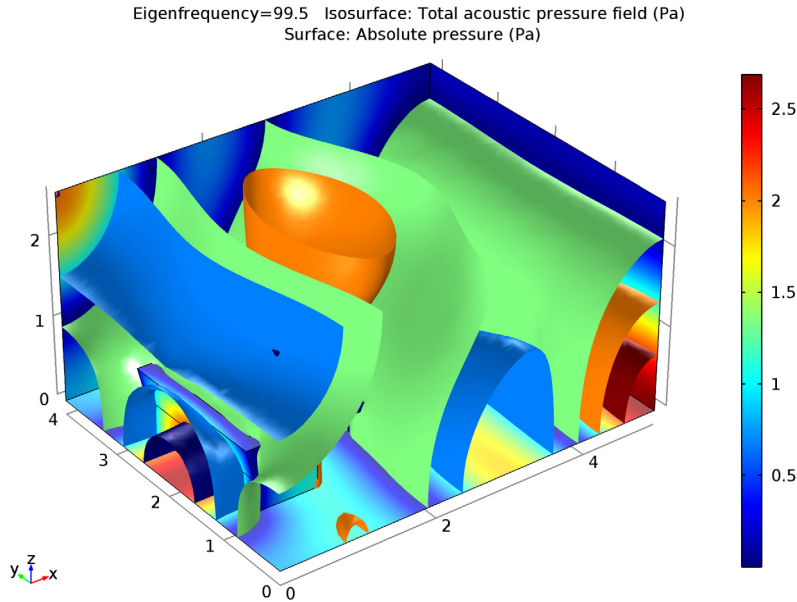
MODE INDEX	FREQUENCY	MODE INDEX	FREQUENCY
2,0,0	68.6	2,0,1	95.2
1,0,1	74.3	3,0,0	103

### *Results and Discussion*

The relevant quantity when it comes to placing the loudspeakers is the amplitude of the standing pressure wave. A sound source excites an eigenmode the most if it is placed in one of the pressure antinodes for the mode. Conversely, with the source in a pressure node, the eigenmode remains silent.

All modes have local maxima in the corners of an empty room so speakers in the corners excite all eigenfrequencies. This simulation predicts eigenmodes that strongly resemble those of the corresponding empty room. The higher the frequency, the more the placing of the furniture matters. For instance, some of the high-frequency eigenmodes are located behind the couch.

In the strictest sense, the results of this simulation only apply to a room with perfectly rigid walls and nonabsorbing furniture. The prediction that speakers placed in the corners of the room excite many eigenmodes and give a fuller and more neutral sound, however, holds for real-life rooms.



*Figure 1: The sound pressure distribution for an eigenfrequency of 99.5 Hz. The real part of the pressure is visualized as an isosurface plot, and the absolute value of the pressure as a boundary plot. Note that this mode does not correspond to any of the analytical modes listed above.*

### *Notes About the COMSOL Implementation*

When simulating acoustics, or in general any wave phenomenon, it is important to resolve the expected wavelength properly. As a rule of thumb, 6 second-order element per wavelength is a reasonable trade-off between computational effort and accuracy. For this example, where wavelengths of interest are below 100 Hz, this implies a maximum element size of

$$h_{\max} = \frac{\lambda}{6} = \frac{c}{6f} = \frac{343}{6 \cdot 100}$$

Therefore, an element size of 57 cm or less, corresponding to at least five elements between floor and ceiling, provides sufficient accuracy. The default mesh for this geometry meets the requirement. However, in general it is advisable to specify a maximum element size explicitly in the mesh settings.

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**Application Library path:** COMSOL\_Multiphysics/Acoustics/  
eigenmodes\_of\_room

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### *Modeling Instructions*

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From the **File** menu, choose **New**.

#### **NEW**

**1** In the **New** window, click **Model Wizard**.

#### **MODEL WIZARD**

- 1** In the **Model Wizard** window, click **3D**.
- 2** In the **Select physics** tree, select **Acoustics>Pressure Acoustics>Pressure Acoustics, Frequency Domain (acpr)**.
- 3** Click **Add**.
- 4** Click **Study**.
- 5** In the **Select study** tree, select **Preset Studies>Eigenfrequency**.
- 6** Click **Done**.

#### **GEOMETRY 1**

The geometry in this model can be created within COMSOL Multiphysics. Here it is imported for convenience.

#### *Import 1 (impl)*

- 1** On the **Home** toolbar, click **Import**.
- 2** In the **Settings** window for Import, locate the **Import** section.
- 3** Click **Browse**.
- 4** Browse to the application's Application Library folder and double-click the file eigenmodes\_of\_room.mphbin.
- 5** Click **Import**.
- 6** Click the **Wireframe Rendering** button on the **Graphics** toolbar.

## MATERIALS

### *Material 1 (mat1)*

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for Material, type Air in the **Label** text field.
- 3 Select Domain 1 only.
- 4 In the **Settings** window for Material, locate the **Material Contents** section.
- 5 In the table, enter the following settings:

Property	Name	Value	Unit	Property group
Density	rho	1.25	kg/m <sup>3</sup>	Basic
Speed of sound	c	343	m/s	Basic

## PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

The boundary conditions are all Sound hard boundary (wall) by default.

