# Simulation of the Spread of Epidemic Disease Using Persistent Surveillance Data

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**Abstract:** This paper proposes a novel data-mining framework, which is called susceptible-infected-removed method based on heat-transfer mechanism (SIR-HT), to simulate the spread of epidemic diseases using persistent surveillance data (A. J. Plaza and C.I Chang, 2009). SIR-HT is formulated by merging the persistent surveillance data about epidemics, geographic information, and the dynamics of disease into a heat transfer model (Dewitt, 2006) according to the theory of statistical mechanics (R.K.Pathria, 2001). In the implementation of this framework, geographic conditions are used to define the heat transfer media, which is featured by heat conductivity and thermal capacitance; persistent surveillance data about epidemic disease is used as the initial conditions; the susceptible-infected-recovered (SIR) model (Daley, 2005), one of the most fundamental dynamic models about epidemic disease is employed as Neumann boundary conditions. As a result, the spread of epidemics can be simulated by solving the corresponding transient heat-transfer problem (D.K. Gartling and J.N. Reddy, 2000) (Y. Liang et al, 2002). Using COMSOL Multiphysics (Pryor, 2010) as the major platform, SIR-HT is assessed by computing the spread of a flu epidemic at a sample site in the Minneapolis (Minnesota, USA) region.

**Keywords:** SIR, heat-transfer, persistent surveillance, epidemic disease, data-mining.

## 1. Introduction

Modeling and simulation of the spread of epidemic disease plays a significant role in human society. Accurate mathematical models can be used to explore the transmission mechanism of epidemic disease so that we can obtain insight into potential costs, benefits, and the effectiveness of prevention and control strategies.

Epidemic models are generally classified into stochastic models and deterministic models. A stochastic model (Daley, 2005) is formulated as a stochastic process with a collection of random variables. The solution to a stochastic model is a probability distribution for each of the random variables. A stochastic epidemic model is generally employed when fluctuations are non-ignorable, as in small populations. Therefore, stochastic model is not suitable for the scenarios concerned by this paper.

A deterministic model (Daley, 2005) is generally formulated in terms of a system of differential equations. The solution to a deterministic model is a function of time or space and is generally uniquely dependent on the initial data. Susceptible-infected-recovered (SIR) (Daley, 2005) model is the most fundamental deterministic model, most of other deterministic models (e.g., SIRS, SEIR, and MSEIR, etc) are regarded as extensions of SIR model. The above deterministic models have the following limitations:

- Insulated society. Namely, the above deterministic models are based on the assumption that there is no entry into or departure from the population. The community has no personnel exchange with neighboring communities;
- No spatial variable. The spread of disease is independent of “distance”.

As a remedy to the shortcomings of above deterministic epidemic models, an extensive SIR model constructed on the heat-transfer (Dewitt, 2006) platform, denoted as SIR-HT, is developed in this paper.

As illustrated in Figure 1, SIR-HT is derived from following three sources:

(1) geographic information such as population, residential environment, transportation network, and weather conditions, etc., which are used to define the disease transimission media;

(2) dynamic behavior of epidemic disease, which is described by SIR model in this work;

(3) persistent surveillance data (Nicoll A, et al, 2010), which is used to define the initial value for the transient solution of SID-HT model.

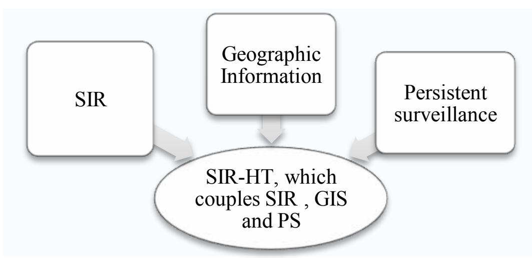


Figure 1. Framework of SIR-HT

The remainder of this paper is organized as follows. Section 2 introduces the priciple of SIR-HT model. Section 3 mathematically discusses the SIR-HT model. Section 4 briefly discusses the implementation flowchart of SIR-HT framework. Section 5 applies SIR-HT framework to simulate the spread of epidemic flu in a sample site near Minneapolis, and the simulation results are assessed. Finally, Section 6 summarizes the paper and discusses the future work.

