Quasielectrostatic Induction on Stationary Vehicles under High Voltage Power Lines

Jon T. Leman[1] and Robert Schaerer[2]
[1]POWER Engineers Inc., Clarkston, WA, USA, [2]POWER Engineers Inc., San Diego, CA, USA

Abstract: The National Electrical Safety Code (NESC) requires that high voltage power lines in the United States be designed to limit the impact of electrostatic effects on nearby equipment. An example is that of a large vehicle parked underneath a transmission line. The rubber tires insulate the vehicle's body from ground. If a person standing on the ground makes contact with the vehicle surface, 60 Hz alternating current can flow through the person to ground. The NESC states that the short-circuit current between the equipment and ground must be limited to 5 milliamperes (often referred to as the 5 mA Rule). Common methods in use today for analyzing this phenomenon make use of geometrical simplifications, empirical formulas, and look-up tables. The author used COMSOL's AC/DC module to perform three-dimensional finite element analysis of a semi-trailer parked underneath high-voltage conductors. The study utilized COMSOL physics interfaces for Electric Currents and Electrical Circuits to calculate the steady-state open circuit voltage induced on the vehicle and to determine the resulting current flow when the vehicle surface was short-circuited to ground. Parametric sweeps provided insight into how the vehicle size, vehicle position, human body resistance, and overhead conductor height affected the short-circuit results. COMSOL significantly reduced the geometric uncertainty inherent in previous analysis methods and allowed the authors to assess the consequence of geometric simplifications in terms of personnel safety and transmission structure costs.

Keywords: High Voltage, Quasielectrostatic Induction, Overhead Transmission, Electrical Safety, National Electrical Safety Code, 5 mA Rule.

1. Introduction

The National Electrical Safety Code (NESC) is a voluntary standard adopted by most electric utilities in the United States of America. The NESC is published and maintained by the Institute of Electrical and Electronics Engineers (IEEE). It provides basic provisions for safeguarding of persons from hazards arising from the installation, operation, or maintenance of electrical supply and communication equipment. A portion of this standard is dedicated to establishing guidelines for clearances to energized electrical supply conductors. Rule 232.C.1.c of the NESC states, "*For voltages exceeding 98 kV ac to ground, either the clearances shall be increased or the electric field, or the effects thereof, shall be reduced by other means as required to limit the steady-state current due to electrostatic effects to 5 mA if the largest anticipated truck, vehicle, or equipment under the line were short-circuited to ground.*" This requirement is often referred to as the "5 mA Rule". A 5 mA current is below that for sustained muscular contraction (see Table 1).

Table 1: Physical effects of 60 Hz electric shock.

Current	Physiological Effect
0.5-1 mA	Threshold of perception
10-20 mA	Sustained muscular contraction
>100 mA	Ventricular fibrillation

A common application of this rule is design of overhead transmission line highway crossings. If a driver parks their vehicle under an energized transmission line, electric coupling from the overhead conductors induces a voltage on the vehicle's metallic surface which is insulated from ground by rubber tires. If the operator comes in contact with both earth and the vehicle body, they provide a path to ground and can experience an electric shock. In properly designed crossings the conductors are high enough that the shock would be harmless.

The physics of this scenario is well understood, but the complex geometry makes analysis difficult. Traditional methods involve simplified electric field equations and empirical testing of various shapes to arrive at a solution. In this study the authors used the finite element method with COMSOL to calculate the short circuit currents of a large vehicle under a high voltage transmission line and compared those results with the traditional semi-empirical methods.

2. Study Scenario

Figure 1 shows the three-phase transmission line selected by the authors for this study. The structures are designed to accommodate two three-phase circuits with conductors arranged in a vertical configuration. Often only one circuit is initially installed. The second circuit would be installed if justified by energy demand.

The nominal voltage is 345 kV line-to-line, a common voltage for bulk power delivery systems in the U.S. The height of the lowest conductor is 30 feet above ground at the point of lowest sag. This is a typical minimum for 345 kV circuits. Each of the three conductors or conductor bundles is referred to as a "phase". The steady-state voltages on the phase conductors are equal in magnitude and have angles that are 120 degrees apart (see the inset of Figure 1).

