# Designing a Smart Skin with Fractal Geometry

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**Abstract:** Recently, the concepts of fractal geometry have been introduced into electromagnetic and plasmonic metamaterials. With their self-similarity, structures based on fractal geometry should exhibit multi-band character with high Q factors due to the scaling law. However, there exist few studies of phononic metamaterials having fractal geometry. COMSOL is used to investigate vector elastic and scalar acoustic wave propagation in two and three dimension systems possessing fractal geometries. The simulations of these systems, guided by our recently developed general design framework, help to further optimize the structures to reduce the loss and enhance the desired properties. Proposed structures will be fabricated via standard lithographic techniques. The wave behavior of the structures will be characterized using different techniques. Fractal phononic structures show potential to be used for a wide range of applications, such as a "smart skin", where multifunctional components can be fabricated within the same platform due to the sparse spatial arrangement of fractal geometry.

**Keywords:** fractal, phononic metamaterials.

## 1. Introduction

Fractal geometry has attracted interest in antenna community since they allows for more compact design, with their lower resonant frequency compared to conventional antenna. Fractals possess self-similarity, the scaling law then indicates that it would exhibit multi-band character and high confinement of the EM field, hence high Q factors. Studies also showed how the geometric factor, fractal order would influence the performance of the fractal resonators. Besides fractal antennas, the small particles fractal aggregates also demonstrate interesting behavior. Due to a lack of long range field on the fractal cluster leads to huge field enhancement, which hence leads to nonlinear optical phenomena.

Recently, fractal geometry (mostly H shaped unites) has also been introduced in electric, magnetic and plasmonic metamaterials. With the self-similarity, the structures possess multiband EM responses covering a broad frequency. By stacking a fractal patterns, it exhibits a polarization and incidence angle-independent stop band. The fractal plasmonic metamaterials supports both TE and TM polarized Surface Plasmonic Polariton (SPP). It also possesses multiple resonances in both x and y direction. All these unique properties gives opportunities for applications.

There exists fewer studies regarding phononic metamaterials with fractal geometry. Although fractal structure has been used in architectures for acoustic purpose, more often it is for atheistic reasons. With a growing interest in phononic metamaterials with frequencies range from ultrasonic to heat, a general framework would be valuable to both understand the mechanism and guide the design. The design of framework proposed by Koh, elt, it could be utilized to guide and unify the field.

## 2. Methods and Use of COMSOL

### 2.1 Draw the fractal patterns

Generate the fractal pattern in COMSOL or draw the patterns in Autocad and imported to the COMSOL software

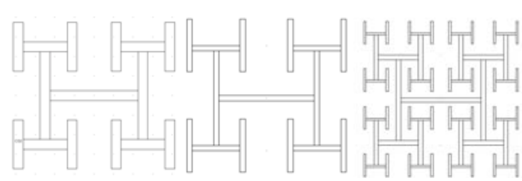

**Figure 1:** H pattern with different level of iterations and thickness/length ration a)N = 2, t/l = 1/9 (only for 1st level, t remain constant); b) N = 2, t/l = 1/18 (only for 1st level, t constant) c) N = 3, t/l = 1/18 (t is the same for 1st level as N =1,2 and halved for N = 3)

**2.2 Define the physics**

Model selection: plain stress or plain strain has been used given the 2D nature of the simulation.

Subdomains: set the H patterns as scatters, set the background as matrix materials, the material properties are listed as Table 1 below:

**Table 1:** Material Properties used in the simulation

| | Scatter (eSc) | Matrix (eM) |
|---|---|---|
| Modulus Pa | 40.8e9 | 4.35e7(4.35e6) |
| Poission Ratio | 0.25 | 0.25 |
| Density kg/m$^3$ | 11600 | 1180 |

Boundaries: Set the rectangular as unit cell of larger entity, such that apply periodic condition on both left/right and top/bottom boundaries. i.e.

$$u_r = u_l * \exp(i * a * (k_{1x} + k_{2x}))$$
$$u_t = u_b * \exp(i * a * (k_{1y} + k_{2y}))$$

Where $u_r$, $u_l$, $u_t$, $u_b$ are the displacements of right, left, top and bottom edge respectively, $a$ is the lattice constant, and $k_{1x}$, $k_{2x}$, $k_{1y}$, $k_{2y}$, are the wave vectors on x and y directions.

