Xylophone Bar Magnetometry and Inertial Grade MEMS Optimisation

A Multiphysics Approach

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Excerpt from the Proceedings of the 2011 COMSOL Conference in Stuttgart
MEMS and microsystems research group with world-leading expertise in dynamics and control, biosensors
- Based in the North East of England
- A major aim is to demonstrate the feasibility of inertial grade navigation, similar to aviation-grade ILS, using an integrated microsensor array – the 9 DOF “holy grail”

Consistent publications and current EPSRC sponsored work on high-Q resonant sensors and associated parametric control techniques

The work presented here today is a subset of my doctoral research
- My work is focused on developing a resonant Lorenz micromagnetometer
- In an inertial navigation context, would be employed in a Kalman filter to implement drift nulling
An XBM is a resonant high sensitivity Lorenz magnetometer

- When a DC sense current is applied, any B field component transverse to the plane of vibration generates a Lorenz force and corresponding deflection of the structure
- When the sense current is made to oscillate at the resonant frequency of a chosen mode of the XBR structure, the static deflection is amplified by the Q factor of the mode

- Differs from other Lorenz magnetometers in that the suspension beams are attached at the node points of the sense beam, decoupling transverse motion between the two
- This gives extremely high Q factors and hence sensitivities
- Q factor can be pushed higher using parametric drive techniques
XBM control relation is known

- Quantitatively expresses dependence of device sensitivity on $Q$, static compliance
- Experimental work gives the achievable limiting parametric gain at $\sim 100$
- Implies a classical resonator $Q$ of $10^4$ would yield an effective parametric $Q$ on order $10^6$ without affecting compliance

It implies that the $Q$ factor is critical

- Need an understanding of $Q$ factor and dissipation mechanisms to optimise design
- Gas damping, surface losses, TED, etc. well characterised in the literature.
- TED sets a hard limit on $Q$ at $\sim 10^6$
- No such results exist for support loss at present in the literature

Strategy: Develop and validate a model of support loss in XBRs

- Model the support losses in COMSOL.
- Use a Rayleigh-Ritz method to obtain the forces of constraint at the distal ends of the XBR support beams
- Use 2D analytical model of elastic wave radiation in a semi-infinite plane to estimate corresponding support radiation
- Cross-validate simulation and analytical results

How can we maximise the performance of an XBM?

$$A_0 = \frac{iF\omega}{2} [Q \times G_T]$$
Model I: Joule Heating

XBR uses an AC sense current to generate Lorenz force

- Scaled with field strength to give wide dynamic range
- The larger the current, the smaller a field can be detected

The sense current amplitude is limited by Joule Heating of the resonator

- Use Joule Heating model, Stationary study type, to find steady state temperature distribution

Under vacuum, only radiation and conduction important

- Surface to surface radiation ignored
- Thermal BCs specify radiation to ambient environment and a prescribed temperature on the distal ends of the supports
- Electrical BCs specify a potential at the distal ends of the supports parametrically, with the symmetry boundary taken as a ground

\[ \nabla \cdot J = Q_j \; ; \; J = \sigma E + J_E \]

\[ E = -\nabla V \; ; \; \rho C_p u \nabla T = \nabla \cdot (k \nabla T) + Q \]
Model 2: Structural Mechanics

Variable density, Young’s modulus
- An analytical expression for the temperature dependence of $\rho$ and $E$ is given in _ref_
- Implemented as a variable in COMSOL and evaluated pointwise over the computational domain
- Used to define the material properties for Model 2

Eigenvalue study of Solid Mechanics model
- At ambience, support and sense beams mode-matched
- This minimises constraint forces and hence optimises support $Q$
- Sensitivity to geometric mistuning studied using COMSOL in Grigg et al., IMAPS DPC, 2011.

Heating-induced mistuning
- Domain rendered inhomogeneous by spatial dependence of temperature
- Wave propagation and consequentially normal modes and their frequencies altered
- This problem is analytically challenging, but made straightforward by COMSOL.
Perfectly Matched Layers are a numerical technique used to simulate infinite domains. They can be best viewed as an analytical continuation of the constitutive equations to the complex plane. The mechanism of support loss is elastic wave radiation. PML allows numerical closure and hence simulation.

Model solved using Frequency Domain study. Resonant frequency of the XBR and force distribution at the support interfaces used as model inputs. Separate geometric model employed.
F3: Job sequence for coupled multiphysics analysis of XBR support loss. This sequence was iterated in a 2-parameter parametric array.

- Geometry modelled parametrically in COMSOL
- The steady-state temperature distribution deriving from an applied sense current and Joule heating studied first
  - The above output was combined with an experimental result from the literature was used to define a variable Young’s modulus and density for the resonator material
- Mode shapes and natural frequencies found using a linear elastic eigenfrequency analysis with material properties defined by the previous result
  - Constraint forces found -> PML model input
  - In addition, the stored strain energy in the resonator was determined using a volume integration probe (Result 1).
- The constraint forces were then coupled to a PML model approximating the resonator substrate as large (and hence entirely dissipative).
  - The total energy flux arising from elastic wave propagation into the substrate could thus be determined (Result 2).
- Combining results 1 and 2 yields an estimate for the device Q factor, as desired.
Parametric Geometric Optimisation

Temperature-modified mode shapes found

Parametric optimisation of geometry facilitated

Optimal tuning is a function of applied sense current – retuned for small-field sensitivity

Validation against literature

<table>
<thead>
<tr>
<th>Nodal Mistuning(%)</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
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<tbody>
<tr>
<td>V0=0</td>
<td>26</td>
<td>41</td>
<td>29</td>
</tr>
<tr>
<td>V0=0.02</td>
<td>28</td>
<td>39</td>
<td>28</td>
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<tr>
<td>V0=0.04</td>
<td>32</td>
<td>31</td>
<td>25</td>
</tr>
</tbody>
</table>

Q vs Geometry

Q (’000) vs Parameters

Analytical vs. Simulated Q for a Cantilever
Two prototypes to date
- Characterised optically and acoustically
- Q around 10,000 at 1 bar!!
- Results of the present study suggest thermal limitation of performance
- As a result, a third prototype is under production in copper – superior heat dissipation
Conclusions

- Support loss, natural frequencies, and static compliance efficiently modelled using COMSOL
  - Analytical work cross-validated against the results with satisfactory agreement for the case without heating effects
  - Model predictions extended using coupled analysis to include heating
- Leads to adjustment of the optimal geometric tuning for an XBR
  - Improved performance in the real world
- Q factor under vacuum implies inertial-grade performance possible

Further Work

- Complete prototyping, obtain experimental validation of modelling => Proof of concept
- Obtain funding, make the device on the microscale
- Analytically model heating effects – finite difference method?
  - Model Parametric Drive
- Realise the magnetic component of 9-DOF IMU
Many thanks for listening to my talk!

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Any Questions?