Simulation of Acoustic Energy Harvesting Using Piezoelectric Plates in a Quarter-wavelength Straight-tube Resonator

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1. Acoustic energy

(1) Low energy density;
(2) Clean, renewable and abundant in our life;
(3) Currently wasted.

**Various sound sources**

- SPL 180 dB — Stun grenades
- SPL 170 dB — Rifle fired at 1m
- SPL 160 dB — Jet engine at 30m
- SPL 150 dB — Rock concert speaker at 1m
- SPL 140 dB — Jet engine at 100m
- SPL 130 dB — Vuvuzela at 1m
- SPL 120 dB — Siren at 30m
- SPL 110 dB — Jack Hammer at 1m
- SPL 100 dB — Traffic on a busy roadway at 10m
- SPL 90 dB — Loud speech at 1m
- SPL 80 dB — Passenger car at 10m
- SPL 70 dB — TV (set at home level) at 1m
- SPL 60 dB —
2. Objective

Develop an acoustic energy harvester:

- Low operating frequency (~190Hz)
- High efficiency
3. Acoustic resonator

Helmholtz resonator

Reduce noise (architectural acoustics, aircraft engines)

42 cm quarter-wavelength tube resonator

1st eigemode shape

Magnitudes of normalized first three acoustic pressure eigenmodes
4. Acoustic energy harvesting mechanism

(a) Piezo cantilever plates in a 42 cm long quarter-wavelength tube resonator (operating frequency is $\sim 190$ Hz)

(b) Eigenmode shape with 3 piezo plates

(c) Normalized pressure with 3 piezo plates
5. Piezo energy conversion (PZT)

Kirchoff’s Voltage Law:
\[ \sigma_{in} = \sigma_i + \sigma_d + \sigma_s + \sigma_p, \]

Output power:
\[
P = \frac{(\omega_n dt_p / \varepsilon)^2 RC_p^2}{(\omega_n R^2 C_p^2 (4\zeta^2 + k^4) + 4\zeta^2 + 4k^2 \zeta \omega_n RC_p} \times \left(\frac{t_c l^2 b}{6l} \Delta p \right)^2,
\]

\[
\frac{\partial P}{\partial R} = 0,
\]

Optimized loading resistance:
\[
R_{opt} = \frac{1}{\omega_n C_p} \frac{2\zeta}{\sqrt{4\zeta^2 + k^4}}.
\]
6. Use of COMSOL Multiphysics and experiment apparatus

(a) Finite element model

(b) Experimental apparatus

(c) Boundary conditions
7. 1 piezo plate placed inside the tube resonator

Output voltage of the piezo plate decreases when moving to the tube closed end
7. 8 piezo plates placed along the whole tube

The summation of power:

$$\sum \frac{V_i}{R_{opt}}, i=1, 2,...$$

Experiment and simulation reach maximum 0.311 mW (at 194Hz) and 0.369 mW (at 190 Hz) for 4 PZT plates from A to D positions.
7. 7 piezo plates placed inside the first half of the tube

Experiment and simulation reach maximum $0.509 \text{ mW}$ (at 191Hz) and $0.605 \text{ mW}$ (at 188 Hz) for 4 PZT plates from A to D positions.
8. Conclusion

(1) Acoustic energy harvesting mechanism is developed at low operating frequency (~190 Hz) using a quarter-wavelength straight-tube resonator with multiple piezoelectric cantilever plates;

(2) The number of plates to generate the maximum voltage and power are limited by the interruption of acoustic air particle motion caused by the presence of plates;

(3) Experiment and simulation reach maximum 0.509 mW (at 191 Hz) and 0.605 mW (at 188 Hz) for 4 PZT plates placed in the first half of the tube resonator.
Table 1: Structure and material properties of PZT piezoelectric plate and polycarbonate.

