Study of Effect on Resonance Frequency of Piezoelectric Unimorph Cantilever for Energy Harvesting

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Overview

• Introduction
• Piezoelectric Effect
• Piezoelectric Cantilever
• Theoretical analysis using Matlab Simulink
• Modeling using COMSOL
• Conclusion
• References
At an average existing mobile Li Batteries has shelf life of 3-4 days.

To investigate renewable power “scavenging” technologies.

Piezoelectric materials can provide a direct transduction mechanism to convert signals from mechanical to electrical domains and vice versa.

Piezoelectric materials are high energy density materials that are suitable for miniaturization. Therefore, this has led to a growing interest in piezoelectric thin films for MEMS applications.
Piezoelectric Effect

• Appearance of an electric potential across certain faces of a crystal when it is subjected to mechanical pressure
• The word originates from the greek word “piezein”, which means “to press”
• Discovered in 1880 by Pierre Curie in quartz crystals.
• Conversely, when an electric field is applied to one of the faces of the crystal it undergoes mechanical distortion.
• Examples --- Quartz, Barium titanate, tourmaline
Electric dipoles in Weiss domains; (1) unpoled ferroelectric ceramic, (2) during and (3) after poling (piezoelectric ceramic)
Piezoelectric Effect

- displacement of electrical charge due to the deflection of the lattice in a naturally piezoelectric quartz crystal
- The larger circles represent silicon atoms, while the smaller ones represent oxygen.
Why Piezoelectric in MEMS

- Suitable for vibrational energy Harvesting
- Compatible with Microfabrication
- Voltages of 2-10V are obtained
- High energy density
- No separate external energy source needed
- Low maintenance
- Good efficiency
Piezoelectric Conversion

33 Mode

31 Mode
Piezoelectric Cantilever

Piezoelectric

Strain in piezoelectric material causes a charge separation
Design and Modeling Considerations

- Good quality material
- Low resistance
- Thermal management
- Higher power and frequency of operation
Theoretical Analysis
Piezoelectric Unimorph Cantilevers

Figure: The schematic of a PUC with the NPL/PL length ratio (a) >1, (b) =1 and (c) <1, and the corresponding induced voltage distribution ((d), (e) and (f)) in the piezoelectric layer with a concentrated force, $F$, applied at the tip. Note that in (a)-(c), the dashed lines in Section-1 and Section-2 indicate the positions of the strain neutral plane.
Piezoelectric materials are characterized by several coefficients:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^S_3$</td>
<td>All strains in the material are constant or</td>
<td>Electrodes are perpendicular to 3 axes. Relative dielectric constant.</td>
</tr>
<tr>
<td></td>
<td>mechanical deformation is blocked in any</td>
<td></td>
</tr>
<tr>
<td>$K^T_1$</td>
<td>All stresses on material are constant or no</td>
<td>Electrodes are perpendicular to 1 axis. Relative dielectric constant.</td>
</tr>
<tr>
<td></td>
<td>external forces.</td>
<td></td>
</tr>
<tr>
<td>$k_P$</td>
<td>Stress or strain is equal in all directions</td>
<td>Electrodes are perpendicular to 3 axis. Electromechanical coupling factor.</td>
</tr>
<tr>
<td></td>
<td>perpendicular to 3 axis</td>
<td></td>
</tr>
<tr>
<td>$d_{33}$</td>
<td>Hydrostatic stress or stress is applied equally</td>
<td>Electrodes are perpendicular to 3 axis. Piezoelectric charge coefficient.</td>
</tr>
<tr>
<td></td>
<td>in all directions. Electrodes are perpendicular</td>
<td></td>
</tr>
<tr>
<td>$g_{15}$</td>
<td>Applied stress, or piezoelectrically induced</td>
<td>Electrodes are perpendicular to 1 axis. Piezoelectric voltage coefficient.</td>
</tr>
<tr>
<td></td>
<td>strain in shear form around 2 axis.</td>
<td></td>
</tr>
<tr>
<td>$d_{31}$</td>
<td>Applied stress, or piezoelectrically induced</td>
<td>Electrodes are perpendicular to 3 axis. Piezoelectric charge coefficient.</td>
</tr>
<tr>
<td></td>
<td>strain is in the 1 direction.</td>
<td></td>
</tr>
<tr>
<td>$S^E_{36}$</td>
<td>Compliance is measured with closed circuit.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stress or strain is shear around 3 direction.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strain or stress is in 3 direction.</td>
<td></td>
</tr>
<tr>
<td>$S^D_{11}$</td>
<td>Compliance is measured with open circuit.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stress or strain is in 1 direction.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strain or stress is in 1 direction.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Elastic compliance.</td>
<td></td>
</tr>
</tbody>
</table>
Resonant frequency \((f_r)\)

\[
f_r = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{K}{m_e}}
\]