Solver: eigenfrequency solver is used to generate the eigenmodes of the structures.

**2.3 Study the relevant factor that affects the dispersion behavior**

Define the physics and save the model as m file and link it to MATLAB to calculate the dispersion curve along high symmetric direction in the reciprocal space. The obtained dispersion curve would provide guidance to investigate the displacement field at point of interest.

## 3. Theory

Phonons are present in all media with a frequency range cover 12 orders of magnitude and different polarization degree of freedom. Koh, et. al. developed a governing framework to guide the design of metamaterials utilizing a combination of global symmetry principles with conservation principles and broken symmetry concept. This framework removed artificial distinction between metamaterials and phononic crystal, hence unified the design of various structures. It developed physical understanding behind the relations between the physical topography and phononic propagation behavior. Due to the general applicability of the framework with only a few assumptions, the framework can be applied to guide the design of structure with fractal geometry, which are different from conventional periodic structures.

The framework identifies that the symmetry of the polarization of the displacement field is determined by the symmetry or irreducible representations of relevant wave vector k; and avoid crossing happens between/among eigenmodes with the same symmetry. Since the fundamental bandgap opening mechanism is avoided crossing, the symmetry of the eigenmodes hence plays an important role in manipulating the phononic properties of the structure.

## 4. Results and Discussion

Through the case studies of Phononic metamaterials based on H shaped fractal pattern, it would demonstrate that the generalized requirement for gap opening is i) correct plane groups symmetry for scatter motif and lattice; ii) controlling the avoided crossing.

**4.1 General Framework**

To prove the first point, track the polarization field along the irreducible representation's boundary Γ→X→M →Γ for the binary system.

As shown in Figure 3, the H pattern has a symmetry of p2mm. At Γ point, the symmetry is C2, mx,y and E, since the two mirror plane are not equivalent, the two mode are non-degenerate. Along ΓX direction, the symmetry is mx and E, the displacement is either symmetry (mode 2) or antisymmetric (mode1) with respect to the wave vector $k_{\Gamma X}$. The polarizations remain the same along the band. For bands with different symmetry (with respect to $k_{\Gamma X}$,) they would cross each other with avoid crossing happen, as happened between band 2 and 3 and 0.7 $k_{\Gamma X}$.
Along XM direction, the symmetry is my and E, the displacement is either symmetry or antisymmetric with respect to the wave vector $k_{XM}$.

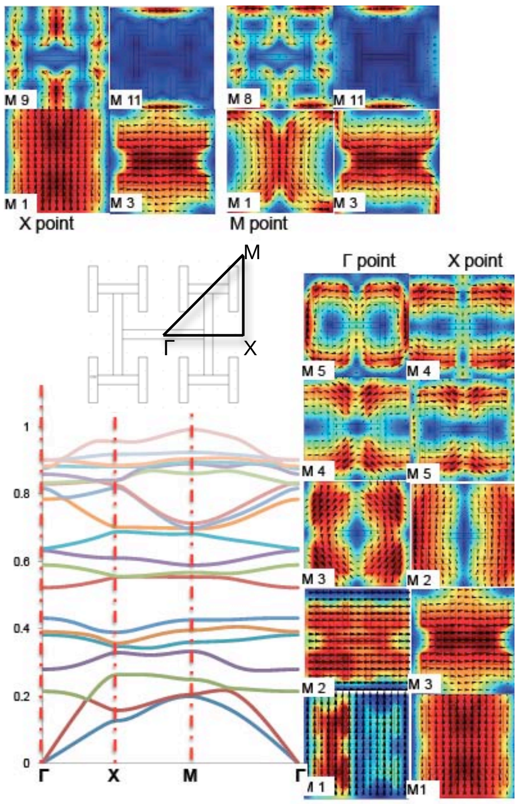


**Figure 3:** Dispersion curve and the corresponding eigenmodes with plain strain model for phononic metamaterials at Γ, X and M point based on H patterns with N = 2, eM = 4.35e7 and t/l = 1/9.

For avoided crossing, at point X, avoided crossings happen between mode 1 and 9 as well as mode 3 and 11 since they are symmetric and antisymmetric with respect to mirror plane along a direction. Hence the bands that appear to bound the bandgap are not necessarily the bands that are interacting to form the bands.