<table>
<thead>
<tr>
<th>Type</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezo plate size</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>$l$</td>
<td>4 cm</td>
</tr>
<tr>
<td>Width</td>
<td>$b$</td>
<td>2 cm</td>
</tr>
<tr>
<td>Total thickness</td>
<td>$t$</td>
<td>0.7 mm</td>
</tr>
<tr>
<td>Piezo plate structure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT layers</td>
<td>$t_p$</td>
<td>0.48 mm</td>
</tr>
<tr>
<td>Carbon fiber</td>
<td>$t_c$</td>
<td>0.22 mm</td>
</tr>
<tr>
<td>Piezo plate’s capacitance</td>
<td>$C_p$</td>
<td>75 nF</td>
</tr>
<tr>
<td>PZT’s Piezo constant</td>
<td>$d_{31}$</td>
<td>750 pC/N</td>
</tr>
<tr>
<td>PZT relatively permittivity</td>
<td>$\varepsilon/\varepsilon_0$</td>
<td>4500</td>
</tr>
<tr>
<td>PZT Young’s modulus</td>
<td>$E_p$</td>
<td>40 GPa</td>
</tr>
<tr>
<td>PZT density</td>
<td>$\rho_p$</td>
<td>7400 kg/m$^3$</td>
</tr>
<tr>
<td>PZT damping ratio</td>
<td>$\zeta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Carbon fiber’s Young’s modulus</td>
<td>$E_c$</td>
<td>2 GPa</td>
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<tr>
<td>Polycarbonate’s density</td>
<td>$\rho_p$</td>
<td>1175 kg/m$^3$</td>
</tr>
<tr>
<td>Polycarbonate’s Young’s modulus</td>
<td>$E_{pc}$</td>
<td>2.2 GPa</td>
</tr>
</tbody>
</table>
Backup slide

\[ \sigma_{im} = \sigma_i + \sigma_d + \sigma_s + \sigma_p \]

\[ \sigma_{in} = \frac{1}{l} \int_{0}^{l} M_p(x) \frac{t_c}{I} dx \]

\[ \nu(\tau, x) = \frac{bx^2 (6l^2 + 4lx + x^2)}{24EI} \Delta p \]

\[ dG_k = \frac{\dot{v}(\tau, x)^2}{2} \frac{mdx}{l} \]

\[ G_k = \int_{0}^{l} dG_k = \frac{\dot{v}(\tau, l)^2}{2} 0.257m \]

\[ \sigma_i = \frac{m_{eq}}{c_i} \ddot{\delta} \]

\[ \sigma_d = \frac{\eta}{c_2} \dot{\delta} \]

\[ \sigma_s = E\delta \]

\[ \sigma_p = -\frac{dE}{\ell_p} V \]

\[ V_{mag.} = \frac{\omega_n RC_p dt_p / \varepsilon}{\sqrt{R^2 C_p^2 \omega_n^2 (4\zeta^2 + k^2) + 4\zeta^2 + 4\zeta k^2 \omega_n RC_p \left(\frac{t_c l^2 b}{6I}\right)}} \Delta p \]

\[ P = \frac{V_{mag.}^2}{R} \frac{(\omega_n dt_p / \varepsilon)^2 RC_p^2}{\omega_n^2 R^2 C_p^2 (4\zeta^2 + k^4) + 4\zeta^2 + 4k^2 \zeta \omega_n RC_p \left(\frac{t_c l^2 b}{6I} \Delta p\right)^2} \]

\[ R_{opt} = \frac{1}{\omega_n C_p \sqrt{4\zeta^2 + k^4}} \]
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Graph 1: Voltage (V) vs. Sound pressure (Pa) and Power (mW)
- Black dots: Experimental voltage
- Blue squares: Experimental power
- Black line: Calculated voltage
- Blue dashed line: Calculated power

Graph 2: Voltage (V) vs. Time (sec)
- Line A
- Blue line B
- Red line C
- Pink line D
Piezoelectric nanowires

MEMS-based piezoelectric Harvesting devices

Falconi (2009)

Wave Energy Piezoelectric Converter

Zurkinden (2007)

Kamel (2010)