Expressed in terms of Bending modulus per unit width \(D_p\)

\[
f_n = \frac{v_n^2}{2\pi} \frac{1}{l^2} \sqrt{\frac{D_p}{m}}
\]

\[
m = \rho_p t_p + \rho_s t_s
\]

\[
D = \frac{E_p^2 t_p^4 + E_s^2 t_s^4 + 2E_p E_s t_p t_s (2t_p^2 + 2t_s^2 + 3t_p t_s)}{12(E_p t_p + E_s t_s)}
\]
The induced voltage unit force $V_{\text{in,ave}/F}$ is given by

$$\frac{V_{\text{in,ave}}}{F} = \frac{1}{2} L g_{31} \frac{E_p}{wD} \left( t_{n1} t_p + \frac{1}{2} t_p^2 \right)$$  \hspace{1cm} (14)

$$K = \frac{2wD}{L^3}$$  \hspace{1cm} (15)

The induced voltage per tip displacement $V_{\text{in,ave}/h_{\text{tip}}}$ is given by

$$\frac{V_{\text{in,ave}}}{h_{\text{tip}}} = \frac{3}{4} \frac{g_{31} E_s t_s E_p t_p (t_s + t_p)}{L^2 (E_s t_s + E_p t_p)}$$  \hspace{1cm} (16)
## Material properties of piezoelectric unimorph cantilever

<table>
<thead>
<tr>
<th>Inputs to the model</th>
<th>ZnO</th>
<th>Pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length(µm)</td>
<td>2500</td>
<td>2500</td>
</tr>
<tr>
<td>Width(µm)</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>thickness(µm)</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Young’s modulus[GPa]</td>
<td>123-210</td>
<td>168</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>--</td>
<td>0.38</td>
</tr>
<tr>
<td>Strain Coefficient(10^{-12} m/v)</td>
<td>-5.4 - 11.67</td>
<td>--</td>
</tr>
<tr>
<td>Density[Kg/m³]</td>
<td>3980</td>
<td>21450</td>
</tr>
<tr>
<td>Dielectric Constant(ε_r)</td>
<td>9-12.64</td>
<td>--</td>
</tr>
</tbody>
</table>
Simulink Model

Figure. Simulink model of piezoelectric unimorph cantilever
Simulation using COMSOL Multiphysics
Use of COMSOL Multiphysics

- Application modes:

  **piezoelectric**: Mechanical / Electrical behavior

  - Generated charge / Electrical potential
  - Vertical vibrations application

  **Moving Mesh**: Varying Length
Geometry

- **3D cantilever**
  - length $L = 2500 \mu m$;
  - width $w = 500 \mu m$;
  - piezoelectric layer thickness $t_{ZnO} = 2 \mu m$.
  - Substrate layer $t_{sub} = 4 \mu m$. 

![Diagram of 3D cantilever with layers labeled](image)
Subdomain and Boundary settings

- **Subdomain**

- **Mechanical boundary conditions**
  - fixed end

- **Electric boundary conditions (piezo layer)**
  - Free end: grounded
  - fixed end: floating potential
  - other surfaces: zero charge
Governing equations

- Piezoelectric Equations in strain-charge form

\[ S = s^E T + dE \]
\[ D = \varepsilon^T E + dT \]

- \( S \) = mechanical strain
- \( T \) = mechanical stress [N/m\(^2\)]
- \( s^E \) = elastic compliance [Pa\(^{-1}\)]
- \( d \) = piezoelectric coefficient [C/N]
- \( D \) = electric displacement [C/m\(^2\)]
- \( E \) = electric field [V/m]
- \( \varepsilon^T \) = dielectric permittivity [F/ m]

\[ \rho = 5680 \text{Kg} / \text{m}^3 \]
Meshing

- Mapped mesh Parameter
Simulation Results

Eigen Frequency Analysis

Figure 1. Model frequency of piezoelectric unimorph cantilever.
Stationary Analysis

Figure. Tip displacement due to applied Acceleration
Frequency Analysis

Figure: Frequency Response of $d_{31}$

Figure: Frequency Response of $d_{33}$
Time dependent Analysis

- Harmonic vibration of 50 N/m² amplitude with frequency from 450Hz to 510Hz is applied on the top surface of beam, so as to produce vibration. The resonant frequency for both $d_{31}$ and $d_{33}$ structure is 585 Hz.

