Coil Systems to Generate Uniform Magnetic Field Volumes

S. Magdaleno-Adame¹, J.C. Olivares-Galvan²*, E. Campero-Littlewood², R. Escarela-Perez², E. Blanco-Brisset¹

¹Cipress No. 88, Col. Las Arboledas, C.P.59386, La Piedad Michoacan, Mexico.
²Universidad Autonoma Metropolitana - Azcapotzalco, Departamento de Energia, Av. San Pablo 180, Col. Reynosa, C.P. 02200, Mexico D.F., Mexico.

*Corresponding author: Guerrero Pte. 114, Col. Centro, Zamora, Michoacan, Mexico, 59600, jolivare_1999@yahoo.com

Abstract: This paper analyzes different types of coil systems to produce volumes of uniform magnetic field. Some types of coil system are presented in this paper such as Helmholtz, Merritt, and Ruben coil systems. In the study coil systems are intended to produce a magnetic flux density of 2µT in the center of the coil. The calculation of the magnetic flux density using analytical formulas and finite element simulations are compared. Finally, a cube volume of uniform magnetic field in each coil is calculated.

Keywords: Helmholtz coil system, Merritt coil systems, Ruben coil system, Bio-electromagnetic.

1. Introduction

Many bio-electromagnetic experiments need to generate uniform magnetic fields [2], [3], [5], and [10]. To study living systems, scientists and engineers use electric coils to generate electromagnetic exposure to high or low frequency. Actually, Helmholtz, Merritt and Ruben coil systems are used to generate volumes of uniform magnetic field in different frequencies. Helmholtz coil system is used at laboratories to generate magnetic fields in small volumes and Merritt and Ruben coil systems are used to produce magnetic fields in large volumes [1], [2].

With these coil systems we could generate a homogeneous magnetic field in a considerable volume surrounding the center of the coil system. Helmholtz coil system has circular coils and Merritt and Ruben coils have square coils.

On the other hand, the effect of low and high frequencies electromagnetic fields on the human body have been studied using coil systems: Electromagnetic fields on human cells to study carcinogenic effects; impact of electromagnetic fields due to of nearby power lines in humans [11]; exposure to electromagnetic fields of incubators during in-vitro studies; exposure of larger animals and humans [12], [13], [14], [17], and germination studies [15].

1.1 Helmholtz Coil System

A Helmholtz coil is a two circular coil system, see Figure 1. These circular coils are commonly used to produce small volumes of uniform magnetic fields [2], [7]. The circular coils have the same number of turns and their ratio of the ampere-turns is 1. Their coils are connected in series. Their building is easy.

The magnetic flux density $B$ in the center of the Helmholtz coil system is given in Teslas by [2]:

$$B = \frac{8.9917 \times 10^{-7} NI}{r}$$ (1)

where: $N$ is the number of turns, $I$ is the current (A) and $r$ the Radius of each coil and separation between coils (m)

1.2 Merritt Coil System

Merritt coils could be a three or four square coil system [1], [2]. These square coils are used to produce large volumes of uniform magnetic

Figure 1. Helmholtz system coil.
Their building is difficult but a Merritt coil system can produce a large volume of uniform magnetic field compared with a Helmholtz coil system. The square coils are connected in series. A Merritt coil system of three square coils is showed in the Figure 2.

**Figure 2.** Merritt coil system -three square coils.

The side length of each coil in this system is \( d \) and the separation of the three coils is \( d/2 \). The distance \( s \) is the separation of the outer coils (see Figure 2).

The ratio of the distance of the outer square coils \( s \) to the side length of the square coils \( d \), is [1], [2]:

\[
\frac{s}{d} = 0.821116
\]  

(2)

The ratio of ampere-turns in the inner coil \( N'T' \) to the outer coil pair \( NI \) is [1], [2]:

\[
\frac{N'T'}{NI} = 0.512797
\]  

(3)

For this system the magnetic flux density \( B \) at the center is given in Teslas by [1], [2]:

\[
B = \frac{1.749 \times 10^{-6} \ NI}{d}
\]  

(4)

where: \( N \) is the number of turns, \( I \) is the current (A) and \( d \) the side length of the square coils (m).

Figure 3 shows a Merritt coil system of four square coils.

