Simulation of Unidirectional Interdigital Transducers in SAW Devices using COMSOL Multiphysics

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Abstract: Surface acoustic wave (SAW) devices based on Rayleigh wave, shear wave, love wave, acoustic plate mode (APM) wave and flexural plate wave have been explored for sensors, actuators and telecommunication applications. An interdigital transducer (IDT) is a metallic comb-like structure fabricated over piezoelectric substrate. SAWs are generated over the substrate by applying electrical signal to the IDT. A unidirectional IDT (UDT) is a type of IDT where the propagation of SAW is restricted to one direction. In this paper we have modeled a floating electrode unidirectional IDT (FE UDT) using COMSOL Multiphysics. In this simulation YZ lithium niobate is chosen as the substrate with mass less electrodes. The frequency response analysis is performed and admittance of the FEUDT is evaluated. Further the simulation results can be extended to extract coupling of mode (COM) parameters, which are vital in designing SAW devices.

Keywords: SAW, NEMS, SAW actuators, UDT, Rayleigh waves, MEMS.

1. Introduction

Surface acoustic waves (SAWs) propagate on the surface of the elastic medium with most of the energy concentrated near the surface. An interdigital transducer (IDT) which is a metallic electrode, fabricated over piezoelectric substrate, performs the function of conversion of electric energy to mechanical energy and vice versa. Single electrode IDT generates waves with equal amplitudes on both the directions and hence they are bidirectional. However triple transit and multiple transit signals arising due to reflections from the single electrode IDT drastically deteriorate the performance of the SAW devices. Unidirectional IDTs (UDT) overcome these reflections [1]. An UDT has directivity, as it generates unequal amplitude waves in two directions. Directivity becomes stronger if the transducer is a complete two port device one being acoustic and other being electrical. Various UDTs such as single phase unidirectional IDT (SPUDT) has been reported in literatures. To behave as a SPUDT, a transducer needs to have internal reflectivity, and the structure must have some asymmetry which can be achieved by sequence of cells, each with electrodes of different width. The SPUDT can be regarded as a sequence of localized reflectors and transducers, taken to be located at reflection centers (RC) and transduction centers (TC). These are the two important points to be considered while designing an SPUDT. RC is the point where the reflection coefficient of the incident waves from both the directions are equal. TC is the point where the generated waves in both directions have same magnitude and phase. Finite element simulations of SPUDT structures can be utilized to extract vital coupling of mode (COM) parameters and optimal design of UDTs can be achieved. In this paper FEM simulation of a floating electrode unidirectional IDT (FEUDT) using COMSOL Multiphysics is reported. FEUDT is a type of SPUDT. Yamanouchi et al [2] and Campbell et al [3] has reported the design of an six electrode FEUDT and SPUDT for low loss SAW filter applications. COM analysis of FEUDT was performed by Masao et al [4]. Various novel FEUDT structures were reported by Morgan [5]. Significance of different width of electrodes in an FEUDT using was analyzed impedance method by Biryukov et al [6].

Figure 1. A Typical FEUDT structure.
FEM simulation methodology of FEUDT in COMSOL Multiphysics can be extended to design optimal UDTs. Further the applied boundary conditions are verified by performing a time domain analysis of an FEUDT and directivity of travelling SAW is observed.

1.1 FEUDT principle

The principle of FEUDT is well reported in literature [4]. A single cell of a six electrode FEUDT is shown in figure 1, where \( p \) is the period and \( \lambda \) is the wavelength of SAW. Electrode 1, 4 are active electrodes, electrode 3, 6 are open floating electrodes and 2, 5 are shorted floating electrodes. The electrodes 3 and 5 have higher acoustic impedance than the free propagating space, which will reflect the generated SAW by the active electrodes, which results in higher amplitude SAW in the forward direction than in the reverse direction. The electrodes 2 and 5 are short strips and they have less acoustic impedance than the propagating free space which results in higher amplitude SAW in forward direction. The combination of both short and open floating electrodes makes the IDT structure to directivity. Thus the six electrodes generate four travelling waves and their combination give rise to a single wave in one forward direction.

