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Simulation of an Ultrasonic Immersion Test for the Characterization of Anisotropic Materials

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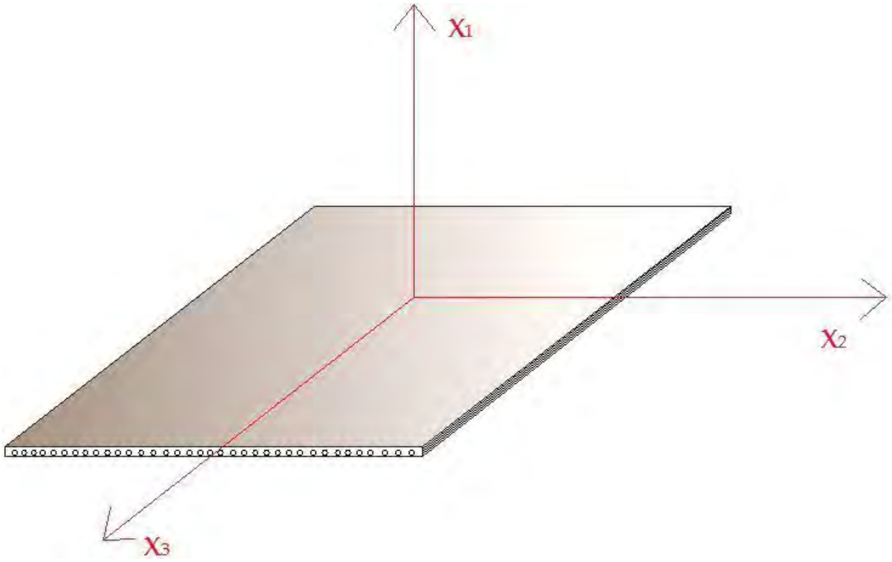
Wave propagation in anisotropic elastic materials

We study the propagation of ultrasonic waves in fiber-reinforced composite materials with a single layer of carbon fibers (CFRP): this material is modeled as ***linearly elastic trasversely isotropic***, with an axis of transverse isotropy coincident with the axis of the fibers



$$\mathbb{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

with $C_{12} = C_{11} - 2C_{66}$

Wave propagation in anisotropic elastic materials

The propagation of elastic waves is described by the equation of motion

$$\mathrm{div}\left(\mathbb{C}\left[\nabla \mathbf{u}\right]\right) = \rho\,\ddot{\mathbf{u}}$$

where ρ is the mass density, $\mathbf{u}(\mathbf{x}, t)$ is a planar elastic wave propagating in direction $\mathbf{n}$

$$\mathbf{u}(\mathbf{x},t) = \mathbf{a}\,\varphi(\mathbf{p}\cdot\mathbf{n} - v\,t)$$

a: direction of motion
n: direction of propagation
v : speed of propagation

and $\mathbb{C}$ is the elastic tensor.
The condition for elastic wave propagation is

$$\left[\boldsymbol{\Gamma} - \rho v^2 \mathbf{I}\right]\mathbf{a} = \mathbf{o}$$

i.e. *Fresnel – Hadamard's condition or Christoffel's equation*.

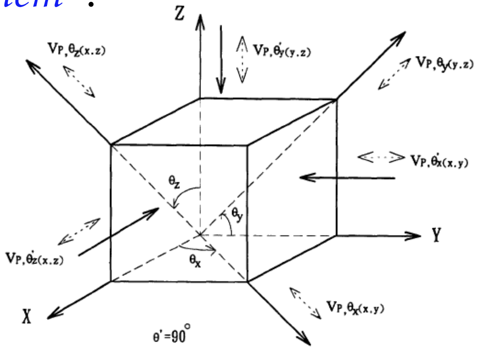
The second order tensor $\boldsymbol{\Gamma}$ is called *Christoffel's tensor* or tensor of propagation. This tensor is given by

$$\Gamma(\mathbf{n}) = \mathbb{C}^t\left[\mathbf{n} \otimes \mathbf{n}\right]$$

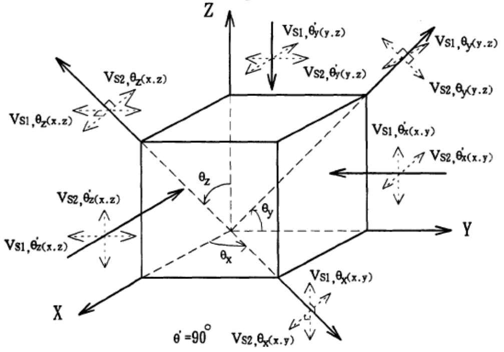
and is strictly related to the elastic tensor $\mathbb{C}$, to the mass density ρ and to the direction of propagation $\mathbf{n}$.

Wave propagation in anisotropic elastic materials

On the theoretical base of the Fresnel – Hadamard's condition it is possible to experimentally evaluate the elastic constants of a material in a non-destructive way by measuring the phase velocities of ultrasonic waves propagating along suitable directions: this is a so-called "*inverse problem*".