Figure 1. Transmission Line Selected for Study

The authors selected a semi tractor-trailer as the vehicle for the study. Nominal dimensions are: total length = 65 feet, width = 8.5 feet, and height = 14 feet. This is representative of the largest vehicle allowed without special permit on highways in most states in the U.S. The vehicle is oriented as shown in Figure 2. The impact of conductor arc due to sag is minor and was neglected in the study. Conductors were modeled with a straight-line approximation.

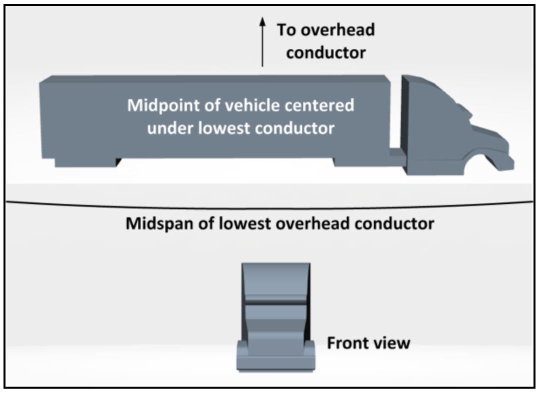

Figure 2. Study Vehicle and Orientation to Conductor

3. Semi-Empirical Approach

Due to the historical difficulty of calculating electric field interactions with complex shapes, a common approach for the 5 mA Rule has been to use a semi-empirical method described by the following steps.

1. Calculate the two-dimensional electric field profile of the transmission line at ground level. Calculating electric fields at ground level allows significant simplification of field equations.
2. Calculate the average electric field over the equivalent length of the vehicle. The equivalent length is the actual length plus some additional distance to account for fringe affects (about 1.5-2 meters at each end of a large vehicle).
3. Calculate the effective surface area of the vehicle. This is typically done using a rectangular box approximation involving empirically generated look-up tables or a geometric approximation where the edges of the top surface are projected down to ground at a 45 degree angle and the area projected on the ground is used as the equivalent charge collecting surface area.
4. Finally, calculate the short circuit current from the vehicle to ground using (1):

$$I = 2\pi f \varepsilon_0 E S \qquad (1)$$

where:
I is the short circuit current,
f is the power system frequency in Hz,
ε_0 is the permittivity of free space,
E is the average electric field, and
S is the effective surface area

Since the 5 mA Rule is associated with safety the authors' goal in performing this study was to eliminate or reduce uncertainty regarding the following semi-empirical process assumptions.

1. The analysis is relatively insensitive to the detailed surface geometry of the vehicle.
2. The electric field at the ground plane is very close to that on any parallel plane in the space that would be occupied by the vehicle.
3. The additional length used to calculate the average electric field appropriately accounts for fringe effects.

3. Use of COMSOL Multiphysics

The following characteristics of this problem were important for selection of the appropriate physics interface.

1. The transmission system is an alternating current system, operating in steady-state at a frequency of 60 Hz. A power system frequency of 60 Hz is in the extremely low frequency (ELF) range. The wavelength for a 60 Hz signal is about 5,000 kilometers (about 3,100 miles), much larger than the largest dimension in the study which is about 200 feet.
2. For typical values of soil resistivity and at 60 Hz the time it takes charge in the earth to redistribute under the influence of externally applied electric fields (charge relaxation time) is very small with respect to the period of the transmission line voltage and current waveforms. Therefore, the earth can be treated as a large conductor. All other model components are metallic conductors with charge relaxation time much smaller than a 60 Hz period. This means that earth, the transmission conductors, and the vehicle each have equipotential surfaces.
3. The long cylindrical geometry of the transmission line conductors results in concentric magnetic flux lines. The orientation of the flux lines with respect to the vehicle are such that magnetic induction as described by Faraday's law and Lenz's law would not result in a significant voltage difference between earth and the vehicle. Also, the short circuit current to ground will be very small so magnetic fields associated with conduction and displacement currents are negligible.
4. Space charges due to ion production from transmission line corona effects are negligible for alternating current systems. Therefore, the electric fields can be calculated solely using the voltage and conductor geometry. Also, the AC electric fields are not influenced by environmental factors such as wind which can cause space charge drift.

