SIMULATION OF PCM MELTING PROCESS IN A DIFFERENTIALLY HEATED ENCLOSURE

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INTRODUCTION

- This study deals with a numerical investigation on the melting process of a PCM in a rectangular enclosure differentially heated;
- A FEM-based code is used in order to solve Navier-Stokes and energy equations in the considered system;
- The industrial framework of the study is the thermal storage for thermodynamic solar power plants;
- The main goal of this study consists, at the present step of the work, in validating the numerical tool by comparison with experimental results previously published.
MATHEMATICAL MODEL

Homogeneous method \(^{(1)}\) (Enthalpy method): one single equation is used to solve the temperature field both in the solid and in the liquid domain of the system:

\[
\frac{\partial H}{\partial t} = k \nabla^2 T + \dot{Q} + \rho C U \cdot \nabla T
\]

where

\[
\begin{align*}
\frac{dH}{dT} &= \rho C(T) dT \\
H(T) &= \int_{T_1}^{T_2} \rho C(T) dT
\end{align*}
\]

MATHEMATICAl MODEL

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where

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\frac{dH}{dt} &= \rho C(T)dT \\
H(T) &= \int_{T_1}^{T_2} \rho C(T)\,dT
\end{align*}
\]

- During solid phase heating, melting and further heating of liquid phase, the enthalpy function can be written as in following:

\[
H(T) = \int_{T_1}^{T_s} \rho C_s(T)\,dT + \int_{T_s}^{T_l} \rho \left[ C_{sl}(T) + \frac{dL}{dT} \right] dT + \int_{T_l}^{T_2} \rho C_l(T)dT
\]

MATHEMATICAL MODEL

The previous formulation is based on the hypothesis that phase change in the considered medium happens with a small temperature variation, therefore \((T_l - T_s)\):

\[
H(T) = \int_{T_1}^{T_s} \rho C_s(T) dT + \int_{T_s}^{T_1} \rho \left[ C_{sl}(T) + \frac{dL}{dT} \right] dT + \int_{T_1}^{T_2} \rho C_l(T) dT
\]
MATHEMATICAL MODEL

➢ By defining the thermal capacity as:

\[
\frac{\partial H}{\partial t} = \frac{\partial H}{\partial T} \frac{\partial T}{\partial t} \quad C_{eq} = \frac{\partial H}{\partial T}
\]

the equation energy:

\[
\frac{\partial H}{\partial t} = k \nabla^2 T + Q + \rho C U \cdot \nabla T
\]

becomes:

\[
C_{eq} \frac{\partial T}{\partial t} = k \nabla^2 T + Q + \rho C_{eq} U \cdot \nabla T
\]
MATHEMATICAL MODEL

Being:

\[ \begin{align*}
H(T) &= \int_{T_i}^{T_s} \rho C_s(T) \,dT + \int_{T_s}^{T_l} \rho \left[ C_{sl}(T) + \frac{dL}{dT} \right] \,dT + \int_{T_l}^{T_2} \rho C_l(T) \,dT \\
C_{eq} &= \frac{\partial H}{\partial T}
\end{align*} \]

we can write:

\[ \begin{align*}
C_{eq} &= \rho C_s & \iff & (T < T_s) \\
C_{eq} &= \rho C_{sl} + \rho \frac{L}{T_l - T_s} & \iff & (T_s \leq T \leq T_l) \\
C_{eq} &= \rho C_l & \iff & (T > T_l)
\end{align*} \]
REFERENCE EXPERIMENTAL SET-UP

- Rectangular enclosure filled by paraffin and differentially heated

**Liquid phase**
- \( \rho = 1100 \text{ kg m}^{-3} \)
- \( c = 2260 \text{ J kg}^{-1} \text{ K}^{-1} \)
- \( k = 0.188 \text{ W m}^{-1} \text{ K}^{-1} \)

**Solid phase**
- \( \rho = 1120 \text{ kg m}^{-3} \)
- \( c = 2260 \text{ J kg}^{-1} \text{ K}^{-1} \)
- \( k = 0.188 \text{ W m}^{-1} \text{ K}^{-1} \)

\[
\beta = 7.6 \times 10^{-4} \text{ K} \quad h_{sl} = 150.5 \text{ kJ kg}^{-1} \quad T_m = 34^\circ \text{C}
\]
REFERENCE EXPERIMENTAL SET-UP

- Time evolution of the “hot wall” temperature during 5 different test-cases (the “cold wall” is kept at constant temperature)
REFERENCE EXPERIMENTAL SET-UP