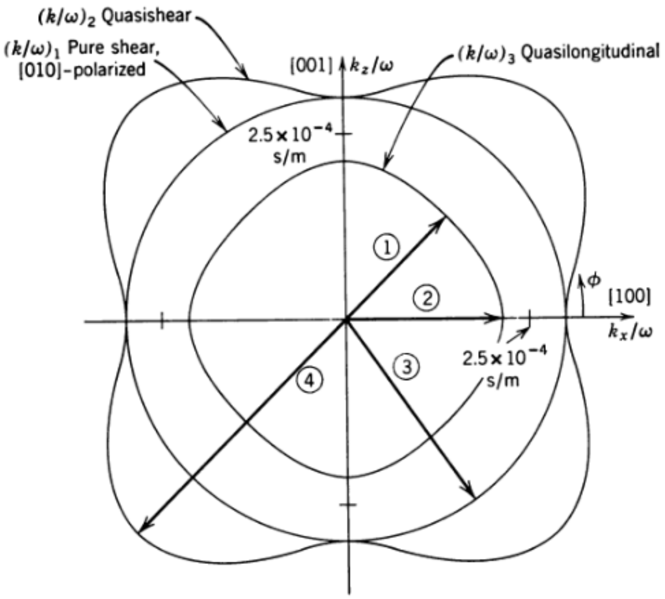
(a) Compressive wave



(b) Shear wave

Polarizations of waves

Directions of wave propagations



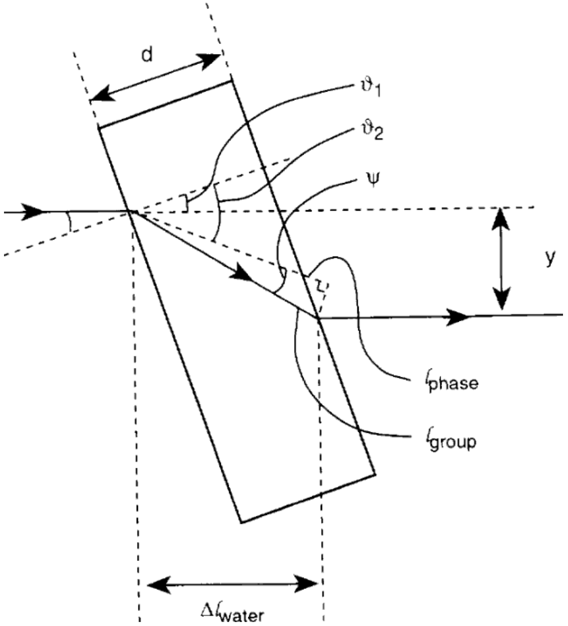
① $\left(\frac{\rho}{c_{11}+c_{44}(1-1/A)}\right)^{1/2}$