The items above allow this problem to be characterized as quasielectrostatic. Therefore, the Electric Currents physics interface with frequency domain study mode was sufficient for performing the analysis in COMSOL. As will be discussed hereafter, the Electrical Circuit interface also provided a means of obtaining vehicle short circuit current results.

3.1. Governing Equations

Equations (2) through (6) describe the fundamental mathematics associated with this physics problem. Equation (2) is the basic relationship between the electric potential and the electric field. An important implication of this relationship is that the electric field lines are always orthogonal to equipotential lines.

Equation (3) is Gauss' law which is that the electric flux through an enclosed surface is proportional to the enclosed charge.

Equation (4) is the continuity equation associated with Gauss' law. This equation defines the relationship between the surface charge density of the conducting body and the external electric field, E^a. The internal electric field of a conductor, E^b, is zero.

Equation (5) is a mathematical description of the law of conservation of charge which requires that any current into or out of a volume be equal to the rate of change of charge within that volume. Equation (6) states that the current density is equal to the sum of the electric field times the surface charge density, the displacement electric field term adapted for steady state frequency domain, and an externally applied current density.

$$\boldsymbol{E} = -\nabla \boldsymbol{V} \tag{2}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \tag{3}$$

$$\mathbf{n} \cdot (\varepsilon_0 \boldsymbol{E^a} - \varepsilon_0 \boldsymbol{E^b}) = \sigma_s \tag{4}$$

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t} \quad (5)$$

$$\boldsymbol{J} = \sigma \boldsymbol{E} + j\omega \boldsymbol{D} + \boldsymbol{J_e} \quad (6)$$

Implementation of this design problem in COMSOL involved two methods. This served as a way to verify reasonableness of results. The authors also used a non finite element program to perform validating simulations.

3.2 Common Modeling Characteristics

The following is a list of model components and characteristics that were the same for both COMSOL approaches used for the study.

3D Solution Space: The solution space consists of a rectangular volume of air surrounding the transmission conductors and vehicle (refer to Figure 3). The bottom surface is the ground plane and represents the surface of the earth. The top and sides are default electric insulation. The volume is large enough that the top and side boundaries do not influence the electric field immediately around the truck.

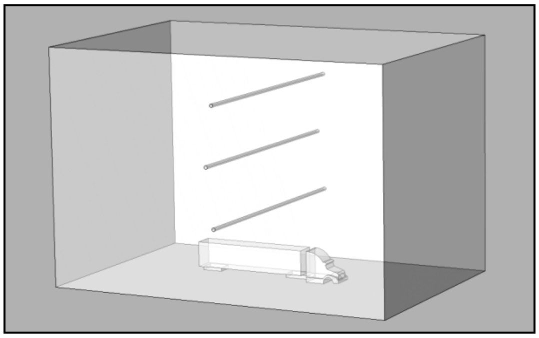

Figure 3. COMSOL Model Geometry Overview

Transmission Line Conductors: The real transmission line from which this model was developed has two conductors per phase that are each 1.5 inches in diameter and 18 inches apart. This is referred to as a conductor bundle. The small conductor diameter compared to the overall geometry (~200 ft) represented a significant challenge for meshing. To reduce the mesh node count each bundle was modeled as a single equivalent conductor. The authors used parametric sweeps in a 2D COMSOL model of the conductors to identify the appropriate parameters for a single conductor equivalent and to verify that the electric field profiles of the equivalent conductor matched that of the actual conductor bundles. Electric potentials were applied to the surfaces of the equivalent single conductors. Conductor voltages were 120 degrees out of phase with each other and each had voltage magnitudes that were 110% of nominal for a 345 kV system.

Vehicle: The authors used a vehicle model prepared in CAD software which was imported into COMSOL. The study also includes sensitivity cases with simplified models of the vehicle (see Figure 4). In each case one surface near the front of the vehicle served as a connection terminal to ground potential. The next sections discuss details about the vehicle ground connection.