## 2. Principle of SIR-HT Model

Assuming that there is no entry into or departure from the population and that the population size in a compartment is differentiable with respect to time, SIR model divides the population into three distinct classes; the susceptible, *S*, who can catch the disease; the infective, *I*, who have the disease and can transmit it; and the removed class, *R*, namely those who have either had the disease, or are recovered, immune or isolated until recovered. The progress of individuals is schematically described by $S \rightarrow I \rightarrow R$.

Let $s(t)$, $i(t)$ and $r(t)$ be the fraction of the number of individuals in each of the corresponding class at time *t*. The dynamic of the spread of an epidemic disease can be governed by the following system of nonlinear ordinary differential equations (Daley, 2005):

$$\begin{aligned} \frac{ds(t)}{dt} &= -\beta s(t) i(t) \\ \frac{di(t)}{dt} &= \beta s(t) i(t) - \gamma i(t) \\ \frac{dr(t)}{dt} &= \gamma i(t) \\ s(t) + i(t) + r(t) &= 1 \end{aligned} \quad (1)$$

In Equation (1), scalar $\beta$ is contact rate; scalar $\gamma$ is the mean recovery rate (Daley, 2005).

| SIR-HT model | Counterpart in Heat transfer model |
|---|---|
| Fraction of infective population | temperature (*T*) |
| Change rate of infective population | Heat-flux(*Q*) |
| Personnel exchange between neighboring community | Conductivity (*k*) |
| Road (including local, high-way, free-way) | Highly conductive layer |
| Terrain conditions (lake, mountain, etc) | Boundary conditions |
| Persistent surveillance input | Initial condition |
| Conservation of infective | Law of conservation of energy |
| The transmission of infectious between neighboring communities is proportional to the difference of their infective rate | Fourier law |

Table 1: SIR-HT vs. Standard Heat-transfer model

As addressed in previous section, standard SIR model does not involve spatial variable and is only applicable to an insulated society. Considering the intrinsic similarity between the spread of epidemic disease and heat-transfer problem, a innovative epidemic model, which is denoted as SIR-HT, is formulated by merging (or translating) the standard SIR model into heat-transfer framework.

Table 1 shows the relation between SIR-HT and generic heat-transfer model. An accurate mathematical description about SIR-HT will be discussed in the next section.

## 3. Mathematical Description about SIR-HT Model

### 3.1. Governing Equations

As addressed in previous section, the application of SIR model is limited because it does not involves a spatial variable, which is denoted as X in many literature. Considering that the spread of epidemic disease is analogous to a heat and mass transfer problem; SIR-HT, an innovative SIR extension is developed in this paper.

Similar to general heat-transfer model, which is constructed according to energy conservation law and Fourier law, SIR-HT model is constructed according to conversation of infective and following assumption: the transmission of infectious disease between neighboring communities is proportional to the difference of their infective rate

As a result, SIR-HT epidemic model is formulated using the following governing equations:

$$\rho C_p \frac{\partial i(X,t)}{\partial t} + \nabla \cdot (-K(X)\nabla i(X,t)) = Q_{\inf} - Q_{rec} \quad (2)$$

$$Q_{\inf} = \beta(X)s(X,t)i(X,t) \quad (3)$$

$$Q_{rec} = \gamma(X)i(X,t) \quad (4)$$

In above equations, *β(X)* is location-related contact rate; *γ(X)* is recovery rate; *s(X,t)* is susceptible fraction, for simplicity, it is assumed that recovered patients will automatically join the susceptible population in this paper; $\rho$ is population density; $C_p$ is a time-scaling coefficient (dimensionless); $Q_{int}$ is the incremental infective caused by contact infection; $Q_{rec}$ is the decremented infective caused by recovery; $\nabla \cdot (-K(X)\nabla i(X,t))$ indicates the infective change caused by the exchanging personnel between the local and neighboring community.