② $(\rho/c_{11})^{1/2}$

③ $(\rho/c_{44})^{1/2}$

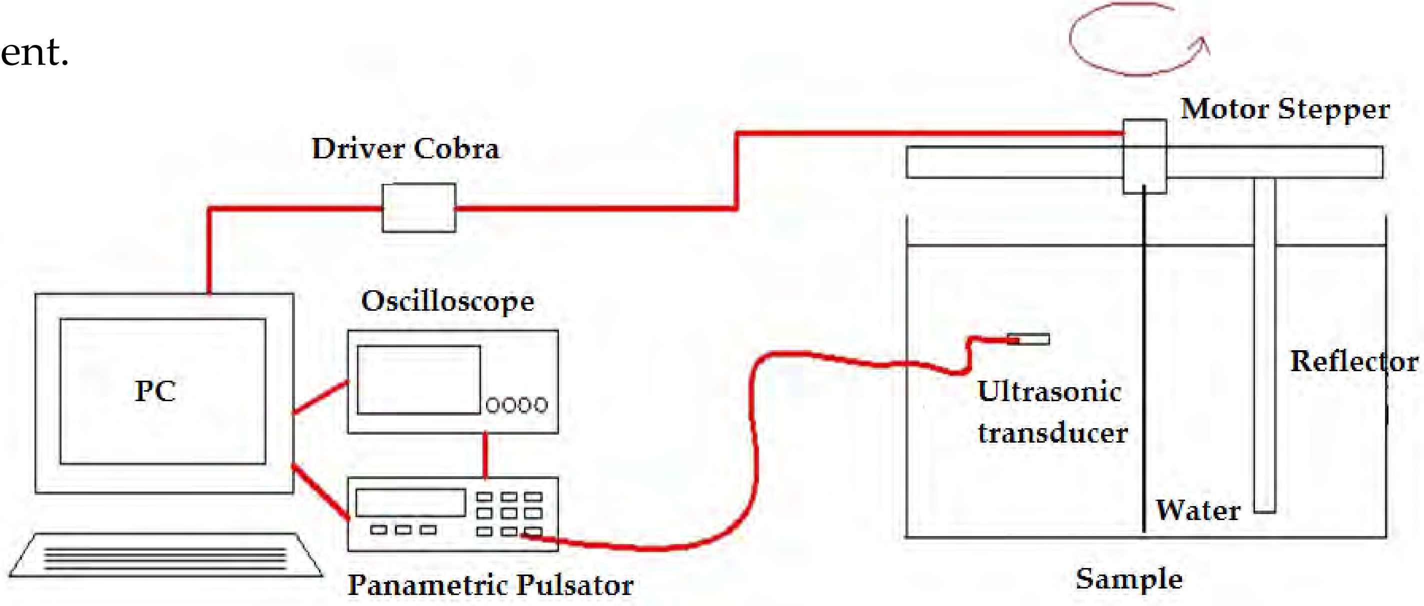
④ $(\rho A/c_{44})^{1/2}$

Anisotropy factor $A = \frac{2c_{44}}{c_{11}-c_{12}}$



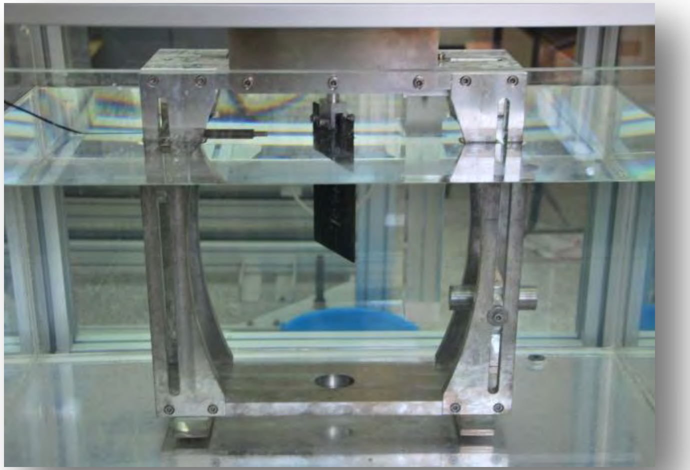
Experimental results: Ultrasonic immersion tests

We determine the elastic constants of a fiber – reinforced composite (CFRP) specimen by ultrasonic immersion tests using an ***innovative experimental device***, designed and built at *Laboratorio Ufficiale Prove Materiali "M. Salvati" (Politecnico di Bari)* for the ultrasonic mechanical characterization of anisotropic materials. The fundamental components of this device are a special immersion tank with suitable seats for the probes, a goniometer tool and an advanced data acquisition system. The goniometer allows to rotate the sample during the experiment.



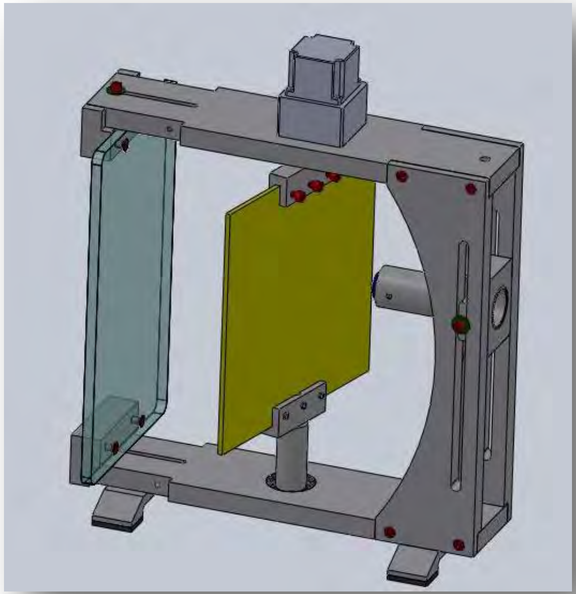
Experimental results: Ultrasonic immersion tests

By the rotation of sample, it's possible measure the time-of-flight of ultrasonic waves through the sample for different angle of propagation. This allows to determine the ultrasonic speed of longitudinal and transversal waves and, then, for the calculation of the elastic constants of the material.



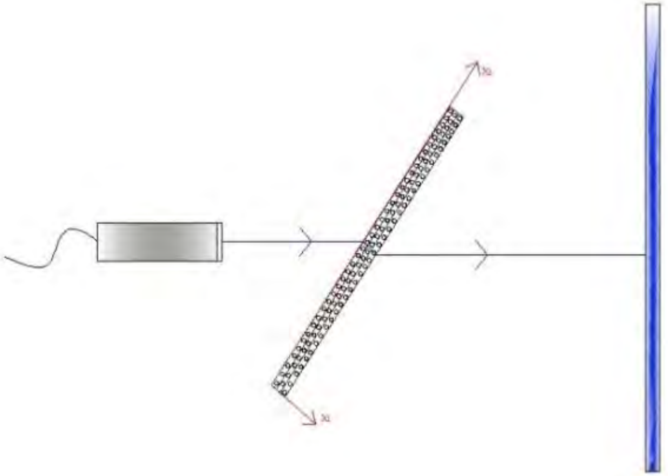
Experimental results: Ultrasonic immersion tests