## STUDY 1

### *Step 1: Eigenfrequency*

- 1 In the **Model Builder** window, expand the **Study 1** node, then click **Step 1: Eigenfrequency**.
- 2 In the **Settings** window for Eigenfrequency, locate the **Study Settings** section.
- 3 In the **Search for eigenfrequencies around** text field, type 90.  
This setting will get you at least 6 solutions with eigenfrequencies in the vicinity of 90 Hz. For this model you typically get additional eigensolutions that the eigenvalue solver finds.

On the **Home** toolbar, click **Compute**.

## RESULTS

### *Acoustic Pressure (acpr)*

The first default plot shows the pressure distribution on the exterior boundaries of the geometry. To see what goes on inside the room, you need to suppress some of the boundaries. This is most conveniently done by first selecting all boundaries and then removing a few of them (number 1, 2, and 4) from the selection.

*Selection*

On the **Results** toolbar, click **Selection**.

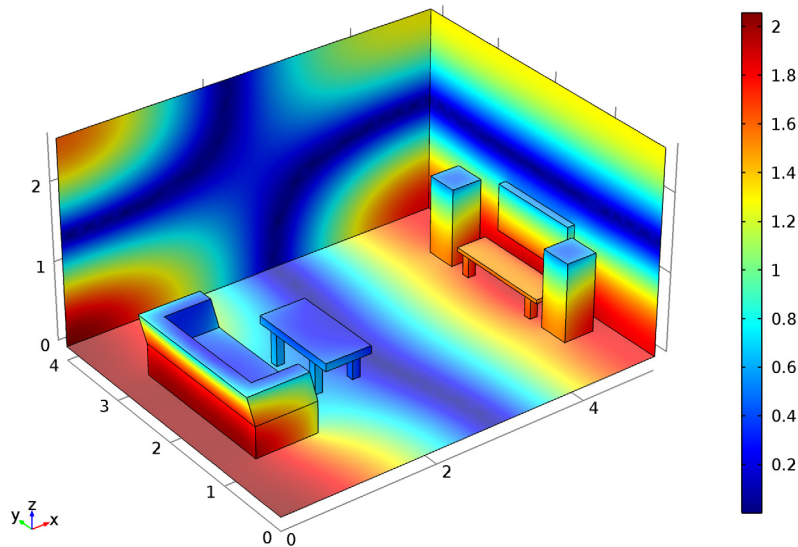
*Data Sets*

- 1 In the **Settings** window for Selection, locate the **Geometric Entity Selection** section.
- 2 From the **Geometric entity level** list, choose **Boundary**.
- 3 From the **Selection** list, choose **All boundaries**.
- 4 Select Boundaries 3 and 5–79 only.

*Acoustic Pressure (acpr)*

- 1 In the **Model Builder** window, expand the **Results>Acoustic Pressure (acpr)** node, then click **Surface 1**.
- 2 In the **Settings** window for Surface, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1>Pressure Acoustics, Frequency Domain>Pressure and sound pressure level>acpr.absp - Absolute pressure**.
- 3 On the **Acoustic Pressure (acpr)** toolbar, click **Plot**.
- 4 Click the **Zoom Extents** button on the **Graphics** toolbar.

Eigenfrequency=74.883 Surface: Absolute pressure (Pa)



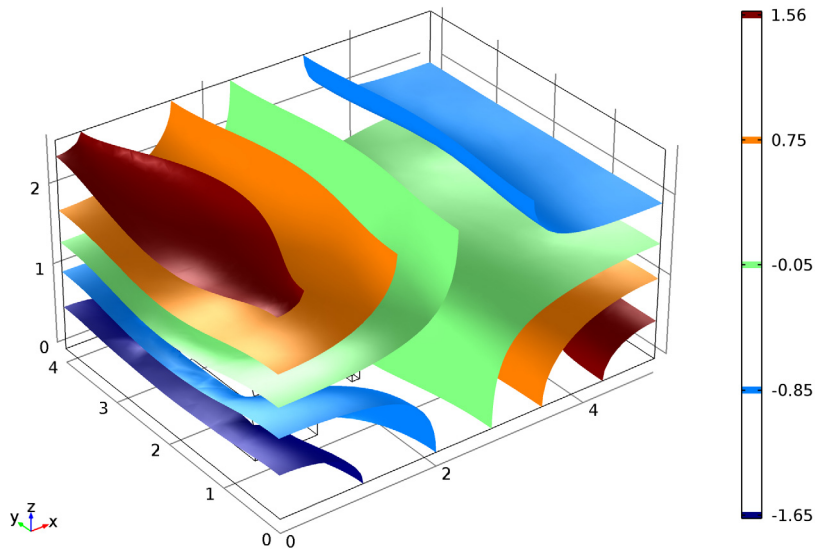
The first plot group now shows the distribution of the absolute value of the pressure for the lowest one of the eigenfrequencies, 74.9 Hz. This appears to be the (0,1,1) mode.

*Acoustic Pressure, Isosurfaces (acpr)*

The third default plot shows the isosurfaces for the same frequency.

