Printed Acceleration Sensor

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Abstract: In this paper we want to figure out the development of a capacitive acceleration-sensor system with the FEM-Method using Comsol Multiphysics. The sensor-system is in the position to detect accelerations in the range of ± 20 g. Furthermore, the sensor-element contains a printed RF-inductance, which is used for contactless data transfer.

On the one hand the simulation of the L-C-oscillating circuit using the RF-modul of Comsol is shown, on the other hand the simulation of the sensor itself was done. The deflection of the sensor by an acceleration load was calculated and the change in sensor’s capacitance by the resulting deflection was evaluated (nonlinear-structural-mechanics and electrostatics coupled by the ALE-method). The changed capacitance of the sensor leads to a change in resonance frequency of the oscillating circuit, which could be detected.

The sensor itself and the coil should be printed with nanosilve-ink.

Keywords: Radio frequency, micromechanics, printed electronics, ALE, nonlinear-structural mechanics

1. Introduction

The idea was to develop an acceleration sensor system with a RF-readout (Fig. 1,2). The sensor itself is a capacitive acceleration sensor. This capacitance combined with the coil L₂ results in an oscillating circuit. The readout system is inductively coupled to this circuit by the coil L₁. The components R_mess, U_mess and U₀ present the evaluating circuit. This circuit is currently developed in a further project. Based on the simulation data we want to decide about the design of the acceleration sensor-system. We want to define the system’s inductance, capacitance and sensor’s mechanical design.

Using Comsol Multiphysics it is possible to reduce the costs of building prototypes. A first layout could be developed easily and cheaply.

2. Use of COMSOL Multiphysics

2.1 Radio-Frequency Calculation

2.1.1 Theory

In a simple parallel plate capacitor it is easy to calculate the capacitance. The capacitance was evaluated by

\[ C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d_0} \]

where A is the area of the capacitor-plates and d₀ is the distance between the plates.
The inductance of the coils can be calculated approximately by

\[ L \approx c_1 \cdot \mu_0 \cdot \frac{N^2 d_m}{1 + c_2 \rho_f^2} \]  

where \( \rho_f \) is the ratio between the outer diameter of the coil \( d_a \) and the inner diameter of the coil \( d_i \)

\[ \rho_f = \frac{d_a - d_i}{d_a + d_i} \]

and \( d_m \) is the median diameter of the coil:

\[ d_m = \frac{1}{2} \cdot (d_a + d_i) \]

The constants \( c_1 \) and \( c_2 \) are geometry based (Tab 1.).

Table 1: Definition of the constants.

<table>
<thead>
<tr>
<th></th>
<th>( c_1 )</th>
<th>( c_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>round coil</td>
<td>2.25</td>
<td>3.55</td>
</tr>
<tr>
<td>quadratic</td>
<td>2.34</td>
<td>2.75</td>
</tr>
</tbody>
</table>

It was shown, that the accuracy of the formula is about 4 % [3].

The resonance frequency is given by

\[ f_0 = \frac{1}{2\pi \cdot \sqrt{LC}} \]

2.1.2 Simulation

In this study the resonance frequency of the system was calculated. The system consists of the two coupling coils (\( L_1, L_2 \)) with the sensor at the end of one coil. The sensor was replaced by two parallel plates. As a first approach a PSPICE simulation of the ideal schematics (Fig. 3) was done. PSPICE is a simulation program which is based on the ideal physical equation of the electrical parts.

![Figure 3: Result of the calculation using PSPICE. The complex impedance (light green), the real part of the impedance (dark green), the imaginary part of the impedance (blue) and the phase of the impedance (red, right y-axis) were calculated.](image)

The simulation by PSPICE showed that the phase and imaginary part of the impedance have their zero crossing point at resonance where the real part of impedance has its maximum (Fig. 3).

In Comsol Multiphysics the coil without the sensor was driven via a lumped port setup. Via the lumped port a voltage of 1V was applied to the coil. The second coil was coupled through the magnetic field. Finally the inductance (Z parameter) was evaluated through the lumped port. The outer mantle of air was modeled as ‘perfect magnetic conductor’. This should gain a maximum of reality.

The problem was solved by the BiCGStab-Solver with right preconditioning. The material of the coils and sensor was silver; the other domains were modeled as air. All domains were meshed with tetrahedrals. There were about 100,000 to 400,000 elements. High mesh quality was ensured. Meshing the two parallel plates was problematic. Some work had to be done to guarantee that the mesh quality was okay in and between the plates.

The magnetic field looked commonly (Fig. 4). It was concentrated in the coils.
In resonance the imaginary part and the phase of impedance should be equal to zero, the real part has its maximum \[1\]. These expectations were fulfilled in both PSPICE and Comsol simulation (Fig. 3,5). Because of these prospects it was possible to evaluate resonance easily.