Table 2 shows the correlation between variables and parameters in the governing equations (2)-(4) and their counterpart in heat-transfer model.

| **SIR-HT model** | **Counterpart Terms in heat transfer model** |
|---|---|
| *i(X,t)* | Temperature (*T*) |
| $\rho$ | Material density |
| *K(X)* | Conductivity (*k*) |
| $C_p$ | Thermal capacity ($c_{p)}$ |
| $Q_{int}$ | Heat source |
| $Q_{rec}$ | Heat loss |

Table 2: Counter terms of governing equations in heat-transfer model

### 3.2. Boundary Conditions Introduced by Transportation Network

The boundary conditions introduced by transportation network (e.g., local, highway, railway, etc.) have a counterpart concept in heat-transfer model --- thin highly conductive layer.

In the similar way as heat-transfer problem, this boundary condition is formulated by the following partial differential equations:

$$d_{rd}\rho_{rd}C_{rd} + \nabla \cdot \left(-d_{rd}\kappa_{rd}\nabla i(X,t)\right) = -n \cdot q \quad \text{on } \partial\Omega_{rd}$$

$$q = -K(X)\nabla i(X,t)$$

In the above formula, $d_{rd}$ is transportation bandwidth; $\rho_{rd}$ is the passenger density; $C_{rd}$ is a coefficient; the transportation network is translated into boundary condition, which is denoted as $\partial\Omega_{rd}$.

| **SIR-HT model** | **Counterpart Terms in heat transfer model** |
|---|---|
| $d_{rd}$ | Thickness of highly conductive layer |
| $\rho_{rd}$ | Material density of highly conductive layer |
| $K_{rd}$ | Conductivity of highly conductive layer |
| $C_{rd}$ | Thermal capacity of highly conductive layer |
| $\partial\Omega_{rd}$ | Boundary condition introduced by highly conductive layer |

Table 3: Counter terms of road-oriented boundary conditions in heat-transfer model

Table 3 shows how to translate road-induced boundary conditions into thin highly conductive layer boundary conditions for heat-transfer model.

### 3.3. Initial Conditions

As a time-dependent problem, the initial conditions *i(X,0)* for SIR-HT model are given according to persistent surveillance data.

## 4. Implementation Flowchart

SIR-HT framework is implemented by formulating and solving the corresponding heat-transfer model.

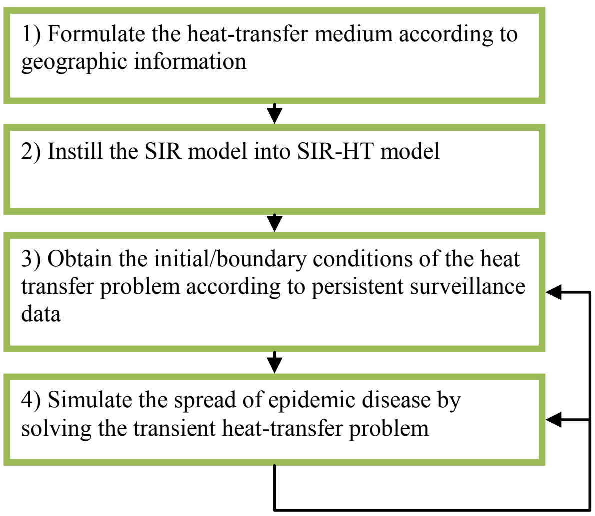


Figure 2. Flow chart of the implementation of SIR-HT

Figure 2 shows that the implementation SIR-HT framework consists of the following four steps:

- Step 1 defines the material properties and boundary conditions.
- Step 2 defines $Q_{int}$ and $Q_{rec}$ , which indicate the change rate of infective introduced by contact infection and recovery respectively.
- Step 3 sets the initial conditions for the transient solution to Equation (2). As shown in Figure 2, the simulation will be restarted when the persistent surveillance data is updated.
- Step 4 solves the Equation (2) using finite-element algorithm (D.K. Gartling and J.N. Reddy, 2000) (Y. Liang et al, 2002)

## 5. Experimental Results

The SIR-HT model is applied in simulating the spread of epidemic flu at a sample site near Minneapolis (Minnesota, USA). Considering the intrinsic consistency between SIR-HT model and general heat-transfer model, heat-transfer module of COMSOL 4.0 is employed in accomplishing the simulation.