We consider two different modes of propagation for the ultrasonic waves



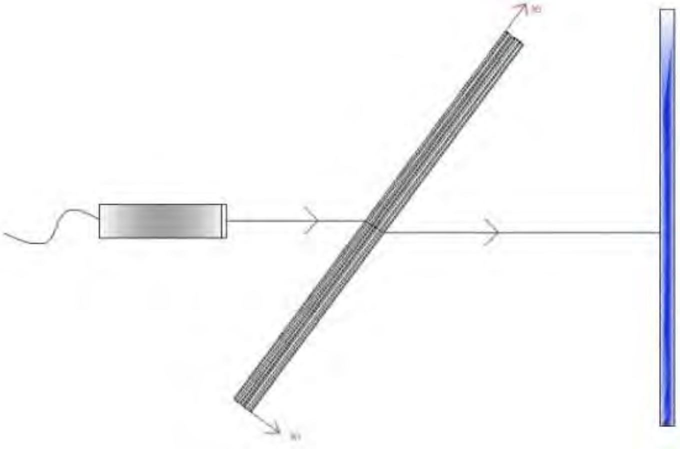
First mode

The specimen has the fibers (x_3 direction) parallel to the axis of rotation of goniometer, thus the ultrasonic waves propagate in the isotropic plane π_{12}.



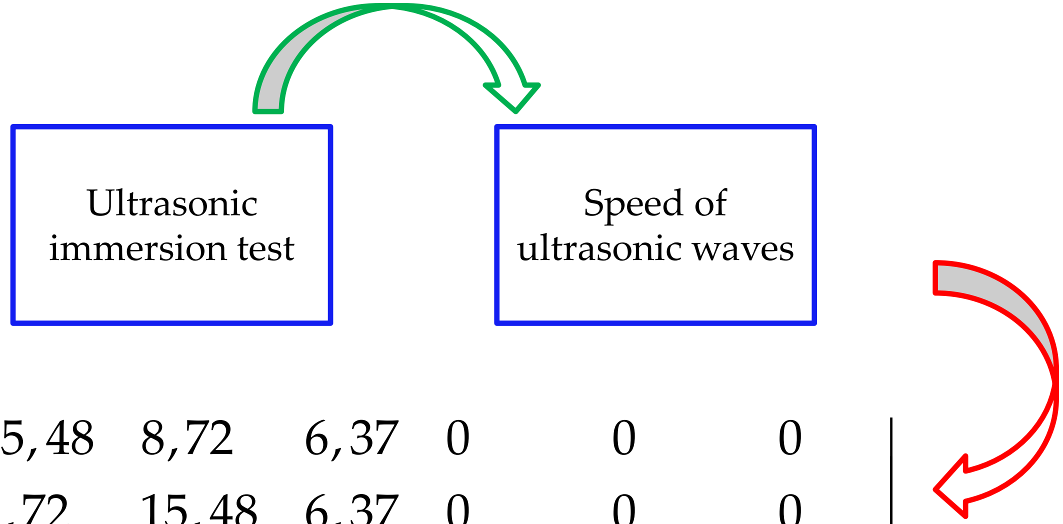
Second mode

The axis of rotation is ortogonal to the fibers (it is coincident with the axis x_2), thus the ultrasonic waves propagate in the anisotropic plane π_{13}.



Experimental results: Ultrasonic immersion tests

Once evaluated the speed of ultrasonic longitudinal and trasversal waves (starting from the evaluation of their time-of-flight across the sample of CFRP), we can solve the so-called *"inverse problem"*. In this way, we determine the five independent elastic constants of the CFRP sample.



Ultrasonic immersion test

Speed of ultrasonic waves

$$\mathbb{C}_{\mathrm{CFRP}} = \begin{bmatrix} 15{,}48 & 8{,}72 & 6{,}37 & 0 & 0 & 0 \\ 8{,}72 & 15{,}48 & 6{,}37 & 0 & 0 & 0 \\ 6{,}37 & 6{,}37 & 181 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4{,}098 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4{,}098 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3{,}38 \end{bmatrix} \mathrm{GPa}.$$

