Poroelasticity Benchmarking for FEM on Analytical Solutions

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Abstract

In poroelastics, hydraulic and mechanical processes are coupled, which allows the simultaneous modeling of these processes (HM-coupling) in porous media. Applications for this type of modeling are generally necessary in all situations in which flow processes through a porous material are accompanied by deformations. The pore pressure as a variable is responsible for fluid flow and movements of the solid, and also depends on both fluid and solid states.
There are several fields of applications, in which poroelastical modeling, as described, is relevant. In geological systems high flow velocities may lead to changes of the solid material, as in the vicinity of pumping or injecting wells. Concerning the exploitation of oil and gas reservoirs poroelasticity may become crucial. The technique of hydraulic fracking utilizes the mentioned coupling. Problems of land subsidence are often related to changes in the subsurface flow system. Aside from soil and rocks the coupling of hydraulic and mechanical features becomes relevant in artificial materials, for example for hydro gels and for swelling, highly pronounced in sponges. Tissue mechanics is a branch of material sciences concerned with such substances.
The model considers Darcy-flow within the porous medium and elastic behavior of the solid material. We compute deformation and pore pressures within the entire domain. The coupling with poroelasticity is achieved using an additional source term in the fluid mass conservation equation.

We present benchmarks in spherical and in cylindrical geometry and compare with analytical solutions (Kirsch 1898, Muskhelisvili 1953, Grandhi et al. 2002, Souley & Thoraval 2011). Meshes are shown in Figure 1. A typical result for stresses is depicted in Figure 2, for the 2D benchmark. In Figure 3 we show the convergence of the numerical solution towards the analytical solution, which is valid for the infinite domain. In COMSOL Multiphysics® a parametric sweep on model extension was used for that purpose. In Figure 4 the numerical error in dependence of grid refinement is examined. From such studies convergence rates are deduced (compare Bradji & Holzbecher 2007, 2008).

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Reference

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Figures used in the abstract

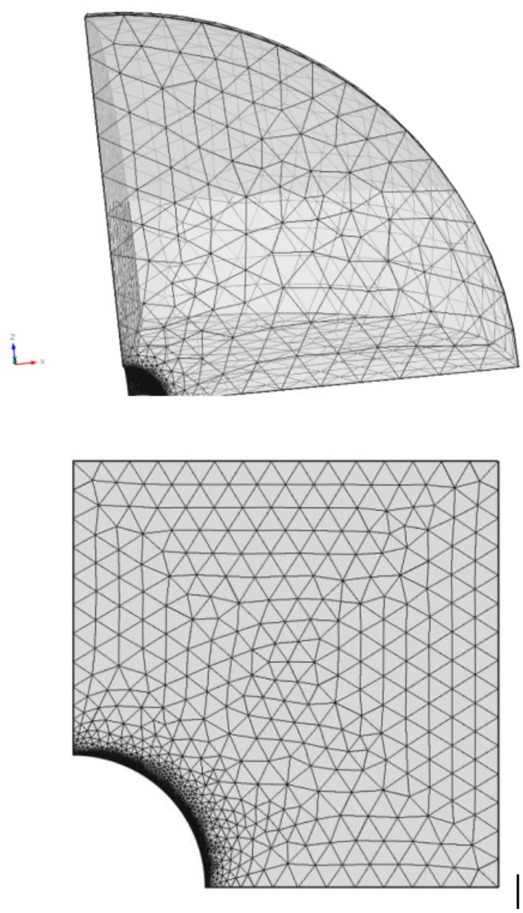

Figure 1: Benchmark model regions with meshes; for spherical cavity in 3D (top) and circular cavity in 2D (bottom)

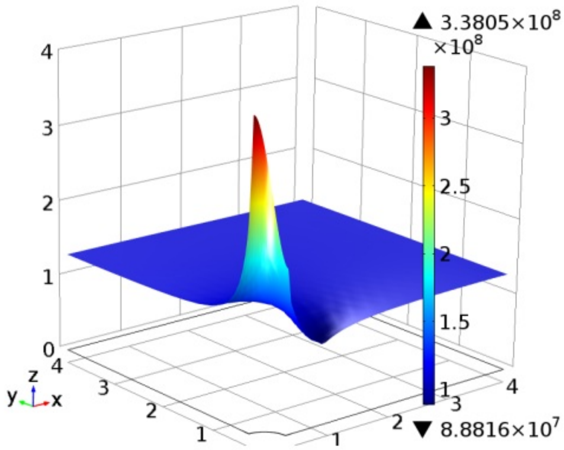

Figure 2: Tresca stress [Pa] of 2D reference problem in deformed mesh

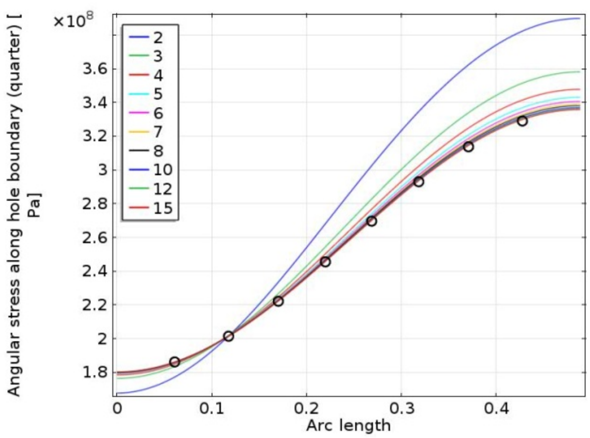

Figure 3: Angular stress along hole boundary in dependence of model extension

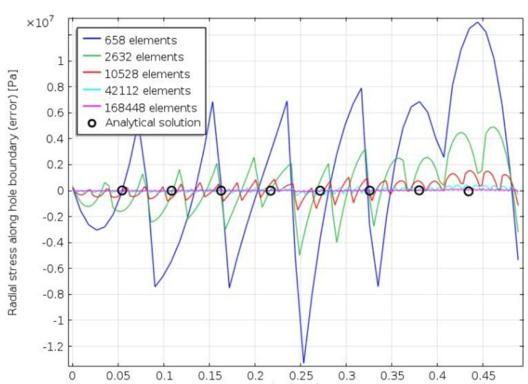

Figure 4: Radial stress error along hole boundary in dependence of grid refinement