- 1 In the **Model Builder** window, expand the **Acoustic Pressure, Isosurfaces (acpr)** node, then click **Isosurface 1**.
- 2 In the **Settings** window for Isosurface, locate the **Levels** section.
- 3 In the **Total levels** text field, type 5.
- 4 On the **Acoustic Pressure, Isosurfaces (acpr)** toolbar, click **Plot**.

Eigenfrequency=74.883 Isosurface: Total acoustic pressure field (Pa)



Modify the third default plot to show the isosurfaces and pressure distribution at 85 Hz, this is the (1,1,1) mode.

- 5 In the **Model Builder** window, click **Acoustic Pressure, Isosurfaces (acpr)**.
- 6 In the **Settings** window for 3D Plot Group, locate the **Data** section.
- 7 From the **Eigenfrequency** list, choose **85.071**.
- 8 Right-click **Acoustic Pressure, Isosurfaces (acpr)** and choose **Surface**.
- 9 In the **Settings** window for Surface, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1>Pressure Acoustics, Frequency Domain>Pressure and sound pressure level>acpr.absp - Absolute pressure**.
- 10 On the **Acoustic Pressure, Isosurfaces (acpr)** toolbar, click **Plot**.

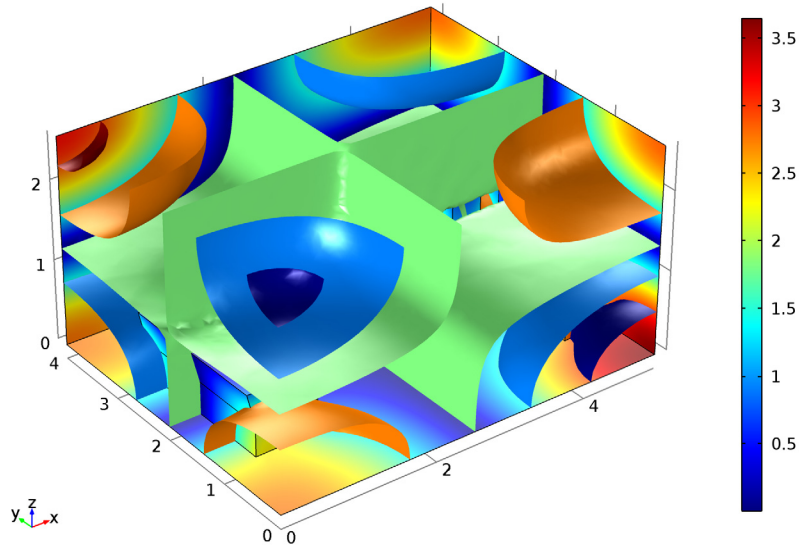


**11** In the **Model Builder** window, under **Results>Acoustic Pressure, Isosurfaces (acpr)** click **Isosurface 1**.

**12** In the **Settings** window for Isosurface, locate the **Coloring and Style** section.

**13** Clear the **Color legend** check box.

Eigenfrequency=85.07 Isosurface: Total acoustic pressure field (Pa)  
Surface: Absolute pressure (Pa)



Finally, reproduce [Figure 1](#).

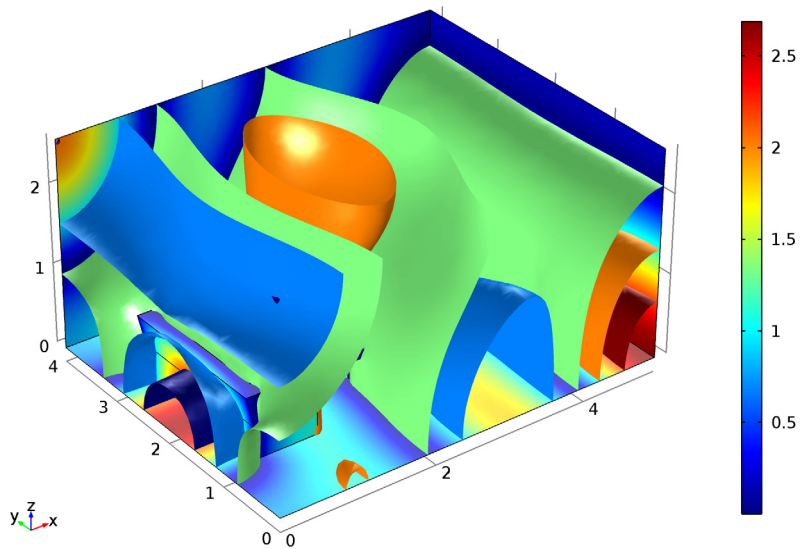
**14** In the **Model Builder** window, click **Acoustic Pressure, Isosurfaces (acpr)**.

**15** In the **Settings** window for 3D Plot Group, locate the **Data** section.

**16** From the **Eigenfrequency** list, choose **99.5**.

**17** On the **Acoustic Pressure, Isosurfaces (acpr)** toolbar, click **Plot**.

Eigenfrequency=99.5 Isosurface: Total acoustic pressure field (Pa)  
Surface: Absolute pressure (Pa)



This mode looks a little different from what you would find in an empty room. It is focused behind the couch.