Air Damping Simulation of MEMS Torsional Paddle

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Abstract: Viscous damping has a significant effect on dynamic performance of the resonators operating within fluid. This work is aimed to find the viscous damping for MEMS torsional paddle operating in air. Interaction of moving structure with the fluid requires a complicated and challenging analysis. The fluid structure interaction of COMSOL Multiphysics is used to study the 2-D interaction of torsional paddle resonator with air and evaluate the damping torque and Quality factor. The effect of resonance frequency by changing the torsional spring on air damping is investigated. The damping coefficient as a function of angular velocity is presented. Prediction of air damping can be used for optimization of such a sensor at design steps.

Keywords: MEMS, Air Damping, Torsional Paddle.

1. Introduction

MEMs resonant devices as biosensors attract a lot of attention, as they are capable to monitor the interaction of the target molecules simultaneously with high resolution. A MEMS torsional paddle can be considered as a potential device for such bio/chemical sensing [1].

One of the important aspects of designing MEMS sensor is having a high performance device, which can meet the required sensitivity. The dynamic performance of the resonator is affected by source of losses such as viscous damping, anchor loss, thermoelastic loss and intrinsic material loss [2]. For resonators the Quality factor which is inversely proportional to energy loss of the resonator can determine the sensitivity as it sets the frequency stability. Quality factor for resonator during one cycle is defined as [3, 4]:

\[ Q = 2\pi \frac{\text{energy stored}}{\text{energy loss}} \]  

Equation 1

It should be considered that investigation and analysis of bio/chemical samples is generally required to be done within fluid. Due to high surface to volume ratio of the MEMS devices, viscous damping is one of the important and dominant sources of losses for resonators operating in fluid which needs to be considered. Prediction of the Quality factor is important in the design steps of such sensors. Viscous damping is associated with area of the moving part.

In this paper, COMSOL Multiphysics is used to estimate the air damping and predict the Quality factor for MEMs torsional paddle resonator. The effect of torsional spring, by varying the dimension of the supports, on its Quality factor is investigated. Following that the sensitivity of the micro paddle is also determined.

2. Micro Paddle Resonator for Mass Detection

The resonant micro-paddle can detect induced mass adsorption onto its functionalized surface by measurement of the resonant frequency shift. Micro paddle can be considered as a potential biosensor for detection of VOCs. The paddle is made from a silicon nitride membrane, which is used as a non-conductive platform.

The micro paddle consists of a suspended plate (8 ×10 μm²), which is anchored via two torsional supports to fixed substrate [Figure 1]. The supports act as springs during the torsional vibration of the micro paddle.

Figure 1. MEMS torsional paddle.

The dynamic equation of motion for micro paddle is described by Equation 2:
\[ I\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + k\theta = \tau \quad \text{Equation 2} \]

Where \( I, D, k \) and \( \tau \) are moment of inertia, damping factor, torsional spring and applied torque on the paddle.

First mode of the paddle is torsional mode. The paddle can be fabricated using the Focused Ion Beam (FIB) and a Pt coil can be deposited on the edge of the paddle to act as a Lorentz force drive method. The micro paddle will be driven to its resonance mode using a sine wave Lorentz force in presence of the magnetic field and alternating current through the coil.

3. Computational method

In this study the geometrical effect (anchors’ length, position and device thickness) on air damping and Quality factor of the torsional paddle are investigated using Fluid-Structure Interaction interface of COMSOL Multiphysics 4.4. Two-dimensional model is developed for simulation and wall of surrounded air is far enough from the resonator to eliminate the effect of any close boundaries on the resonator. Angular displacement (\( \theta = \theta_0 \sin(\omega_n t) \)) is applied on the vertical edges to produce the moment, where \( \omega_n \) is evaluated from the 3D Eigenfrequency simulation. Two different set of simulation is carried out with thickness of 200 and 500 nm and in each set the anchors length is varied from 1-5 \( \mu \)m. Pressure, density and viscosity of air are kept constant for all simulation and are equal to properties of atmospheric air.

Flow properties and boundary conditions play an important role in order to estimate the air damping. Knudsen number (\( k_n \)) is used for characterization of the flow, defined as a ratio between the mean free path of the air molecules (68nm at atmospheric pressure) to the characteristic dimension of the oscillating device [3]. Since \( k_n < 0.01 \), Reynolds number (\( Re \ll 2000 \)) and Mach number (\( M < 0.3 \)) for micro paddle, so the fluid would be in continuum regime and classified as a laminar and incompressible fluid, Navier-Stokes equation with no-slip boundary condition is used for simulation [5, 6].

The Quality factor of the device is determined by energy stored divided by energy loss due to air damping torque on its surfaces during one oscillation cycle. The energy loss can be found by the integration of the dissipated power \( (\text{damping torque} \cdot \text{angular velocity}) \) and can the simplified to Equation 3:

\[ Q = \frac{I\theta_{max}^2}{T_{max}} \quad \text{Equation 3} \]

Where \( T_{max} \) is the maximum damping torque.