Use of COMSOL Multiphysics

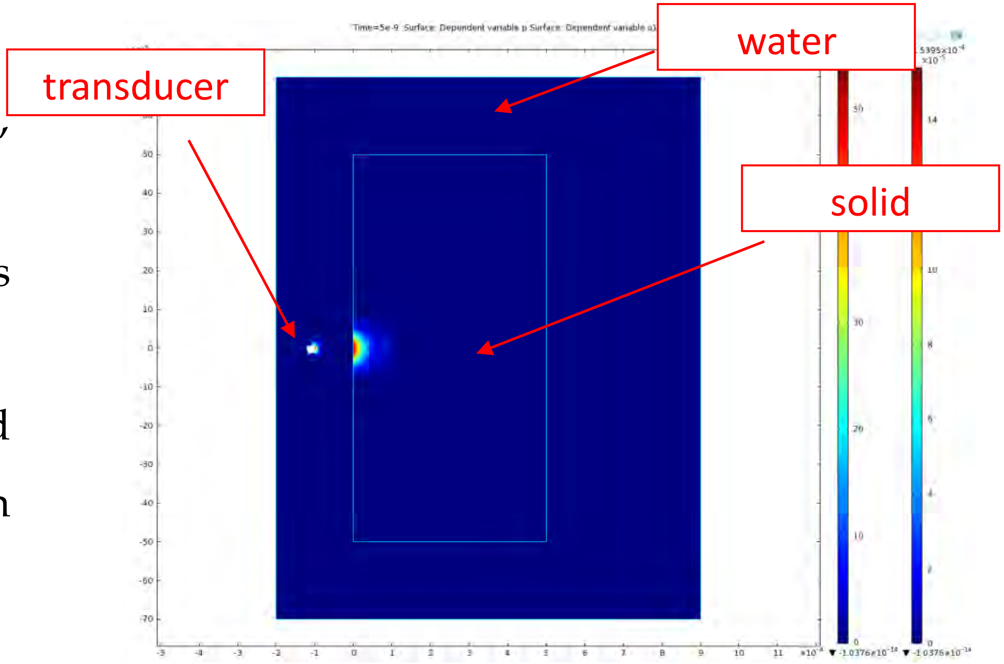
For improving the capability of the test, we simulate the propagation of ultrasonic waves in an immersion test on a fiber-reinforced composite sample using GENERAL FORM PDE module of COMSOL. This module may solve many classical PDE, and allows:

1. to model the fluid (water) and the solid (fiber-reinforced composite) domain;
2. to change the predefined forms of the PDE for adapting them to the elastodynamic mechanical model.

We use the TIME DEPENDENT SOLVER, suitable for elastic wave propagation.

The model is 3D (it follows previous studies on semplified 2D models).

The material parameters for the solid domain come from ultrasonic immersion test on a sample of CFRP described before .



transducer

water

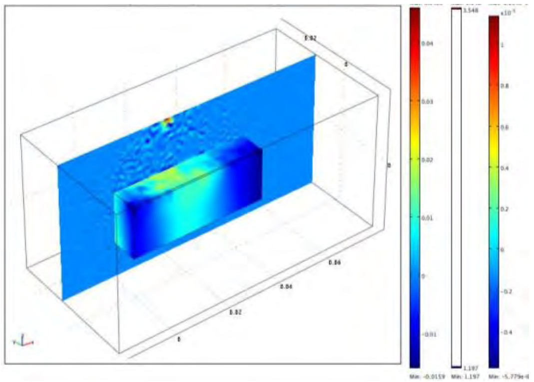
solid

Numerical results: Ultrasonic immersion tests

Given the elastic constants, the numerical analysis is aimed to determine the *speed of ultrasonic longitudinal and trasversal waves* through the evaluation of their time-of-flight across the sample of CFRP: *“forward problem”*.