2. Piezoelectric Model

\[ T_{ij} = C_{ijkl}^{E} S_{kl} - e_{ijkl} E_{k} \]  
\[ D_{i} = e_{ijkl}^{e} S_{kl} - e_{ijkl}^{e} E_{k} \]  
Where, \( T_{ij} \) Represents the stress vector, \( C_{ijkl}^{E} \) is the elasticity matrix, \( e_{ijkl} \) is the strain vector, \( e_{ijkl}^{e} \) is the permittivity matrix, \( E_{k} \) the electric field vector, \( S_{kl} \) is the strain vector, \( D_{i} \) is the electrical displacement. The degrees of freedom (dependent variables) are the global displacements \( u_{i} \), \( u_{o} \), and \( u_{t} \) in the global \( x_{1} \), \( x_{2} \), and \( x_{3} \) directions (figure 2), and the electric potential \( V \) can be obtained by solving the Newton’s and Maxwell equation related to equations (1 and 2) are given below,

\[ \sum_{j\ell} C_{ijkl}^{E} \frac{\partial^{2} u_{j}}{\partial x_{j} \partial x_{k}} + \sum_{j\ell} e_{ijkl}^{e} \frac{\partial^{2} V}{\partial x_{j} \partial x_{k}} = \rho \frac{\partial^{2} u_{i}}{\partial t^{2}} \]  
\[ \sum_{j\ell} e_{ijkl}^{e} \frac{\partial^{2} u_{j}}{\partial x_{j} \partial x_{k}} + \sum_{j\ell} e_{ijkl}^{e} \frac{\partial^{2} V}{\partial x_{j} \partial x_{k}} = 0 \]

for \( i, j, k, l = 1, 2 \) and 3.
The Rayleigh elastic wave displacement has components in surface-normal and surface-parallel directions with respect to the direction of the wave propagation. The particle in the upper surface takes elliptical path having both the components.

3. Simulation Scheme

The various aspects of simulation such as geometry, material properties, and boundary conditions are discussed in this section.

3.1 Material design and structure

![Figure 2](image-url). Coordinate system for the wave solutions.

![Figure 3](image-url). Schematic view of the FEUDT unit cell structure considered for the simulation.
As the Rayleigh waves have no displacement in $x_2$ direction, a 2D plain strain structure along $x_1$, $x_3$ plane is valid to perform the simulation [4]. The amplitude of Rayleigh SAW decays exponentially along the depth of the substrate.

Two different FEM simulations are conducted to study the FEUDT structure. In the first simulation frequency response of a single FEUDT cell is performed and in the later time domain analysis of an FEUDT of length $3\lambda$ is performed and unidirectional action of the FEUDT is visualized. Figure 3 shows the structure considered to perform frequency response analysis of FEUDT. The dimensions of the structure considered for the simulation are as shown in the Table 1.

### Table 1: Dimensions of the structure

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Electrical boundary condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_p$</td>
<td>Applied electric potential $V = -1$ V</td>
</tr>
<tr>
<td>$\Gamma_p^+$</td>
<td>Applied electric potential $V = 1$ V</td>
</tr>
<tr>
<td>$\Gamma_s$</td>
<td>For the part of surface not covered by electrodes, Zero charge symmetry $\mathbf{n} \cdot \mathbf{D} = 0$. $\mathbf{n}$ is unit vector normal to the surface and $\mathbf{D}$ is the electric displacement vector.</td>
</tr>
<tr>
<td>$\Gamma_f$</td>
<td>For floating electrodes electrically not connected floating potential satisfying $\int -\mathbf{n} \cdot \mathbf{J} = 0$. $\mathbf{J}$ is current density vector.</td>
</tr>
<tr>
<td>$\Gamma_{f1}$ and $\Gamma_{f2}$</td>
<td>For the floating electrodes electrically shorted together, Condition is applied such that $\int_{\Gamma_{f1}} -\mathbf{n} \cdot \mathbf{J} = \int_{\Gamma_{f2}} -\mathbf{n} \cdot \mathbf{J} = 0$.</td>
</tr>
</tbody>
</table>