Figure 3. Map of a sample site near Minneapolis

Figure 3 shows the geographic map of sample site. It is observed that the sample site includes residential area, roads, farm land, lakes, forest, etc.

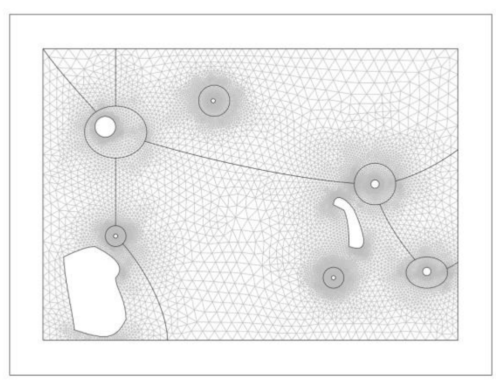

Figure 4. Heat-transfer model derived of Figure 3

Figure 4 shows the two-dimensional heat-transfer model, which is derived from Figure 3. Among the six residential communities in the model, four of them are connected via road, while the other two residential communities are relatively isolated.

To accomplish the simulation using COMSOL heat-transfer module, it is necessary to translate the SIR-HT model into heat-transfer model. Tables 4-6 list the parameter values about the residential community, farm land, and road. It has to be remarked that the parameter values listed in Tables 4-6 are subject to further validation.

In the resulted heat-transfer model, lakes, mountain areas, and forest are considered as perfect insulator.

| SIR-HT model | Parameter value in corresponding heat transfer model |
|---|---|
| $\rho$ | 2700 [kg/m^3] |
| *K(X,t)* | 500 [W/(m*K)] |
| $C_p$ | 900 [J/(kg*K)] |

Table 4: Parameter value for residential society

| SIR-HT model | Parameter value in corresponding heat transfer model |
|---|---|
| $\rho$ | 10 [kg/m^3] |
| *K(X,t)* | 0.038 [W/(m*K)] |
| $C_p$ | 100 [J/(kg*K)] |

Table 5: Parameter value for farm land

| SIR-HT model | Parameter value in corresponding heat transfer model |
|---|---|
| $\rho_{rd}$ | 7000 [kg/m^3] |
| $K_{rd}$ | 0.038 [W/(m*K)] |
| $C_{rd}$ | 300 [J/(kg*K)] |
| $d_{rd}$ | 0.01 [m] |

Table 6: Parameter value for boundary conditions introduced by transportation network

Once the heat-transfer model is formulated and the initial value is given, the spread of epidemic disease can be simulated by solving the transient heat-transfer problem.

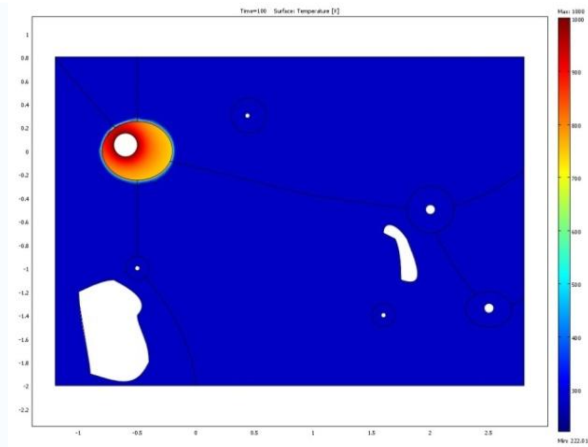

Figure 5. Spread of epidemic flu at day 1

Figure 5 shows the initial status of epidemic flu at the sample site. In this scenario, it is assumed that the epidemic flu breaks out in only one residential society, and the persistent surveillance data shows that 80% of the population getss infected.