Five Elastic constants of CFRP and mass density

Numerical model with **COMSOL Multiphysics**

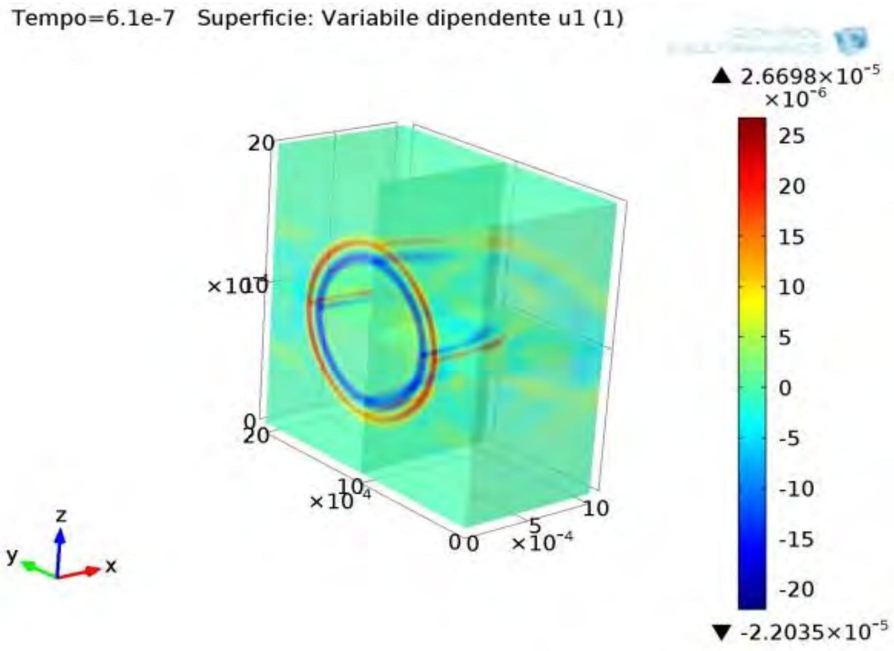


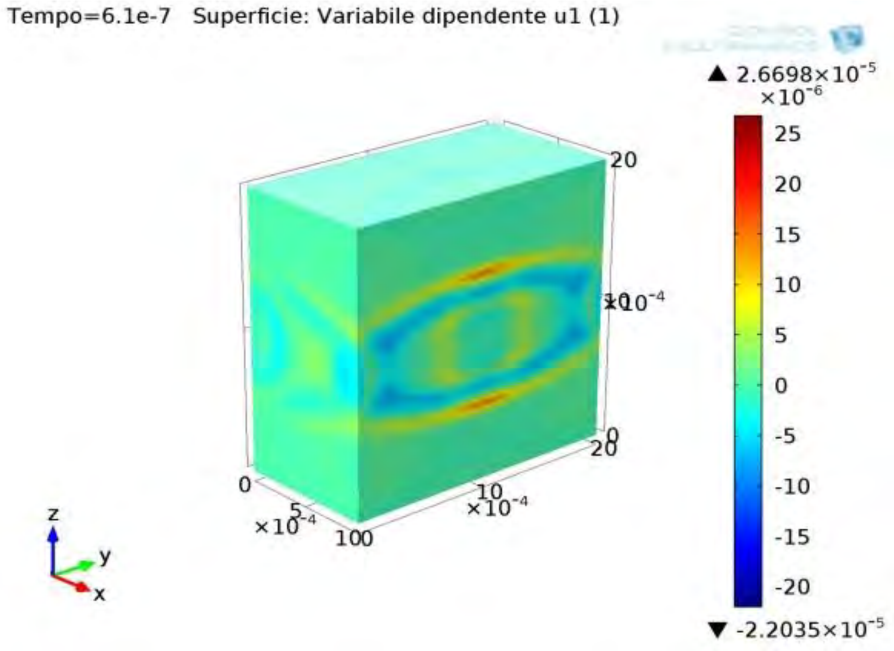
Speed of ultrasonic waves

We study two numerical models, simulating the two modes of ultrasonic wave propagation used in the test.

Numerical results: Ultrasonic immersion tests

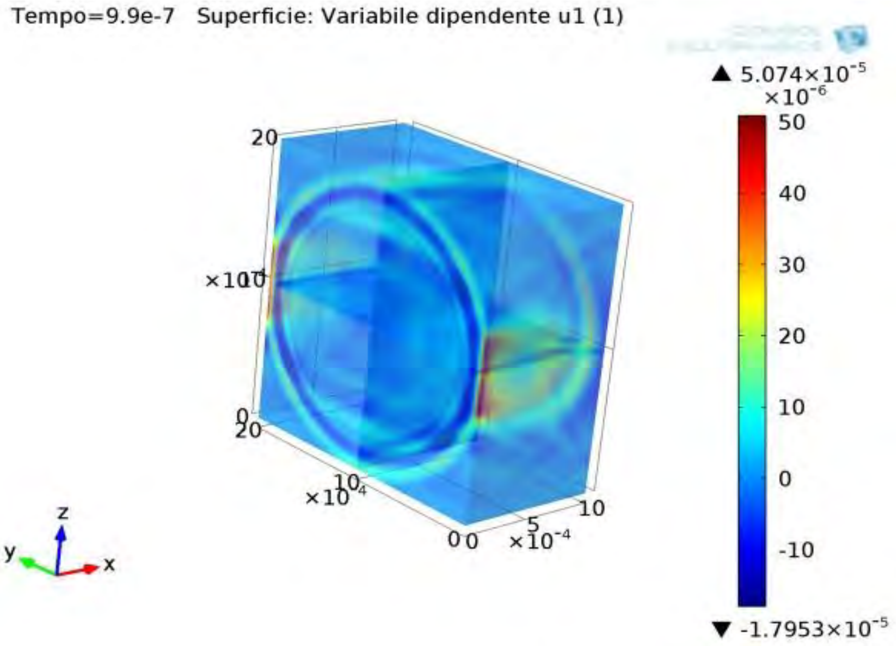
First mode: ultrasonic waves propagation in the isotropic plane π_{12}

