Computational homogenization in with an application on masonry structures

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Part A: The proposed computational homogenization method

Classical configuration

Proposed scheme
Steps

1. Consideration of a masonry RVE FEM with COMSOL Multiphysics,
   - Non-linear perfect plastic law in the mortar joints
   - Linear elastic bricks
   - Linear displacement boundary conditions loading
Steps

2. Consideration of different loading paths and loading levels (parametric analysis)

3. Estimation of the average stress and strain

4. Repetition for the estimation of stiffness information
Steps

5. Creation of two databases:
   a) Stress
   b) Stiffness

6. Incorporation in an overall multi-scale homogenization scheme in MATLAB using interpolation (metamodel)

7. Comparison with direct heterogeneous macroscopic analysis in ABAQUS-MARC
The microscopic analysis

• Scanning the 3d space of loading stains =>
  Determination of several loading paths for the RVE

*Key parameter for the success of the concept*

• Incorporation of a parameter in the linear displacement equations =>
  Determination of several strain loads
Loading paths

- Linear displacements:
  \[ u|_{\partial V_m} = \varepsilon^M x \quad \varepsilon^M = [e_{xx} \quad e_{yy} \quad e_{xy}]^T \]

- Creation of loading paths:
  - Simulation of possible combinations of \( \varepsilon^M \) members =>
  - Introduction of two angles, a, b =>
  - 3d scanning of the strain space:
    - \((a, b) = (a, 90), (a, 60), (a, 30), (a, 0), (a, -30), (a, -60), (a, -90)\), for \( a=0:30:360 \) => 91 loading paths

- Incremental application of (each) loading
Averaging procedure: strains-stresses

- For each load path and load level:
  
  \[
  \langle \epsilon \rangle_{V_m} = \epsilon^M \\
  \langle \sigma \rangle_{V_m} = \frac{1}{V_m} \int_{V_m} \sigma^m dV_m
  \]

- Postprocessing subdomain integration: COMSOL

- Usage of script files to request the output quantities

- Stress database: Saving in MATLAB mat files
Averaging procedure: effective constitutive tensor

- Repetition of analyses for every load path and load level
- For each load path – load level:
  - Three test, incremental strain vectors are considered
  - Three incremental average stress vectors are calculated
- Estimation of the effective elasticity tensor: Hooke’s law

\[
\begin{bmatrix}
\delta\varepsilon^M \\
\delta\varepsilon_1^M \\
\delta\varepsilon_2^M \\
\delta\varepsilon_3^M
\end{bmatrix} =
\begin{bmatrix}
\delta\varepsilon_1^M \\
\delta\varepsilon_2^M \\
\delta\varepsilon_3^M
\end{bmatrix}
\]

\[
\begin{bmatrix}
\delta\sigma^M \\
\delta\sigma_1^M \\
\delta\sigma_2^M \\
\delta\sigma_3^M
\end{bmatrix} =
\begin{bmatrix}
\delta\sigma_1^M \\
\delta\sigma_2^M \\
\delta\sigma_3^M
\end{bmatrix}
\]

\[
[\delta\sigma^M] = C^M [\delta\varepsilon^M] \Rightarrow C^M = [\delta\sigma^M][\delta\varepsilon^M]^{-1}
\]
Overall multi-scale computational homogenization scheme

- **MATLAB FEM\(^2\) code: masonry structures**
- Plane stress, first order, full integration FE
- Obtaining macroscopic information:
  - Stress database → Macroscopic stress
  - Stiffness database → Macroscopic tangent stiffness
  - Repetition for each Gauss point and time step
- An interpolation method is needed
  - Simplest, easier solution: MATLAB function “TriScatteredInterp”
Overall multi-scale computational homogenization scheme

- **Stress interpolation:**
  - Each strain vector (3x1) corresponds to one average stress value
  - 3 repetitions to obtain the (3x1) stress vector

- **Effective elasticity tensor interpolation:**
  - Each strain vector (3x1) corresponds to one value of the tensor
  - 9 repetitions to obtain the (3x3) elasticity tensor
Results: micro simulations

Non-linear average stress-strain behaviour
Results: micro simulations
Plastic strain: gradually increased, in the mortar
Results: overall homogenization scheme

Application 1: small masonry wall

Homogeneous model: Proposed approach (20x20 elements)

Direct heterogeneous model: ABAQUS/MARC software
Results: overall homogenization scheme

Degradation of strength

Homogeneous model: Proposed approach

Direct heterogeneous model: ABAQUS
Results: overall homogenization scheme

Force – displacement diagrams

The previous small wall: 0.52x0.26m

A new, bigger masonry wall: 1.82x1.69m
Results: overall homogenization scheme

Application 2: a bigger masonry wall + distributed displacement of 5mm (20x20 elements)
Results: overall homogenization scheme

Degradation of strength – Displacement distribution
Results: overall homogenization scheme

Application 3: masonry wall + openings

Degradation of strength
Results: overall homogenization scheme

Stresses $S_{xx}$

Multi-scale homogenization

DNS model
Results: overall homogenization scheme

Stresses $S_{yy}$

Multi-scale homogenization  

DNS model
Results: overall homogenization scheme

Stresses $S_{xy}$

Multi-scale homogenization     DNS model
Conclusions

• A method for non-linear homogenization
• Good convergence with direct macroscopic analysis
• General method:
  1) Can be applied to other RVEs
  2) Can be applied to different constitutive RVEs laws
• Future study:
  1) Application to more complex constitutive laws / different RVEs
  2) Different interpolation methods (Neural Networks)
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