### 4.2 Plain Strain Vs Plain Stress

The effect of the plain stress vs plain strain model on the H pattern has also been studied as shown in Figure 4. No significant changes have been observed for the polarization of the displacement field except that the frequency are relatively lower than the corresponding mode at plane strain model. This is predictable given reduced rigidity of the plane stress model, which leads to lower eigenfrequencies. The general framework is still applicable for this model.

### 4.3 Material Properties

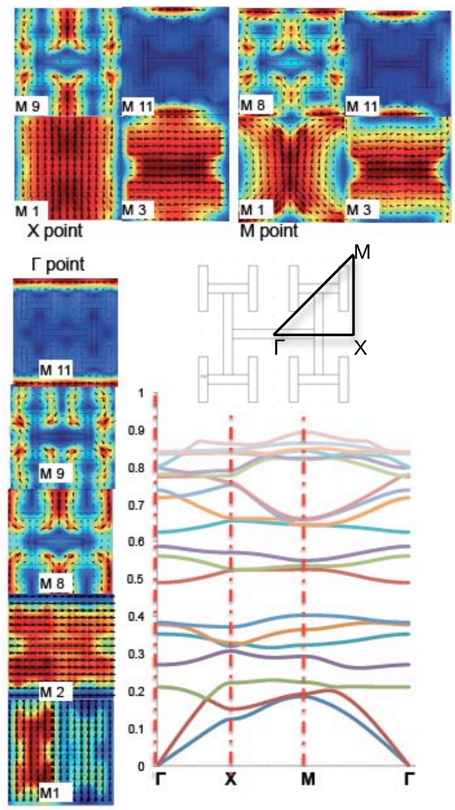


**Figure 4:** Dispersion curve and the corresponding eigenmodes for phononic metamaterials with plain stress model at Γ, X and M point based on H patterns with N = 2, eM = 4.35e7 and t/l = 1/9.

With the framework for the global behavior, the local interactions can be tuned by vary the material properties, i.e lower the modulus of the matrix materials, as shown in Figure 5. The eigenfrequencies are further reduced due to the deceasing modulus of the system. From the Eigenmode plots, it is clear that the displacement field is more confined for as compared to the harder matrix. For eM = 4.35e7 Pa, the displacement mainly focused on the matrix phase, but still spread into the boundary of higher level H arm (the scaled down H pattern), especially the corner of the H arms. For eM = 4.35e6 Pa, the field are well confined in the matrix phase. Given the larger contract of the modulus, higher frequency would prefer to stay at more rigid phase, i.e, the matrix in this case.

### 4.4 Thickness/Length ratio

The effect of the t/l ratio has also been studied for the H pattern, as shown in Figure 6.

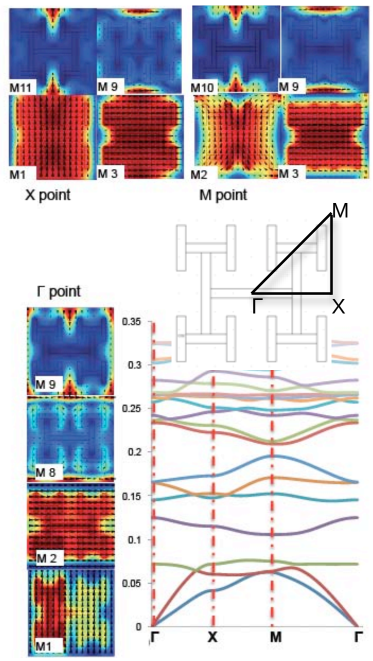

**Figure 5:** Dispersion curve and the corresponding eigenmodes for phononic metamaterials with plain stress model at Γ, X and M point based on H patterns with N = 2, eM = 4.35e6 and t/l = 1/9.

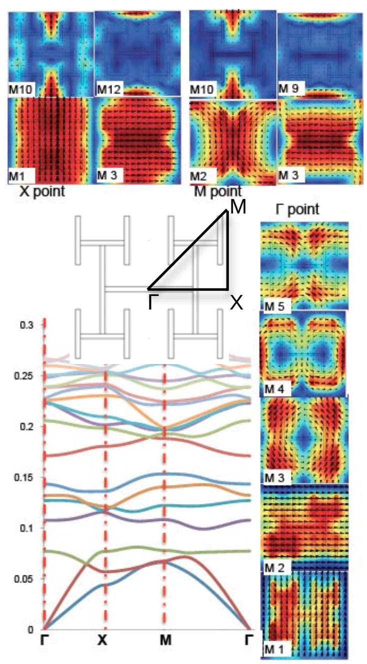

**Figure 6:** Dispersion curve and the corresponding eigenmodes for phononic metamaterials with plain stress model at Γ, X and M point based on H patterns with N = 2, eM = 4.35e6 and t/l = 1/18.

