Heat and Mass Transfer Modelling During Freezing of Foodstuffs

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Abstract: The objective of this study is to develop a mathematical model to determine the weight loss and the freezing rate during the freezing of unwrapped foodstuffs in a very cold environment. The model allows comparing two freezing processes applied to methylcellulose gel: the first uses nitrogen gas at -80°C and the second is the classical method using cold air. In order to take into account the phase change, the model includes thermophysical properties that are temperature dependent and a model based on Schwartzberg’s model is used for the ice formation. The numerical study reveals that the time step has to be chosen carefully in order to preserve the heat balance during the simulation of freezing and that the mesh grid has to be refined near the surface to take into account strong gradients of water content. Numerical results agree well with experimental data and confirm that the weight loss and freezing time can be reduced if low temperatures are used.

Keywords: freezing, cryogenic, weight loss, apparent heat capacity.

1. Introduction

Freezing is a preservation method widely used in the food industry and frozen foods can be obtained by several techniques. Among them cryogenic freezing is useful for increasing the freezing rate thanks to low temperatures. The water to ice transition induces mechanical stress due to expansion during phase change and experimental results confirm that freezing produces structural damages regardless of the type of freezing [1,2]. However numerous and smaller ice crystals result in reduced freeze damage. Because the number of ice crystals is known to be inversely dependent to the freezing rate, freezing in cryogenic conditions can improve the quality of food [1,2].

A rapid reduction of the surface temperature of unwrapped food results in a rapid reduction of partial vapour pressure at the surface of food. Hence in case of rapid freezing, a reduction of the mass loss by evaporation is expected in comparison with slow freezing conditions [1,3].

There are many models developed for coupled heat and mass transfer in food processing but few of them are yet available for freezing, especially if mass transfer is considered. Efforts remain to be made in modelling if we want to understand and predict the behaviour of food quality factors [4,5]. The main objective of this study is to develop a general model that can predict the weight loss and the freezing time according to freezing conditions, especially in very cold environments.

2. Governing equations

The study concerns the freezing of a plate in a cryogenic system. For non porous food, water evaporates from the surface and is replaced by water diffusing from the centre to the surface until freezing occurs. Thereafter there is no significant water diffusion inside the food and sublimation will occur at the surface [5]. We assume that moisture exists as liquid water and ice and that the moisture flux is only due to liquid phase diffusion. If we define X as the dry basis moisture content, Fick’s diffusion law can be written as:

\[ \rho_{\text{app}} \frac{\partial X}{\partial t} = \nabla \left( \rho_{\text{app}} D_{\text{eff}} \nabla X \right) \]  

Where \( \rho_{\text{app}} \) is the apparent density of dry matter, and \( D_{\text{eff}} \) is the effective diffusivity of liquid moisture. \( X_i \) is the liquid part of the moisture and the quantity of ice is assumed to be dependent on the temperature and the water content as follows:

\[ X_{\text{app}} = (X - X_i) f(T) \]  

and

\[ X_i = X - X_{\text{ice}} \]
Where \( X_b \) is the bound water (ie unfreezable water), \( T_f \) the initial freezing temperature and \( f(T) \) a temperature dependent function. Introducing value of \( X_b \) eq(1) can be modified to be in a convenient form for modelling in Comsol:

\[
\frac{\partial X}{\partial t} =
\nabla \left( \rho_{\text{app}} \left[ D_{\text{eff}} \left( 1 - f(T) \right) \nabla X + \left( X_b - X \right) \frac{\partial f}{\partial T} \nabla T \right) \right)
\]

(4)

The PDE interface in coefficient form is used to implement eq(4) in Comsol 4.2a. Symmetry and water flux are considered at the boundaries:

\[
\begin{align*}
\rho_{\text{app}} D_{\text{eff}} \nabla X &= 0 \quad \text{at } x = 0 \\
\rho_{\text{app}} D_{\text{eff}} \nabla X &= -h_{\text{m}} \left( P_{\text{sat}} a_w - P_{\text{sat,amb}} \right) \quad \text{at } x = L / 2
\end{align*}
\]

(5)

The mass transfer coefficient \( h_{\text{m}} \) is obtained from the Lewis analogy, \( P_{\text{sat}} \) and \( P_{\text{sat,amb}} \) are the saturated vapour pressures at the surface and ambient temperatures respectively, \( a_w \) is the water activity and RH is the ambient relative humidity (RH = 0 in the \( N_2 \) environment).