The piezoelectric equations are implemented using COMSOL Multiphysics. $YZ$ lithium niobate elastic constants, piezoelectric stress constants, permittivity constants and density are taken from [9] and assumed for the substrate. The electrodes are assumed to be mass less.

### 3.2 Boundary conditions

Traction free, $\mathbf{n} \cdot \mathbf{T} = 0$ boundary is given to the top surface of the substrate, where $\mathbf{T}$ denotes the stress vector. The bottom surface is fixed, $\mathbf{u} = 0$ in its position and the displacement, where $\mathbf{u}$ is the displacement vector. As the IDTs are periodic in nature, appropriate periodic boundary conditions are applied for the boundaries $\Gamma_L$ and $\Gamma_R$ using the equations (5).

$$\Gamma_R (u, V) = \Gamma_L (u, V) \exp (-j2\pi \gamma m)$$

where, $n$ is any positive integer, $\gamma$ is complex prorogation constant and $V$ is electric potential. With reference to figure 1, the various electrical boundary conditions applied is summarized in Table 2. Frequency response analysis is performed using available UMFPACK solver in COMSOL Multiphysics.

### 4. Results and discussions

From frequency response analysis, complex charge $Q$ (or current in the electrode) is extracted from post processing, the admittance ($Y$) of the device can be calculated by equation

$$Y = j\omega Q/V$$

where $\omega$ is the angular frequency. Figure 4 shows the computed frequency response of the FEUDT, where the normalized admittance

![Figure 4](image_url)
\( Y(\omega \varepsilon_0) \) where \( \varepsilon_0 \) is the permittivity of vacuum is plotted with respect to frequency. It can be seen from the figure there are two resonant frequencies at \( \omega_{s1} = 98.208 \text{ MHz} \) \( \omega_{s2} = 98.275 \text{ MHz} \) and two anti-resonant frequency at \( \omega_{01} = 98.209 \text{ MHz} \) and \( \omega_{02} = 98.276 \text{ MHz} \), which means the unidirectional behavior of the structure.

![Figure 5](image)

**Figure 5.** Profile of \( x_3 \) displacement of SAW in \( x_1 \) direction when excited by a sin wave at center frequency.

The phase shift \( \phi \) between transduction center and reflection centre is calculated using equation given below,

\[
\sin^2 \phi = \frac{(\omega_{02} - \omega_{22})(\omega_{s2} - \omega_{01})}{(\omega_{22} - \omega_{s2})(\omega_{01} + \omega_{02} - \omega_{11} - \omega_{22})}
\]

and it is found to be 44.5724°. The optimal value of the phase shift is 45°, which can be achieved by tuning the electrode thickness, electrode width and periodicity of the IDT [7]. In order to visualize the unidirectional phenomenon of FEUDT, time domain analysis of an FEUDT consisting of 18 electrodes which is of 3 wavelengths in length is considered. Time domain analysis is performed by applying a potential of 0.5 V is applied at centre frequency for 20 ns. The periodicity of the electrodes, width of the electrode and electrical boundary conditions are same as discussed in section 3. However periodic boundary conditions are replaced by absorbing boundaries at the ends of the transducers. Figure 5 shows simulated \( x_3 \) displacement at time of 20 ns. It can be noted that displacement amplitudes of the SAW is higher along the \(+x_1\) direction and lower amplitude towards \(-x_1\) direction. Thus unequal amplitude of SAW in both the directions shows the directivity of the simulated FEUDT.

8. **References**