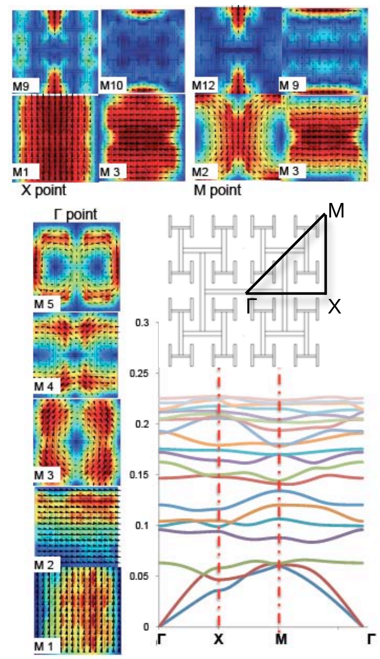

**Figure 7:** Dispersion curve and the corresponding eigenmodes for phononic metamaterials with plain stress model at Γ, X and M point based on H patterns with N = 3, eM = 4.35e6 and t/l = 1/18.

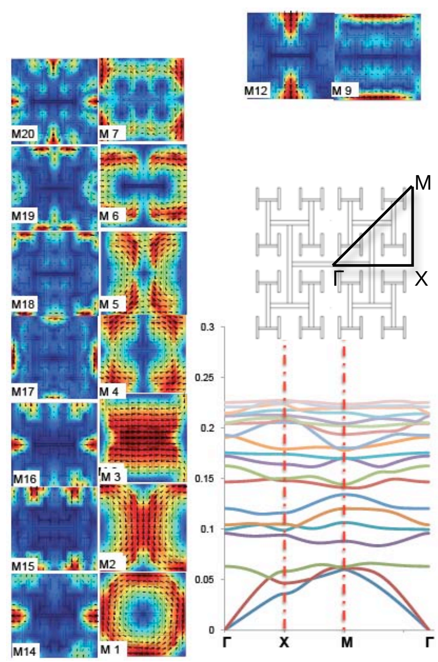

**Figure 8:** Dispersion curve and the corresponding eigenmodes for phononic metamaterials with plain stress model at M point based on H patterns with N = 3, eM = 4.35e6 and t/l = 1/18.

Due to the decrease of filling fraction of more rigid scatter phase, the eigenfrequencies decrease in general. The displacement fields at higher frequency are also less confined compared to the pattern with larger thickness. The general guideline about symmetry and avoid crossing still applies in this case, as demonstrated in the eigenmodes.

### 4.5 Levels of iterations

The effect of the level of iterations has also been studied for the H pattern, as shown in Figure 7.

Despite the increase in the filling fraction of more rigid scatter phase, the eigenfrequencies decrease in general. The eigenmodes of displacement fields showed that the displacements are more tortuous than the pattern with only two iterations. Given the increased fractal dimension with the increased number of iteration, the effective path for the displacement increases too, and the geometry also force the displacement to bend/rotate more as compared with the case for N = 2.

It is noticed that the dispersion bands become flat for the N = 3 case at M point. Figure 8 shows the eigenmodes for different eigenfrequencies at M point. It is found that compared to the N = 2 at M point, the displacement are more localized with a rotating nature, which correspond well with the flat bands.

## 5. Conclusions

The current paper demonstrates the feasibility of apply the general guideline developed by Koh. et. al to phononic metamaterials based on fractal geometries, in this case H pattern. To further verify the theory, other case studies have also been carried out, i.e materials properties, geometric factor, level of iteration. It is found that while the general framework defined where the avoid crossing can happen to open the spectral gap; materials properties and other factor can be introduced to turn the local interaction to optimize the structure for specific applications.

## 6. References


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## 7. Acknowledgements


Institute for Soldier Nanotechnologies, MIT supports this research.