The heat transfer equation takes into account the phase change as follows:

\[
\rho_{\text{app}} C_{\text{p,app}} \frac{\partial T}{\partial t} = \nabla \left( k_{\text{eff}} \nabla T \right)
\]

(7)

The apparent heat capacity depends on the heat capacities of dry matter, water and ice and includes the latent heat of freezing \( H_f \):

\[
C_{\text{p,app}} = C_{\text{w}} + X_i C_{\text{i}} + X_{\text{w,ice}} C_{\text{ice}} - H_f \frac{\partial X_{\text{ice}}}{\partial T}
\]

(8)

Evaporation and sublimation occur at surface and the boundary conditions are:

\[
\begin{align*}
- k_{\text{eff}} \nabla T &= 0 \quad \text{at } x = 0 \\
- k_{\text{eff}} \nabla T &= h(T - T_{\text{amb}}) + F_s \left( H_f + f_{\text{ice}} H_i \right) \quad \text{at } x = L / 2
\end{align*}
\]

(9)

where \( h \) is the heat transfer coefficient, \( T_{\text{amb}} \) the external temperature, \( F_s \) the flux of water at the surface, \( H_i \) the latent heat of evaporation and \( f_{\text{ice}} = 1 \) when ice is present.

The Heat Transfer in Solids module was used for implementing the aforementioned equations in Comsol 4.2a.

3. Results and discussion

3.1 Experiments

Several experiments were carried out to measure the weight loss during freezing in a cryogenic freezer (Figure 1). Samples are made of Tylose®, a gel composed of water (85%), methylcellulose powder (14%) and salt (1%). This model food is widely used in food-related related research because of its thermophysical properties that are close to those of meat. Flat slabs 2 cm thick were built. The other dimensions (20 and 10 cm) are sufficiently large to assume 1D heat and mass transfer.

![Figure 1. Experimental set-up](image)

3.2 Thermophysical properties of food

Knowledge of the thermophysical properties of Tylose is necessary to predict heat and mass transfer during freezing. The density of the food is deduced from measurement carried out in the unfrozen part and from the quantity ratio of components for the frozen part (dry matter, ice and liquid water). The thermal conductivity is calculated by means of a Maxwell type model, including measurement of the thermal conductivity of unfrozen Tylose. The apparent heat capacity and the bound water are deduced from thermal analysis carried out using

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Differential Scanning Calorimetry (DSC). The water activity was measured for several water contents and modelled by a GAB equation and the effective diffusivity comes from the literature [6].

The results from the DSC have to be corrected to include the ice formation beginning at the initial freezing temperature. According to Le Bail et al. [7], a simple relation can be used for the function f(T):

\[
f(T) = \begin{cases} 
1 - \frac{T_r - 273.15}{T - 273.15} & \text{if } T \leq T_r \\
0 & \text{if } T > T_r 
\end{cases}
\]  

(11)

This relation remains an approximation. It can be adjusted knowing the final temperature of freezing \( T_f \) for which it is assumed that 100% of the freezeable water is frozen:

\[
f(T) = 1 - \left( \frac{T_r - 273.15}{T - 273.15} \right) \left( \frac{T - T_f}{T_r - T_f} \right)
\]

(12)

eq(12) is used between \( T_f \) and \( T_r \) and \( f(T) = 1 \) for \( T < T_f \). The final freezing temperature \( T_f \) is deduced from the DSC curve (\( T_f \approx -15^\circ \text{C} \)).

3.3 Numerical procedure

A 1D geometry is created in Comsol4.2a with dimension L/2 (L=2cm). User-controlled mesh with the fine predefined size is selected in the Mesh settings window. Maximum element size of \( 1 \times 5 \text{ mm} \) at the external boundary (where mass transfer takes place) ensures the mesh independence. The Time-Dependent Solver is used with free time steps limited to 1s in order to be able to take into account the latent heat characterised by the wide peak of the curve \( C_p, \text{mL}(T) \). Calculations are performed on a Sun® Microsystems U40 Workstation, equipped with 2xAMD® Opteron processors at 3 GHz, with 20GB of Ram, running on RedHat® Enterprise LINUX 5, 64 bits. Simulating 1500 s of freezing needs 16 s (CPU time).

3.4 Numerical results vs experimental data

The temperature predicted by the model 1cm beneath the surface (symmetry plane) is compared with experimental data (3 replicates) in figure 2. The heat transfer coefficient was set to 30 \( \text{W.m}^{-2}\text{K}^{-1} \), value estimated in the cryogenic chamber, according to the gas velocities.