**Figure 3.** Merritt coil system - four square coils.

For a Merritt coil system of four square coils the ratio of the distance from the center to the inner pair of coils \( a \), and the side length of the coils \( d \) is [1], [2]:

\[
a/d = 0.128106
\]  

(5)

The ratio of the distance \( b \), from the center to the outer pair of coils and \( d \), is [1], [2]:

\[
b/d = 0.505492
\]  

(6)

The ratio of the ampere-turns in the inner coil pair \( N'I' \) to that in the outer coil pair \( NI \) is [1], [2]:

\[
\frac{N'I'}{NI} = 0.423514
\]  

(7)

For this coil system the magnetic flux density \( B \) at the center is given in Teslas by [1], [2]:

\[
B = \frac{1.795 \times 10^{-6} \ NI}{d}
\]  

(8)

where: \( N \) is the number of turns, \( I \) is the current (A) and \( d \) the side length of the square coils (m).

### 1.3 Ruben Coil System

A Ruben coil system is a five square coil arrangement [1], [2]. This system is more complex than Merritt coil systems. In this system a fifth coil is added. Building these systems is more difficult compared with a Merritt and Helmholtz coil systems. A Ruben coil system uses coils with an ampere-turns ratio proportional to the integers 19, 4, 10, 4, and 19,
see Figure 4, [1], [2]. This ratio accomplishes a homogeneous magnetic field in a considerable volume surrounding the center of the system. For this complex coil system the magnetic flux density $B$ at the center is given in Teslas by [1], [2]:

$$B = \frac{1.878 \times 10^{-6} NI}{d}$$  \hspace{1cm} (9)

where: $N$ is the number of turns, $I$ is the current (A) and $d$ the side length of the square coils (m)

Figure 4 shows a Ruben coil system.

2. Finite Element Simulations of Coil Systems

Comsol Multiphysics is used to calculate the magnetic flux density in the center of coil systems: Helmholtz (two circular coils), Merritt (three and four square coils) and Ruben (five square coils). These coil systems were modeled in 3D using the AC/DC Module-Statics and Magnetic of Comsol. We made a magnetic static analysis to generate a magnetic flux density $B=2\mu T$ in the center of each coil system, which is a very common value in bio-electromagnetic studies ($B=0.5\mu T - 2mT$).

In Helmholtz and Merritt coil systems a square conductor of copper (25mm x 25mm) is used in the model. This conductor has a cross section $A=6.25 \times 10^{-4}$ m$^2$. In the Ruben coil systems a square conductor of copper (500mm x 500mm) is used to model the coils. This conductor has a cross section $A=0.25$ m$^2$.

For the Helmholtz coil system 100 turns are proposed in each circular coil and a current of 10mA circulating in both as they are connected in series.

For the Merritt coil systems 100 turns are proposed for the outer coils and a 10mA current circulating in both the inner and outer coils as they are connected in series. The number of turns for the inner coils is calculated in each case.

![Slice: Magnetic flux density, norm [T]](image)

Figure 4. Ruben coil system.

![Slice: Magnetic flux density, norm [T]](image)

Figure 5. a) Slice: Distribution of magnetic flux density, b) Streamline: Distribution of magnetic flux density.

2.1 Finite Element Simulation of Helmholtz Coil System

A Helmholtz coil system was simulated. The distance of separation between the two coils was calculated and in this case, is the radius of each coil. The radius $r$ was obtained from (1) and then the separation of the circular coils was calculated to obtain a magnetic flux density $B=2\mu T$ in the center.

$$r = 0.449585m$$
A 1600A/m² current density was applied in the circular coils. Figure 5 shows the distribution of magnetic flux density in the Merritt coil of three coils. A magnetic flux density of $B = 1.944 \mu T$ was calculated in the center of the coil system. It represents an error of 2.8%. It can be observed that a volume of uniform magnetic field is located in the center of the coil system. A cube of 20cm³ of uniform magnetic field was obtained for this coil system (see Figure 6).

Figure 6. Lines to measure and to calculate an equivalent cube of uniform magnetic field in coil system.

