Optimal placement of piezoelectric plates to control multimode vibrations of rotating beam

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Gas Turbine Blade Vibrations

Fatigue Related Phenomena

Catastrophic Failures and Reduction of the Blades Life
without control

with control
Optimal placement of piezoelectric plates: single mode

\[ F(t) = F_0 \cos(\omega_1 t) \]

Pin Force Model

\[ L^P = M_a \left( a + \frac{h}{2} \right) \]

\[ \frac{i'}{a} \]

\[ \frac{h}{2} \]
Optimal placement of piezoelectric plates: multimode control

\[
\begin{align*}
M\ddot{X}(t) + C\dot{X}(t) + KX(t) &= B(a, h) \sum_{k=1}^{N_s} V_k \cos(\omega_k t) \\
\end{align*}
\]

Piezoelectric effect

\[
\begin{align*}
w(a, h, x, t) &= \sum_{n=1}^{N} X_n(t) \phi_n(x) \\
\end{align*}
\]

\(Ns=2\) → 2 eigenmodes considered: i and j

\[
\begin{align*}
M\ddot{X}(t) + C\dot{X}(t) + KX(t) &= B(a, h) \left[ V_i \cos(\omega_i t) + V_j \cos(\omega_j t) \right] \\
\end{align*}
\]
Optimal placement of piezoelectric plates: multimode control

After a transitory time, the amplitude of the vibrations of the free end is:

\[
|w(a,h,L)| = \frac{M_a}{\left| \begin{array}{c}
V_i' \ a + \frac{h}{2} \\
\frac{h}{2} \ i(L)
\end{array} \right| + \left| \begin{array}{c}
V_j' \ a + \frac{h}{2} \\
\frac{h}{2} \ j(L)
\end{array} \right|}
\]

If \( r \) is the ratio percentage of the \( j \)-mo mode:

\[
|w(a,h,L)| = \frac{M_a}{\left| \begin{array}{c}
(1 \ r) \ a + \frac{h}{2} \\
\frac{h}{2} \ i(L)
\end{array} \right| + \left| \begin{array}{c}
r \ j' \ a + \frac{h}{2} \\
\frac{h}{2} \ j(L)
\end{array} \right|}
\]
Optimal placement of piezoelectric plates: multimode control

\[ a + \frac{h}{2} = L \]

\[ \left| w(a,h,L) \right| = w \cdot a \cdot \frac{h}{2}, L = w(x,L) \]

\[ w(x,L) = \frac{M_a}{r} \left[ (1 - r)[i'(L) \cdot i'(x)] \cdot i(L) + [i'(L) \cdot i'(x)] \cdot i(L) \right] + \left[ j'(L) \cdot j'(x) \right] \cdot j(L) \]
Optimal placement of piezoelectric plates: experimental apparatus for fixing beam
Optimal placement of piezoelectric plates: experimental apparatus for fixing beam

It is possible to find all the experimental results in:

Optimal placement of piezoelectric plates: rotating beams
Optimal placement for coupling of the first and second mode
Optimal placement for coupling of the second and third mode
Optimal placement for coupling of the first and third mode
Optimal placement of piezoelectric plates: experimental apparatus for rotating beam
Conclusions and future work

- The optimal placement of piezoelectric plates to control multimode vibrations of rotating beam has been studied.
- Optimal configurations have been reported for different angular velocities and different mode ratio.
- Experimental prototype has been built and the experimental tests are going on.
- The work must be extended to:
  - more than 2 frequencies
  - torsion effect (numerical studies are going on)
  - real geometry of the blade