- Experimental acquisition > Spot temperature at several locations
REFERENCE EXPERIMENTAL SET-UP

- Experimental acquisition > Solid/liquid interface (T_melting) position
NUMERICAL MODEL

- Test-case model > Geometry

![Diagram of a rectangle with dimensions 15 cm by 10 cm]
NUMERICAL MODEL

Test-case model > Fluid-dynamics

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot \left[ -p \mathbf{I} + \eta \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right] + \mathbf{F}_g \]

\[ \nabla \cdot \mathbf{u} = 0 \]

\[ \mathbf{F}_g = \rho g \beta \Delta T \]

\[ \eta = \eta_l \quad \text{se} \quad T > T_m \]

\[ \eta = \infty \quad \text{se} \quad T < T_m \]
NUMERICAL MODEL

➢ Test-case model > Thermal analysis

\[
\rho \left( C_p \right)_{eq} \frac{\partial T}{\partial t} = k \nabla^2 T + Q + \rho \left( C_p \right)_{eq} \mathbf{u} \cdot \nabla T
\]

\[Q = 0\]
\[T = T_0\]
**NUMERICAL MODEL**

- Test-case model > Thermal analysis

\[
\rho \left( C_p \right)_{eq} \frac{\partial T}{\partial t} = k \nabla^2 T + Q + \rho \left( C_p \right)_{eq} \mathbf{u} \cdot \nabla T
\]

- If \( [(T<(T_m-DT)) \text{ or } (T>(T_m+DT))] \) then \( (C_{p_{eq}}=C_p) \) also \( (C_{p_{eq}}=C_p+H/DT) \)

- \( DT = 0.01 \, ^\circ C \)

- \( B = 0 \) (solid)
- \( B = 1 \) (liquid)
NUMERICAL MODEL

- Test-case model > Numerical solution
  - Continuous equations discretized by a F.E. method on no-structured and no-uniform mesh made of triangular Lagrange elements of order 2
  - Time-marching performed by an Implicit Differential-Algebraic (IDA) solver based on a variable-order and variable-step-size Backward Differentiation Formulas (BDF)
  - Nonlinear system of equations solved at each time step by a modified Newton-Raphson algorithm
  - Algebraic systems of equations coming from differential operators discretization solved by a PARDISO package
  - Output time-step of 60 seconds
RESULTS

- Velocity field (white arrows), physical phase (red for liquid and green for solid) and temperature field (scale of colors):

Test-case #5 – Time: 1800 sec
RESULTS

- Velocity field (white arrows), physical phase (red for liquid and green for solid) and temperature field (scale of colors):

*Test-case #5 – Time: 3600 sec*
RESULTS

- Velocity field (white arrows), physical phase (red for liquid and green for solid) and temperature field (scale of colors):

Test-case #5 – Time: 5400 sec
RESULTS

- Velocity field (white arrows), physical phase (red for liquid and green for solid) and temperature field (scale of colors):

Test-case #5 – Time: 7200 sec
RESULTS

- Velocity field (white arrows), physical phase (red for liquid and green for solid) and temperature field (scale of colors):

Test-case #5 – Time: 9000 sec
RESULTS

- Velocity field (white arrows), physical phase (red for liquid and green for solid) and temperature field (scale of colors):

Test-case #5 – Time: 12000 sec
VALIDATION

- **Temperature distribution** along the upper wall of the enclosure: comparison between experimental and numerical results at several time instants.
VALIDATION

- **Temperature distribution** along the **bottom wall** of the enclosure: comparison between **experimental** and **numerical** results at several time instants.
**VALIDATION**

- **Solid/liquid interface position**: comparison between experimental and numerical results at several time instants.
CONCLUSIONS

- The melting process of a PCM in a differentially heated rectangular enclosure is numerically simulated. The enthalpy method is adopted for modelling heat transfer and the solid phase is regarded as a liquid having an almost infinite viscosity.

- The solid-liquid interface location and the thermal maps obtained for several transient heating conditions well highlight natural convection effects, enhancing heat transfer in the top portion of the cavity.

- Results are successfully compared with experimental data previously published and concerning an analogue system. The shapes of the melt front obtained at various times from computations well fit with experiments. Quantitatively comparison between numerical and experimental results show good agreement.

- From comparisons, the proposed numerical approach appears validated and suitable for use in the pre-design of PCM storage systems.
This study has been developed at:

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