2.2 Finite Element Simulation of Merritt Coil System – Three Square Coils

A Merritt coil of three square coils was simulated. The distances $d$ for this coil system to calculate the side length of the square coils were obtained from (4) for a magnetic flux density of $B = 2 \mu T$ in the center.

$$d = 0.8745 \text{m}$$

The distance between coils is $d/2 = 0.43725 \text{m}$. $s$ is obtained from (2):

$$s = 0.7180 \text{m}$$

The ampere-turns of outer coils are $NI = 1 \text{A} \cdot \text{t}$ and the intensity current in the inner coil is 0.001A, and the number of turns of the inner coil $N'$ is calculated from (3).

$$N' = 82.1116$$

The applied current density in the outer coils is 1600A/m² and the current density in the inner coil is 820.48A/m². Figure 7 shows the distribution of magnetic flux density in the Merritt coil system of three coils.

A magnetic flux density of $B = 1.91 \mu T$ in the center of the coil system was calculated. It represents a 4.5% error. It can be observed that a volume of uniform magnetic field is located in the center of the coil system. A cube of uniform magnetic field for this coil system measured and calculated using the same idea from Figure 6.

2.3 Finite Element Simulation of Merritt Coil System – Four Square Coils

A Merritt coil of four square coils was simulated. Again the distance $d$ (side length of the square coils) for this system was obtained from (8) for a magnetic flux density $B = 2 \mu T$ in the center.
Then distances $a$ and $b$ are calculated:

$$a = 0.11497514 \text{ m}$$

$$b = 0.45367907 \text{ m}$$

The obtained magnetic flux density in the center of the coil system is $B = 2.096 \mu \text{T}$, which represents a percent error of 4.5%.

It can be observed that a volume of uniform magnetic field is located in the center of the coil system that measured and calculated, using the same idea from Figure 6, results in a cube of uniform magnetic field of 50 cm$^3$.

### 2.4 Finite Element Simulation of Ruben Coil System

A Ruben coil of five square coils was simulated. The distance $d$ (side length of the square coils) for this system was obtained from (8) for a magnetic flux density $B = 2 \mu \text{T}$ in the center.

$$d = 17.84 \text{ m}$$

The ampere-turns ratios used for the coils were 19, 4, 10, 4, and 19, as shown in Figure 4. The applied current density to the coils corresponds

Figure 8. a) Slice: Distribution of magnetic flux density, b) Streamline: Distribution of magnetic flux density.

The ampere-turns of outer coils are $NI = 1 \text{ A}$ and the current in the inner coil is 0.001 A, therefore, the number of turns of the inner coil $N'$ from (3).

$$N' = 42.3514$$

The current density applied to the outer coil is 1600 A/m$^2$ and to the inner coil is 677.6224 A/m$^2$.

Figure 8 shows the distribution of magnetic flux density in the four coils Merritt coil system.
to 76A/m² for the 19 ampere-turns, 16A/m² for the 4 ampere-turns and 40A/m² for the 10 ampere-turns. Figure 9 shows the distribution of magnetic flux density in the Ruben coil.

The magnetic flux density in the center of the coil system was $B = 1.85 \mu T$, meaning a 7.5% error. The volume of uniform magnetic field located in the center of the coil system resulted of 500cm³ (Figure 6).

**Table 1.** Obtained cubic volumes of uniform magnetic field and magnetic flux densities in the center of the analyzed coil systems.

<table>
<thead>
<tr>
<th>Coil System- # and form of coils</th>
<th>Volume (cm³)</th>
<th>$B$ (µT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helmholtz-2 Circular Coils</td>
<td>20</td>
<td>1.944</td>
</tr>
<tr>
<td>Merritt-3 Square Coils</td>
<td>50</td>
<td>1.91</td>
</tr>
<tr>
<td>Merritt-4 Square Coils</td>
<td>50</td>
<td>2.096</td>
</tr>
<tr>
<td>Ruben-5 Square Coils</td>
<td>500</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Table 1 shows the volumes of the uniform magnetic field cubes and the values of magnetic flux density obtained in each coil system.

### 3. Conclusions

The analyzed coil systems are very useful to generate volumes of uniform magnetic fields. Helmholtz coil is useful when small volumes of uniform magnetic field are needed. Merritt and Ruben coil system can be used to obtain large volumes of uniform magnetic field.

Merritt coil system of four square coils gives better results than same Merritt coil system with three coils or Helmholtz coil system or Ruben coil system. However, a Ruben coil system generates a very large volume of uniform magnetic field compared with other coil systems. Nevertheless, a Ruben coil system is more difficult to build.

### 4. References