Numerical Study of a DC Electromagnetic Liquid Metal Pump: Limits of the Model

Nedeltcho Kandev

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Introduction

This work presents the results of a fully 3D MHD simulation of an electromagnetic DC pump for liquid aluminum using a rectangular flow channel subjected to an external transversal non uniform magnetic field.
Introduction

Electromagnetic Pump - AC and DC

- The concept of EP was created in 1960’s (pumping Sodium) and later in 1970’s (pumping zinc and aluminum).

- Now EP are used (mainly the AC concept) in many industrial applications: extrusion billet casting, metal refinery for transporting molten metals, alloys production, velocity-meter for molten metal by measuring the Lorentz braking force etc.
Principal of EP and geometry of the model

The physical principal is based on the Lorentz force:

\[ \vec{F} = \vec{J}_e \times \vec{B} \]

Two concepts EMP:
- Pump AC and
- Pump DC.

MHD model based on EMP prototype:
Channel length \( L = 0.3 \text{ m} \), height \( H = 0.02 \text{ m} \) and width \( W = 0.1 \text{ m} \). Magnet length \( Lm = 0.05 \text{ m} \)
Study of the MHD

The present complex MHD problem involving laminar or turbulent metal flow under the influence of a strongly non uniform magnetic field is not necessarily symmetrical and, therefore, only a fully 3D high resolution simulation could grasp all spatial aspects of the electromagnetic and fluid dynamics phenomena.
Study of the MHD

State of the art (2007):

- Richard J. Holroyd (1979) (experimental)
- I. J. Ramos and N. S. Winowich (1990) (2D, laminar)
- M. Hughes and al. (1995) (2D, laminar)
- Suwon Cho (1998) (Analytique)
- Votaykov E. V. and al. (2007) (3D, laminar)

The MHD pumping was not sufficiently investigated, especially for the case of the turbulent flow.
**Classical formulation of the MHD model**

**EM: Ohm’s law for moving media:**

\[ \vec{J} = \sigma (-\nabla \phi + \vec{u} \times \vec{B}) \]

\[ \nabla \cdot \vec{J} = 0 \]

**Conservation of the el. current**

**Navier-Stokes equations for laminar flow**

\[ \rho (\vec{u} \cdot \nabla) \vec{u} - \nabla \cdot \left[ \eta (\nabla \vec{u} + (\nabla \vec{u})^T) \right] = -\nabla P + F_L \]

\[ \nabla \cdot \vec{u} = 0 \]

**Momentum**

**Conservation of the flow**
Modeling of the magnetic field

Usually, the external non-uniform magnetic field $B$ is simulated by using a 3D parametric Function.

Example of 3D function
Simulation using COMSOL-Multiphysics

- The main advantage of using Comsol is that it is not necessary to write the internal source codes since the basic expressions are already built-in.

- It is possible with Comsol to use coupling of different physical modules to carry out simulations of complex physical problems.

- In our MHD simulations we have used: Magnetostatics (emqav), DC conductive media (emdc), Navier-Stokes module for laminar flow and k-ε Turbulence model (chns).
Our model for laminar brake flow 3D:

\[ \nabla \times \left( \frac{\nabla \times \widetilde{A}}{\mu} \right) = \sigma (-\nabla \phi + \bar{u} \times (\nabla \times \widetilde{A})) \]

\[ \bar{J} = \sigma (-\nabla \phi + \bar{u} \times (\nabla \times \widetilde{A})) \]

\[ \nabla \cdot \bar{J} = 0 \]

\[ \rho (\bar{u} \cdot \nabla) \bar{u} - \nabla \cdot \left[ \eta \left( \nabla \bar{u} + (\nabla \bar{u})^T \right) \right] = -\nabla P + F_L \]

\[ \nabla \cdot \bar{u} = 0 \]

Maxwell-Ampère’s law

Ohm’s law

Conservation of the el. current

Navier-Stokes equations

Conservation of the flow
Geometry of the model

The electromagnetic domain is delimited by an air sphere.
Current density distribution (brake flow)