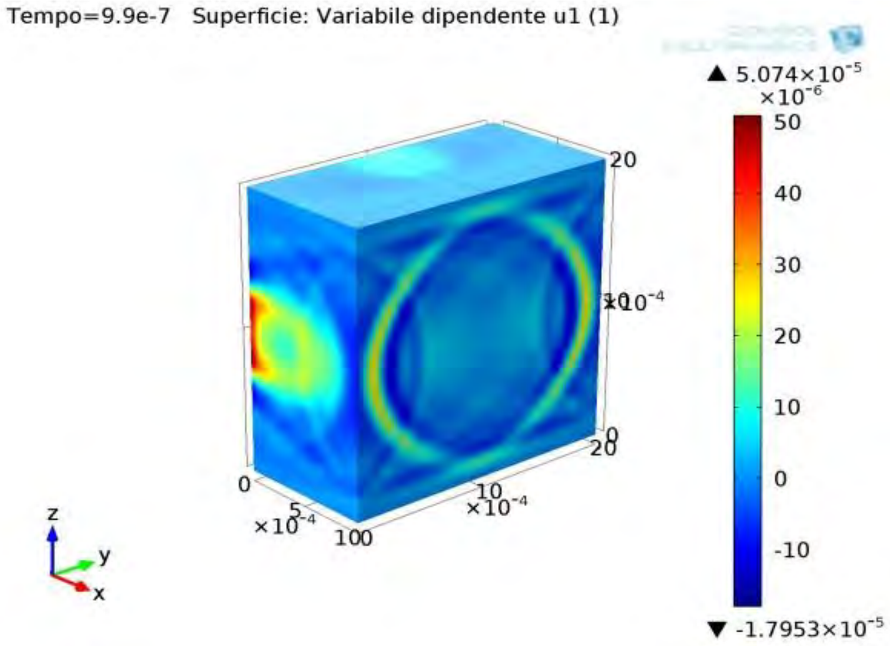




Numerical results: Ultrasonic immersion tests

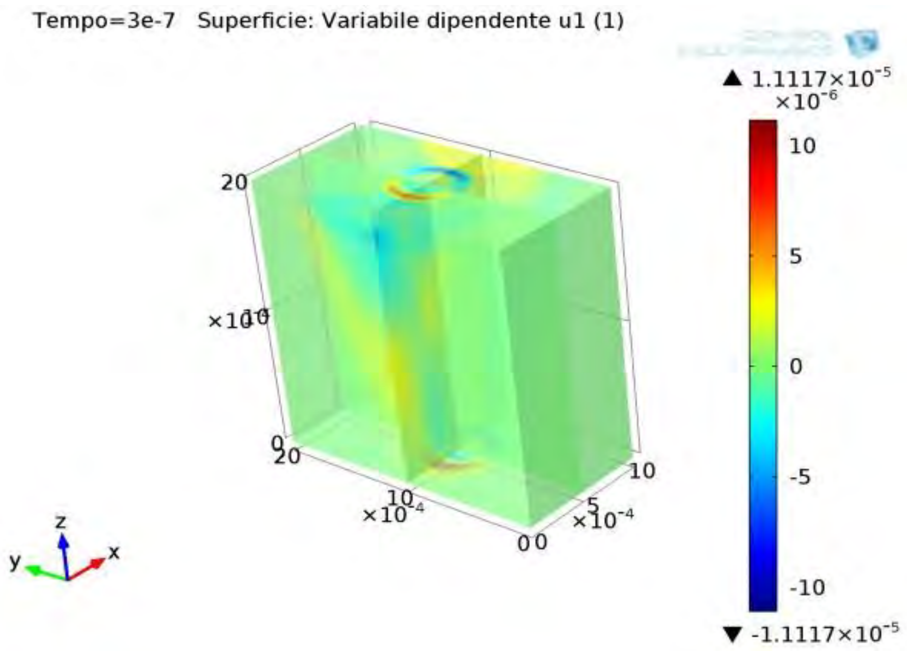
First mode: ultrasonic waves propagation in the isotropic plane π_{12}