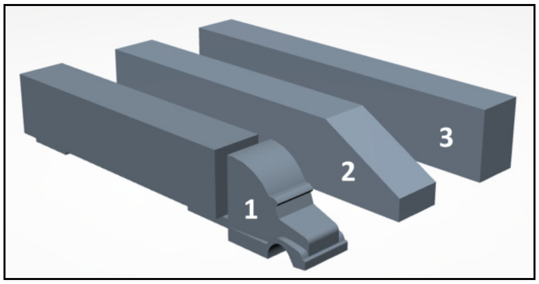

Figure 4. Vehicle Cases

3.3 COMSOL Approach 1

The first approach used only the Electric Currents interface with a frequency domain study mode (60 Hz). The truck was modeled as a solid body of conducting material. The interior volume was meshed and included in the solution set. A solid conducting cylinder connected the truck to the ground plane. The authors calculated the vehicle current to ground by evaluating a surface integral of the normal current density over the surface where the cylinder met the ground plane.

The metal body of a vehicle is thin; however, the solid conducting body approximation is reasonable when the period of the power system frequency is much larger than the conductor charge relaxation time.

This method required a very fine mesh. The degrees of freedom solved for in the model numbered about 1,600,000. The authors used a fully coupled iterative solver which converged to an error of 10^{-5} in three iterations.

3.4 COMSOL Approach 2

The second approach utilized the Electrical Circuit interface in conjunction with the Electric Currents interface. The internal truck volume was excluded from the solution set and a high conductivity material was assigned to the boundary surfaces of the vehicle. The authors then applied electric shielding to these surfaces and linked them to the electrical characteristics of the surface material. A small surface near the front of the vehicle served as a "Terminal" with "Circuit" selected as the Terminal type.

In the Electrical Circuit interface an "External I Vs. U" element provided the coupling to the terminal surface in the Electric Currents interface. A small resistor in parallel with the External I Vs. U element served as the short to ground and was used to model human body resistance in sensitivity cases. The current through this resistor was the vehicle short circuit current. Figure 5 illustrates the setup.

The authors used a fully coupled direct solver for this approach. The computation was more demanding and the mesh resolution had to be reduced to less than 200,000 degrees of freedom in order to reach a solution.

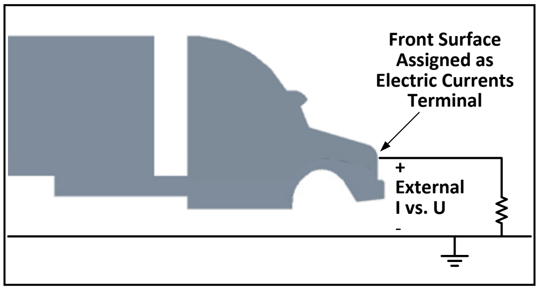

Figure 5. Electrical Circuits Interface

3.5 CDEGS Software

The final method involved software called CDEGS (**C**urrent **D**istribution, **E**lectromagnetic Fields, **G**rounding and **S**oil Structure Analysis). CDEGS computes electromagnetic fields for user defined networks of energized and grounded conductors. This software is used for substation ground grid and transmission line AC interference analysis. The authors prepared a simple case using a network of conductors arranged to approximate the study system.

3.6 Sensitivity Studies

In addition to the primary study goal of using COMSOL to validate traditional 5 mA Rule methods, the authors performed the following sensitivity studies.

1. Evaluation of how geometric simplifications affected the results.
2. Parametric sweeps to evaluate how results were affected by vehicle size, vehicle position under the conductors, conductor height, and human body resistance.

4. Results

Figure 6 shows the electric field magnitudes for one of the cases studied. The results shown on the vertical cut plane were calculated from the coplanar components of the electric field. The values on the surface of the truck and ground plane are the magnitudes of the electric field component that are normal to the surface. The color range was manually adjusted to emphasize the electric field gradients at the truck. Dark red locations denote areas where the electric field strength is 25 kV/m or higher. Though not apparent on the plot, the electric field strength at the conductor is about 10 times higher than the electric field at the surface of the truck.