Figure 2 shows that the model slightly underestimates the freezing time if the latter is measured by the length of the plateau at the temperature \( T_f \). In fact, there is a strong influence of the heat transfer coefficient on the temperature values. As explained by A. E. Delgado and D-W. Sun [4], imprecise predictions of freezing times are often due to imprecise knowledge of the experimental conditions. The influence of the heat transfer coefficient on the central temperature is depicted in Figure 3. The effective freezing time \( t_f \) can be defined as the time that takes the thermal centre of the product to reach the temperature \( T_f \). In this case, Figure 3 indicates that an uncertainty of 10% in the heat transfer coefficient leads to an uncertainty up to 9% on \( t_f \).
The weight loss predicted by the model is compared with experimental data in Figure 4. Numerical results agree reasonably well with the experimental weight loss. However, there is also an influence of the heat transfer coefficient. A 10% higher h leads to 8% lower weight loss because the chilling time decreases (Figure 5).

![Figure 4](image4.png)

**Figure 4.** Numerical and experimental results concerning the weight loss with $T_{\text{sub}} = -80^\circ$C and heat transfer coefficient of 30 W.m$^{-2}$K$^{-1}$.

![Figure 5](image5.png)

**Figure 5.** Influence of the heat transfer coefficient on the weight loss.

### 3.5 Cryogenic freezing vs air-blast freezing

According to the previous results, we can assume that the model is able to predict temperature and weight loss during freezing, despite the uncertainties concerning the heat transfer coefficient. We use the model to compare two freezing methods, the cryogenic method described in the previous section and the more conventional technique using cold air at -30°C. Results shown in Figure 6 present the temperature of Tylose sample at several locations with a heat transfer coefficient set to 30 W.m$^{-2}$K$^{-1}$. The results indicate that the effective freezing time is approximately 3 times higher than in cryogenic freezing.

![Figure 6](image6.png)

**Figure 6.** Temperature distribution inside the food sample during freezing with air at -30°C and heat transfer coefficient of 30 W.m$^{-2}$K$^{-1}$.

The total weight loss at the end of freezing is close to 1.1%. This result is in agreement with experimental results obtained from experiments carried out in an air blast freezer (total weight loss of 1.15%).

The weight loss predicted by the numerical model with different heat transfer coefficient is presented in Figure 7.

![Figure 7](image7.png)

**Figure 7.** Weight loss during freezing with air at -30°C according to different heat transfer coefficients.

Results show that increasing the heat transfer coefficient increases the weight loss during the chilling. However, because the initial freezing
temperature is reached sooner, the total weight loss is lower if high heat transfer coefficients are used. After 4000s, the mass loss represents 0.9, 1.1 and 1.3% of the initial mass for h equal to 40, 30 and 20 W.m\(^{-2}\).K\(^{-1}\) respectively. Please also note that for the last heat transfer coefficient value, the sample is not totally frozen at the end of the simulated process with a central temperature of -6°C. Thus the evaporation continues in this case and the final weight loss is higher than 1.3%.

The influence of the external temperature can also be studied with the numerical model. Weight losses for different ambient air temperatures are presented in Figure 8. As expected, the lower the external temperature is, the lower the weight loss will be.

![Figure 8. Weight loss during freezing according to different ambient temperatures (h = 30 W.m\(^{-2}\).K\(^{-1}\)).](image)

### 3.6 Discussion

Weight loss during freezing is mainly due to moisture evaporation and a high degree of dehydration takes place in the beginning of the freezing process. Numerical results confirm that the moisture evaporation is slowed down considerably after passing the latent heat zone. Thus using a cryogenic freezer can reduce the weight loss. The distribution of the water content in the sample at the end of freezing is presented in Figure 9 for both ambiances with the same heat transfer coefficient. The strong water content gradient close to the surface involves the use of finer grid in this region.

![Figure 9. Moisture content distribution in the food at the end of freezing according to the ambiance (h = 30 W.m\(^{-2}\).K\(^{-1}\)).](image)

4. Conclusion

A mathematical model was developed to predict the freezing time and the weight loss of food during freezing in very low ambient temperature. Such freezing processes are characterised by high heat and mass transfer compared with conventional freezing in cold air. A modified function f(T) was proposed to take into account the ice formation during freezing. Numerical results confirm that using very cold medium for freezing can reduce the freezing time and the weight loss, the latter being mainly due to moisture evaporation before the latent heat zone.
5. References

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