- Force per unit area is taken as 50 N/m² which is equivalent to a proof mass of 0.145 mg deposited on tip of cantilever at 9.81 m/s² acceleration.
Damping

Rayleigh damping for transient analysis

\[
\begin{bmatrix}
\frac{1}{2\omega_1} & \frac{\omega_1}{2} \\
\frac{1}{2\omega_2} & \frac{\omega_2}{2}
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} =
\begin{bmatrix}
\zeta_1 \\
\zeta_2
\end{bmatrix}
\]

\[\zeta_1 = \zeta_2 = 0.1\]

\[\omega_1 = 450\]
\[\omega_2 = 510\]
Output of Transient Analysis

Fig. 1. Frequency Response of $\nu_{31}$ mode

Fig. 2. Frequency Response of $\nu_{33}$ mode
Figure: Extrusion plot showing total displacement of $d_{33}$
Parametric Segregated Analysis Output

**Figure: Plot of total displacement vs length in d31**

**Figure: Plot of total displacement vs length in d33**
Figure: Extrusion plot for maximum voltage.

Figure: Extrusion plot for maximum displacement.
Length(1)=-4e-4
Subdomain: Electric potential [V] Deformation: Displacement amp

Max: 2.289e-3
Min: -8.232e-3
<table>
<thead>
<tr>
<th>Ref</th>
<th>Device</th>
<th>Dimension</th>
<th>$V_{\text{peak}}$</th>
<th>$F$(Hz)</th>
<th>Acceleration $g$</th>
<th>$V/mm^3$</th>
<th>FOM $V/mm^3g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[6]</td>
<td>$\alpha31$ PZT</td>
<td>2mm X 0.6mm X 1.64µm</td>
<td>0.45</td>
<td>608</td>
<td>1</td>
<td>228.7</td>
<td>228.7</td>
</tr>
<tr>
<td>[7]</td>
<td>$\alpha31$ PZT</td>
<td>2mmX3.2mmX1.39µm</td>
<td>16</td>
<td>60</td>
<td>0.79</td>
<td>112.4</td>
<td>142.3</td>
</tr>
<tr>
<td>[8]</td>
<td>$\alpha31$ ZnO</td>
<td>27mm x .3mm x 0.2mm.</td>
<td>$4.7\times10^{-9}$</td>
<td>10</td>
<td>0.1</td>
<td>$2.9\times10^{-4}$</td>
<td>$0.9\times10^{-9}$</td>
</tr>
<tr>
<td>[9]</td>
<td>$\alpha33$ PZT</td>
<td>0.8mmX1mmX10µm</td>
<td>2.2</td>
<td>528</td>
<td>0.39</td>
<td>275</td>
<td>705</td>
</tr>
<tr>
<td>[10]</td>
<td>$\alpha33$ PZT</td>
<td>0.8mmX1.2mmX2µm</td>
<td>1.6</td>
<td>870</td>
<td>2</td>
<td>833.3</td>
<td>416.6</td>
</tr>
<tr>
<td>Proposed</td>
<td>$\alpha31$ ZnO</td>
<td>2.5mm x .5mm x 2µm.</td>
<td>1.05</td>
<td>485</td>
<td>1</td>
<td>420</td>
<td>420</td>
</tr>
<tr>
<td>Proposed</td>
<td>$\alpha33$ ZnO</td>
<td>2.5mm x .5mm x 2µm</td>
<td>4.2</td>
<td>485</td>
<td>1</td>
<td>1680</td>
<td>1680</td>
</tr>
</tbody>
</table>
Conclusion

• The work presents the study of piezoelectric cantilevers with engineered extensions to effectively convert ambient vibrations into electricity.

• Piezoelectric converters appear to be the most attractive for Micro-scale devices with a maximum demonstrated power density.

• Vibration powered systems are being actively pursued and will be up and running shortly.
Acknowledgement

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References


Thank you.