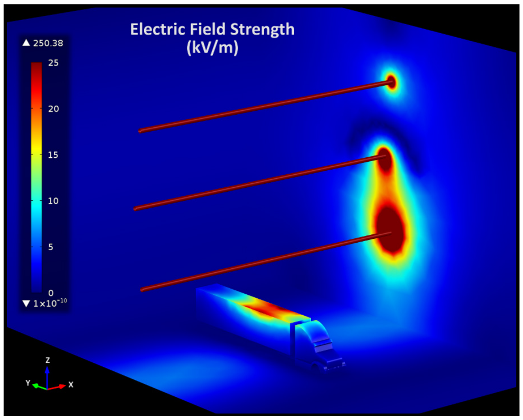

Figure 6. Electric Field Magnitude (kV/m)

4.1 Primary Case

The primary case for comparison is described in Section 2. If the vehicle is not shorted to ground, results show that the open circuit voltage on the surface of the vehicle reaches a potential of about 6,000 volts[1]. Table 2 lists the vehicle short circuit results for the primary comparison case. Table 2 also includes sensitivity results for the different vehicle approximations.

Table 2: Vehicle Short Circuit Current Results

Approach	Short Circuit Current		
	Case 1*	**Case 2***	**Case 3***
Semi-empirical	n/a	n/a	4.2 mA
COMSOL approach 1	3.8 mA	4.1 mA	4.2 mA
COMSOL approach 2	4.0 mA	4.0 mA	4.1 mA
CDEGS	n/a	4.1 mA	4.2 mA

* Vehicle geometry case - refer to Figure 4

There is good correlation among the results of the different approaches. These results indicate that the assumptions associated with the traditional approach are reasonably well optimized. Had the COMSOL results shown significantly larger values then this would indicate a possible safety problem with the assumptions. Had the COMSOL results shown significantly lower short circuit results then this would be indication that transmission structures may be taller than necessary. Every 10 feet of structure height costs about $6,000 per structure for above ground materials and the foundation reinforcements necessary to withstand greater bending moments. Costs can add up if unnecessary height is designed into a large number of structures.

The results also show that the analysis is not very sensitive to detailed geometry. Only about a 10% difference exists between the case with the most geometric detail and the case with the least. A portion of this difference is due to varying mesh resolution.

4.2 Parametric Sweep Sensitivity Studies

Figure 7 shows the result of a parametric sweep of vehicle length. State height and width limits for semi-trailers are reasonably consistent throughout the U.S.; therefore, only the impact of vehicle length was examined. The result indicates that the worst case is not necessarily the longest expected vehicle as implied by the wording of the NESC 5 mA Rule. Shorter vehicles may be the controlling case. This result depends heavily on the configuration of the overhead conductors.

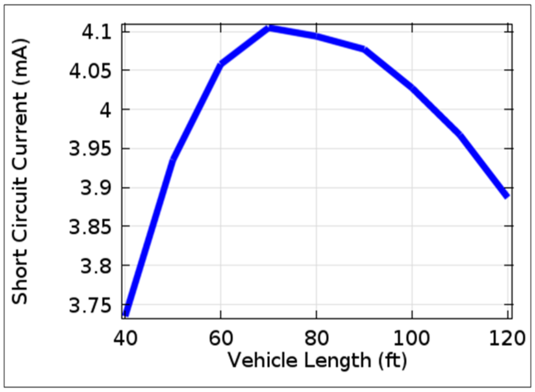

Figure 7. Short Circuit Current vs. Vehicle Length

Figure 8 is the result of a parametric sweep of vehicle position along its length axis. The plot shows that the worst case position is with the vehicle approximately centered under the overhead conductors. This is the expected result given the conductor configuration. Other configurations such as delta or horizontal arrangements would be less obvious. A vehicle orientation parallel to the conductors could result in a greater average electric field and larger short circuit currents. Normally transmission line corridors near highways are wide enough to prevent a truck from parking parallel and directly under the transmission line. The parallel case was neglected in this study, but would be an important consideration for design activities.