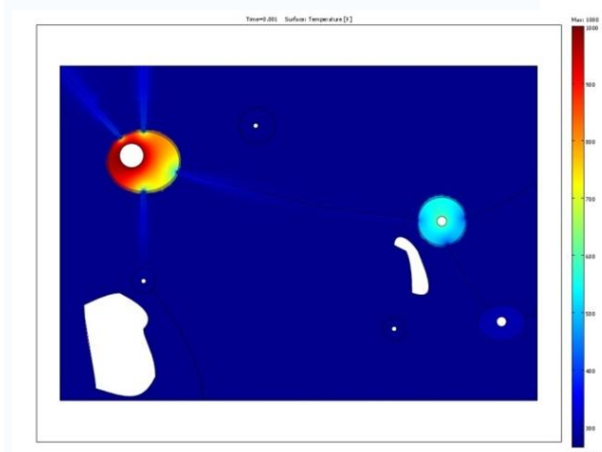

Figure 6. Spread of Epidemic Flu at day 3

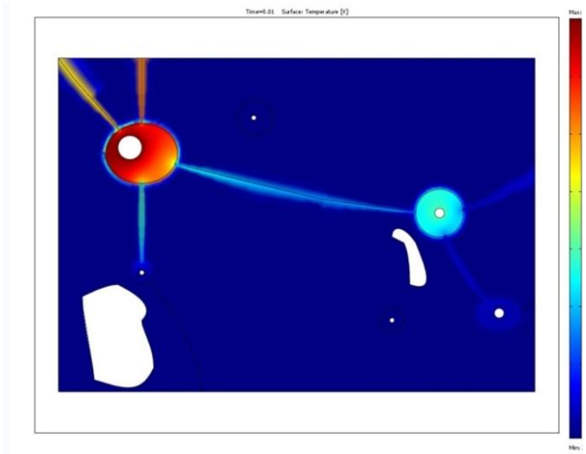

Figure 7. Spread of epidemic flu at day 5

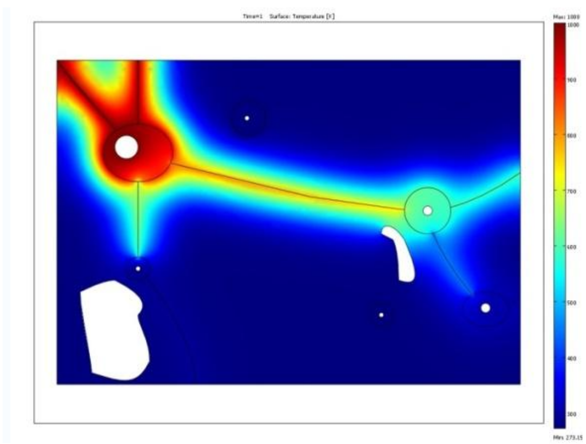

Figure 8. Spread of epidemic flu at day 10

Figures 6-8 shows the simulation results about the spread of epidemic flu from Day 3 through Day 10. It is observed that the epidemic flu spreads mainly within residential area and through transportation network.

## 6. Conclusions

In this work, a novel deterministic epidemic model is developed and tested in simulating the spread of epidemic flu at a sample site near Minneapolis city. The simulation results are basically consistent with our expectation. However, a more critical validation about the SIR-HT model is needed with the support and collaborations of experts in multidisciplinary areas such as medical science, sociology, statistic, optimization, geology science, and public health, etc.

In addition, the paper provides a generic framework that explores time-dependent event out of persistent surveillance data. This framework can be applied in other areas. For example, forecasting the migration of locust group, stopping the spread of forest fire, and predicting enemy movement under conditions of attack.

In the near future, the potential progress should be made in the following directions:

- Validation of proposed mathematical model;
- Effect of public prevention strategy and medical treatment over the SIR-HT;
- Introduction probability into SIR-HT model to achieve stochastic description about the spread of epidemic disease
- Effects of air-line transportation over SIR-HT.

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