Braking force

\[ J \times B \]
Velocity profile (laminar brake flow)

Typical Hartmann destruction of the velocity profile

Velocity field
Our model for turbulent flow 3D:

\[ \nabla \times \left( \frac{\nabla \times \vec{A}}{\mu} \right) = \vec{J} \]

\[ \vec{J} = \sigma (-\nabla \phi + \vec{u} \times (\nabla \times \vec{A})) + \vec{J}_e \]

\[ \nabla \cdot \vec{J} = 0 \]

\[ \rho (\vec{u} \cdot \nabla) \vec{u} - \nabla \left[ (\eta + \eta_T) \left( \nabla \vec{u} + (\nabla \vec{u})^T \right) \right] = -\nabla P + (\vec{J} \times \vec{B}) \]

\[ \rho \vec{u} \cdot \nabla k - \nabla \left[ \left( \eta + \eta_T \sigma_k \right) \frac{\nabla k}{\sigma} \right] = \frac{1}{2} \eta_T \left( \nabla \vec{u} + (\nabla \vec{u})^T \right)^2 - \rho \epsilon \]

\[ \rho \vec{u} \cdot \nabla \epsilon - \nabla \left[ \left( \eta + \eta_T \sigma_k \right) \frac{\nabla \epsilon}{\sigma} \right] = \frac{1}{2} C_{\epsilon_1} \frac{\epsilon}{k} \eta_T \left( \nabla \vec{u} + (\nabla \vec{u})^T \right)^2 - \rho C_{\epsilon_2} \frac{\epsilon^2}{k} \]

\[ \eta_T = \rho C_\mu \frac{k^2}{\epsilon} \]

\[ \nabla \cdot \vec{u} = 0 \]

Maxwell-Ampère’s law

Ohm’s law

Cons. el. current

Momentum

Kinetic energy

Dissipation rate

Turbulent viscosity

Cons. flux
**Pumping** ($I_e=1794\,A$, $B_0=0.4\,T$, $F_t=38.93\,N$, $U_m=0.88\,m/s$)

Revelation of *special swirling* in the induced current

Induced current density
Total Lorentz driving force

\[ F_T = F_e + F_{Lbr} = 69.87 - 30.94 = 38.93 \text{ N} \]
Velocity profile for turbulent pump flow

\[ U_m = 0.88 \text{m/s} \]
**MHD simulation 3D**

**Package**: COMSOL Multiphysics 3.5a  
**Method**: finite element  
**Applications**: coupling of different modules:  
- Navier-Stokes laminar flow model (chns) or  
- k-ε turbulent flow model (chns)  
- Conductive media (emdc)  
- Magnetostatics (emqav)  
**Solver**: Stationary segregated solver  
**Computer**: Double Quad Xeon, 8 processors  
10Gb de RAM, 64 bits.  
**Time**: 6 to 8 hours  
**Meshing**: up to 172 000 elements, 1 725 000 DOFs

Hydro Québec
Institut de recherche
Summarized results

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<th>Application</th>
<th>Ve</th>
<th>Ie</th>
<th>Pinlet</th>
<th>Vo</th>
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$$R_e = U_0 h / \eta$$

$$H_a = B_0 h \sqrt{\frac{\sigma}{\mu}}$$
Conclusion

1. The simulation of laminar and turbulent flow of liquid aluminum accurately represents the formation of an M shaped velocity profile and corresponds with the results of recently published works.

2. Our theoretical model utilizes not only Ohm’s law, but also the Maxwell-Ampere equation, thus representing more precisely the phenomena in a real EM pump.

3. For the turbulent pumping case, our fully 3D simulation revealed for the first time the appearance of two couples of small current loops resulting from the complex spatial interaction of electromagnetic and hydrodynamic phenomena.
Thank you!

Questions???
Model: P/N 345704
The crew