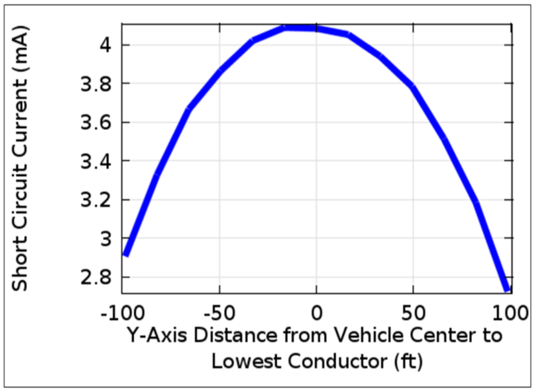

Figure 8. Short Circuit Current vs. Vehicle Position

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[1] All voltage and current results are reported as root mean square (rms) values.

The parametric sweep of Figure 9 examines the impact of conductor height (spacing between conductors is constant). This data could be used to optimize clearance height and meet the 5 mA Rule criteria. In this case the height of the conductors could be reduced to about 25 ft. For this situation other NESC clearance requirements would limit reductions. However, this demonstrates the usefulness of parametric sweeps for optimization of 5 mA Rule clearances when other clearances do not control the design.

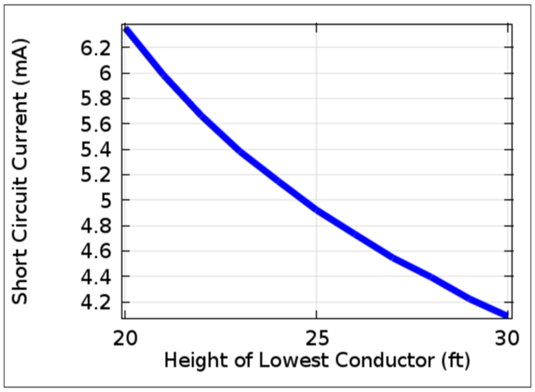

Figure 9. Short Circuit Current vs. Conductor Height

Typical human body resistance with dry skin ranges between about 700 and 2000 ohms. The final parametric sweep in Figure 10 shows that the short circuit current results are essentially unchanged within this resistance range. The body resistance is in series with the very small capacitance (large impedance) of the truck and overhead line system. The impedance of the equivalent capacitance dominates. Any change to overall impedance due to the normal range of body resistance is negligible.

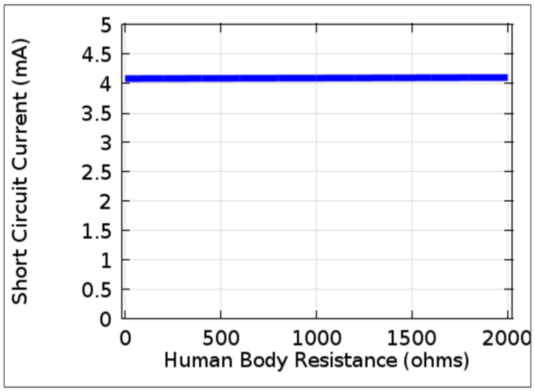

Figure 10. Short Circuit Current vs. Human Body Resistance

5. Conclusions

The results of the comparative study indicate that the assumptions associated with the semi-empirical approach are reasonable. The semi-empirical approach seems to be well optimized for the case studied and geometric simplifications appropriately bias the result toward the side of safety.

The sensitivity studies revealed the following important points.

1. Reasonably accurate results can be obtained using course geometric representations of the vehicle or object under the transmission line. The detailed COMSOL model with a high resolution mesh produced a short circuit current result that was about 10% lower than the semi-empirical method.
2. The NESC 5 mA Rule is phrased in terms of the largest expected vehicle under the power line. The results indicate that the largest vehicle is not always the controlling case.
3. The current delivered through a person who touches the vehicle is essentially independent of resistance in the typical range of the human body.

The COMSOL software was very helpful in overcoming uncertainty associated with the semi-empirical method and improved the authors' understanding of the physics behind this type of power system behavior.

Future work to verify broad applicability of these conclusions includes performing analysis of additional vehicle geometries, varying transmission line configurations and voltages, and evaluating different vehicle orientations.

6. References

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