As a last study the sensor’s capacitance and the numbers of turns of the coils were changed.
A simulation with \(n = 5\) turns of the coils was done and the area of the sensor-plates was changed.
Afterwards sensors were produced in real (with a simple HF-capacitance) and measured at a network-analysator to verify the results of the simulation. It can be seen that the simulation and measurement look the same (Fig. 6). There were only small differences.

2.2 Structural-Mechanics-Calculation

In this study the sensor itself was developed. The geometry for this case is shown in figure 7.

As a first study, the deflection of the sensor’s above plate was calculated. Therefore the other plate was defined as ‘fixed’ and the outer boarders of the above plate were also modeled as ‘fixed’. The acceleration load on the above plate was \(\pm 20\) g defined as a body load on the above plate. The material of the sensor was silver from...
the material library, because there were no parameters for the nanosilver-ink yet. The sensor was meshed using tetrahedrals. About 45,000 of them were used (Fig. 8). Every plate had at least 3 elements over thickness, some even 5.

The deflection was calculated using the nonlinear material and the nonlinear geometry model of Comsol. The resulting deflection was about 4.5 µm as a maximum (Fig. 9).

The deflection has three zones (Fig. 10). The middle zone is the zone with normal linear material behavior. The first and third zones are the zones where the nonlinearity in the material-model and geometry affects the result.

Figure 8: Exemplary tetrahedral surface-mesh of the sensor

Figure 9: Exemplary tetrahedral surface-mesh of the sensor

Figure 10: Deflection of the sensor's plate over acceleration load.

More analyses, like the analysis of the influence of the residual stress on the deflection, should be done in future.

2.3 Nonlinear Structural Mechanics and Radio Frequency Coupled via the ALE-Moving Mesh Method

2.3.1 Theory

The following consideration can be done:

Based on the equation for the resonance frequency in an oscillating circle

$$f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \cdot C}}$$

and the equation of a simple parallel plate capacitor

$$C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d}$$

equalized over C

$$f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \cdot \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d} \cdot \sqrt{d_0}}}$$

from the simple beam theory there is the equation

$$\Delta d = 2 \cdot F \cdot \frac{l^3}{48 \cdot E \cdot I}$$
\[ \Delta d = 2 \cdot m \cdot a \cdot \frac{l^3}{48 \cdot E \cdot I} \]

with
\[ d_0 = d + \Delta d \text{ (Fig. 11)} \]

Figure 11: Capacitor plates not deformed and deformed to show the use of variables. \( d_0 \) is the distance of the not deformed plates. \( d \) is the distance of the deformed plates and \( \Delta d \) is the deformation itself.

all together
\[
f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \cdot \varepsilon_0 \cdot \varepsilon_r \cdot A}} \cdot \sqrt{d_0 + 2 \cdot m \cdot a \cdot \frac{l^3}{48 \cdot E \cdot I}}
\]

and with some constants introduced
\[ f_0 = c_0 \cdot \sqrt{c_1 + c_2 \cdot a} = f_0(\sqrt{a}) \]

It can be seen that there should be a rooty correlation between resonance frequency and acceleration load.

2.3.2 Simulation

In this FEM-study the sensor should get an acceleration load and the aim was to calculate the change in resonance frequency for this system. Therefore a study with three steps is calculated:

- Nonlinear-structural-mechanics:
  - calculation of the deflection
- ALE/Moving-Mesh:
  - Mapping this deflection to the mesh
- Radio frequency (with frequency sweep):
  - Calculate the resonance frequency

The geometry was meshed using tetrahedrals. There were some problems in meshing regarding the small conductive path of the coil and the thin capacitor plates. The mesh resulted in about 600,000 elements. A parametric sweep over the different acceleration loads was calculated. So it was possible to get a curve with the acceleration load over resonance frequency (characteristic curve, Fig. 12). It can be seen that the characteristic curve is nonlinear. A regression of the data was done. It was found that the regression is
\[ f_0 = \pm 0.25 \cdot \sqrt{|a|} + 52.65 \]

with \( a \) as the acceleration.

Figure 12: Plot of the resonance frequency over acceleration load.

Excerpt from the Proceedings of the 2012 COMSOL Conference in Milan
As you can see there is a rooty correlation between the acceleration load and the resonance frequency. This agreed with the theory.

3. Conclusions

The whole system was calculated so that a decision on

- coil’s geometry
- number of the turns of the coil
- inductance of the coil
- capacitance of the sensor
- geometry of the sensor
- …

could be made without any prototyping (Fig. 13). The simulations were verified by measurements, so that it is secure that the calculations are correct.

Figure 13: Resulting sensor system for testing. Sensor itself replaced by a HF-capacitance.

4. Acknowledgements

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5. References