A Weak Formulations for Calculating Spin Wave Dispersion Relation in Magnonic Crystals

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Introduction: We study the spin wave excitation (coherent precession of magnetic moments) in periodically arranged magnetic stripes, i.e., in one-dimensional magnonic crystal (MC).

Computational Methods: Two approaches have been implemented [1,2,3]:

I (red points in figures)

\[ \frac{\partial M(r,t)}{\partial t} = \gamma M(r,t) \times H_{eff}(r,t) \]

\[ \nabla \times h = \alpha E \]

\[ \nabla \times E = -\mu \frac{\partial |h|^2}{\partial t} \]

II (black points in figures)

\[ \nabla \times (\mu^{-1} \nabla \times E) - \omega^2 \mu \times (\mu^{-1} \nabla E) = 0 \]

\[ \mu_{\sigma} = \begin{cases} (\mu_0, H_y, H_x + M_y h_y - 2\pi f) &\text{if } M_y > 0 \\ (\mu_0, H_y, H_x + M_y h_y - 2\pi f) &\text{if } M_y < 0 \end{cases} \]

\[ \psi = \psi e^{i k_x x} \]

Weak formulation

\[ 0 = \int F_i \, dx \, dy \]

Results:

a) \( d < 100 \) nm, small exchange interactions:

\( A = 500 \) nm, \( d = 40 \) nm, \( \sigma = 10 \times 10^{-5} \) S/m, \( H_s = 0.1 \) T, \( M_1 = 1.7 \times 10^4 \) A/m, \( M_2 = 1.7 \times 10^4 \) A/m

<table>
<thead>
<tr>
<th>Mesh points</th>
<th>No of degrees of freedom</th>
<th>time</th>
<th>Exchange Interactions</th>
<th>d &gt; 100nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach I (direct solver)</td>
<td>7410</td>
<td>60565</td>
<td>28 s</td>
<td>✓</td>
</tr>
<tr>
<td>Approach II (iterative solver)</td>
<td>5670</td>
<td>12139</td>
<td>26 s</td>
<td>x</td>
</tr>
</tbody>
</table>

b) \( d < 100 \) nm, large exchange interactions:

\( A = 500 \) nm, \( d = 40 \) nm, \( \sigma = 10 \times 10^{-5} \) S/m, \( H_s = 0.1 \) T, \( M_1 = 0.95 \times 10^4 \) A/m, \( M_2 = 1.05 \times 10^4 \) A/m, \( A = 6 \times 10^{-11} \) J/m

Conclusions: We have presented two approaches to calculate spin wave dispersion relation in magnonic crystals. We have defined a structure that dispersion relation can be obtained using both approaches and compared them. In general, the approach I have to be use for MCs where the exchange interactions are important (small lattice constant), while the approach II is useful in the structures with large thickness and low exchange coefficient.

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References:
3. C. Fietz, Y. Urzhumov, and G. Shvets, Optics Express, 19, 20 (2011)