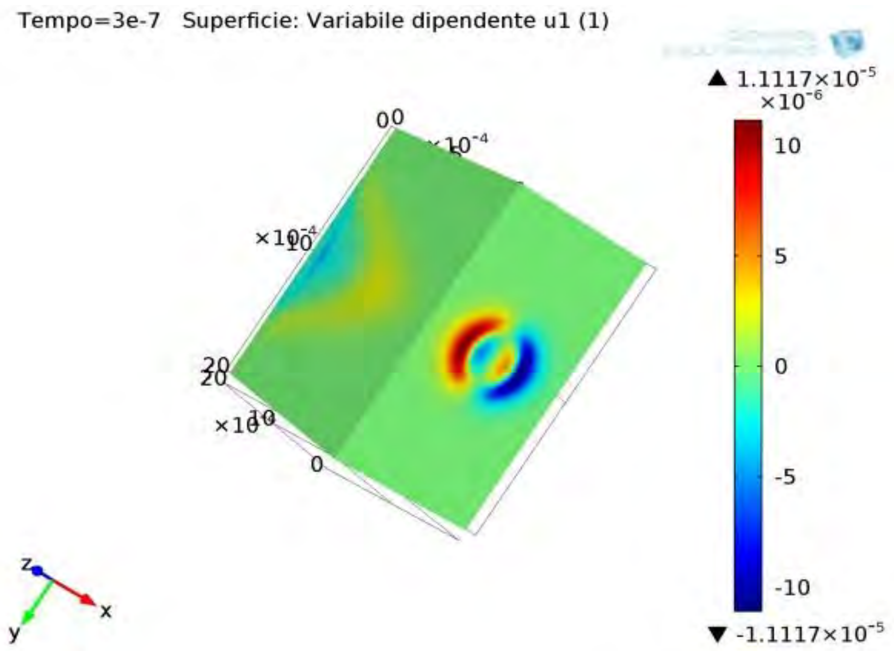




Numerical results: Ultrasonic immersion tests

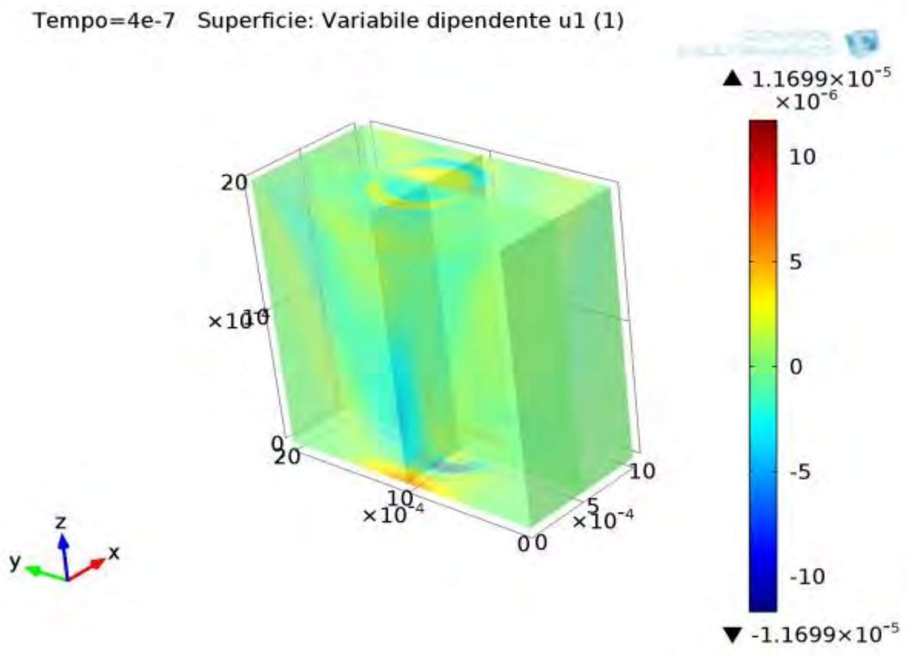
Second mode: ultrasonic waves propagation in the anisotropic plane π_{13}

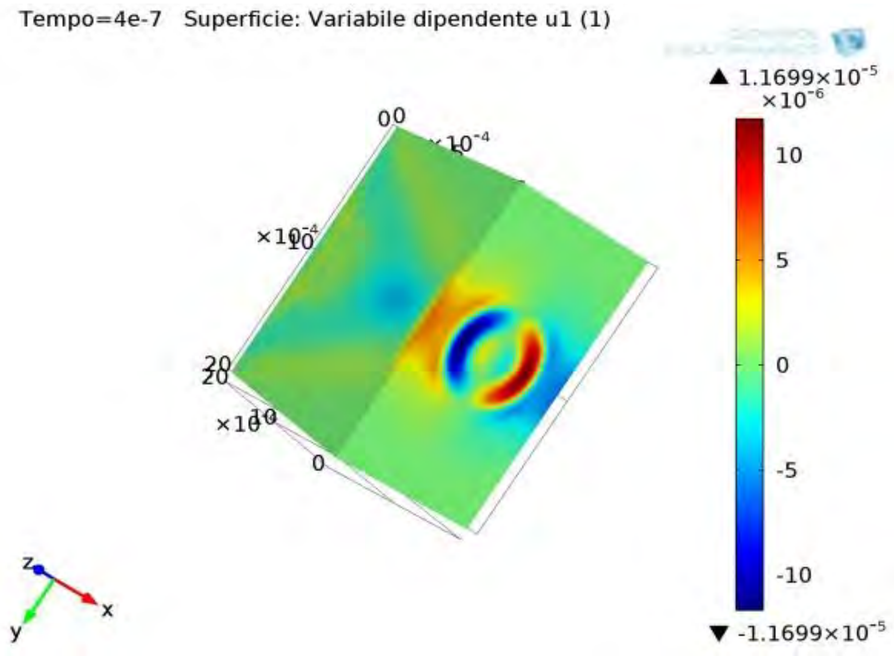




Numerical results: Ultrasonic immersion tests

Second mode: ultrasonic waves propagation in the anisotropic plane π_{13}





Conclusions

Goals of the numerical model

1. evaluation of the ultrasonic speed of longitudinal and transversal waves; this values are very close to those experimentally evaluated;
2. identification of the areas of maximum intensity of the emerging ultrasound beam, and measurement of the phase velocity and the phase angles: this is worthwhile for defining the optimal positions of the transducers in the experiments;
3. determination of the optimal angles of incidence of the ultrasound beam on the surface of the sample for getting the maximum energy for the longitudinal and transverse waves inside the sample;
4. analysis of the effects of rotations of the specimen and/or of the transducers on the propagation of the ultrasonic waves, for improving the handling of the specimen in the tests;
5. determination of the planes of symmetry of the mechanical response.

Conclusions

The proposed numerical model is innovative in the field of ultrasonic NDT since:

1. is a full 3D model;
2. simulates ultrasonic immersion tests and then considers the coexistence of two phases (the fluid and the solid);
3. is able to study the mechanical response of materials with more complex anisotropic behavior;
4. is an important starting point for particular application like the analysis of ultrasound propagation in anisotropic materials with defects, damages, and initial stress (residual and/or applied).