4. Results and discussion

The damping torque is calculated by integration of the damping force about the center of rotation over the surfaces of the resonator. The simulation result of the damping torque about one period for angular velocity \( \dot{\theta} \) is shown in Figure 2.a. There is a phase difference between the damping torque and the corresponding angular velocity, the damping torque can be resolved into two components; in phase and out of phase with the resonator’s angular velocity, which are contributing to the damping and additional mass loading, respectively [Figure 2.b]. The resonance frequency will shift to lower value due to this additional mass loading.

![Figure 2](image_url)

**Figure 2.** (a) Phase difference between the damping torque and angular velocity, (b) Sine and cosine components of damping torque.

The effect of placement of the anchors on Quality factor is studied. It has been observed that anchors at the center of the paddle will improve the Quality factor compared to the
offset one. Figure 3 shows the movement of air around the paddle. The improved Quality factor is due to more uniform flow movement across the structure.

Figure 3. Eigenfrequency and air flow distribution around the paddle (length×width×thickness=10×8×0.2μm³), (a,b) anchors at the centre, (c,d) anchors offset from the centre.

As it is mentioned before, in each set of simulation the main area of the paddle is kept constant and by varying the anchors’ length and thickness, the natural resonance frequency of the paddle will change. Anchors’ for these sets of simulation are placed at symmetry point. The term $D\dot{\theta}$ is the damping part of the equation of motion of the paddle and is equal to $\tau_{\text{damping}}$. Damping torque was achieved using simulation and then damping coefficient $(D)$ can be found for each set of simulation. Figure 4.a and 4.b represent the calculated $D$ values and damping torque versus angular velocity ($\dot{\theta}$), respectively where $\dot{\theta}$ varies for each anchor length given the driving amplitude is kept constant.

Figure 4. (a) Damping coefficient versus angular velocity, (b) Damping torque versus angular velocity.

The thickness of the paddle does not contribute much compared to the top and bottom surfaces of the paddle in air damping and only has an effect on natural frequency. As it is illustrated by Figure 4.a there is a linear relationship between $D$ and $\dot{\theta}$, which can be described by equation 4:

$$D = C_1\dot{\theta} + C_2$$  \hspace{1cm} \textit{Equation 4}

Where $C_1$ and $C_2$ are constant which are equal to 7E-25 N.m.s² and 3E-20 N.m.s. The damping torque can be expressed by damping coefficient times the angular velocity, which would be second order polynomial in term of $\dot{\theta}$ as it is presented in Figure 4.b.

The simulation parameters and corresponding results are summarized in Table 1.

<table>
<thead>
<tr>
<th>$L$ (µm)</th>
<th>$t$ (nm)</th>
<th>$f$ (MHz)</th>
<th>$T_{\text{max}}$ (N.m)</th>
<th>$D$ (N.m.s)</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>2.86</td>
<td>8.03E-16</td>
<td>4.47E-20</td>
<td>170</td>
</tr>
<tr>
<td>2.5</td>
<td>200</td>
<td>2.01</td>
<td>5.31E-16</td>
<td>4.21E-20</td>
<td>127</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>1.47</td>
<td>3.48E-16</td>
<td>3.75E-20</td>
<td>105</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
<td>6.33</td>
<td>2.36E-15</td>
<td>5.92E-20</td>
<td>712</td>
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<tr>
<td>2.5</td>
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<td>4.50</td>
<td>1.47E-15</td>
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</tr>
<tr>
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<td>500</td>
<td>3.34</td>
<td>9.57E-16</td>
<td>4.56E-20</td>
<td>488</td>
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</tbody>
</table>

Table 1. Summarized simulation results ($L$ = anchors’ length, $t$ = thickness, $f$ = resonance frequency, $T_{\text{max}}$ = maximum damping torque, $D$ = damping coefficient and $Q$ = Quality factor).

The crucial goals for a mass sensing resonator are having high Quality factor and sensitivity. The sensitivity is defined by equation 5:

$$S = \frac{\Delta f}{\Delta m}$$  \hspace{1cm} \textit{Equation 5}

The sensitivity is found using Eigenfrequency study in COMSOL with and without any added mass (100pg) on the resonator. The sensitivity for resonator with anchor length of 2.5 µm placed on middle and 2.5 µm far from central line with thickness of 200 and 500 nm are evaluated and summarized in Table 2.

<table>
<thead>
<tr>
<th>$L$ (µm)</th>
<th>$t$ (nm)</th>
<th>$f_{\text{unloaded}}$ (MHz)</th>
<th>$S$ (Hz/fg)</th>
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<tr>
<td>2.5</td>
<td>200</td>
<td>1.37</td>
<td>5.58</td>
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<tr>
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<td>200</td>
<td>2.01</td>
<td>8.17</td>
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<tr>
<td>2.5</td>
<td>500</td>
<td>4.50</td>
<td>10.88</td>
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Table 2. Summarised sensitivity for paddle.
It can be concluded that with the same dimension of resonator by increasing the thickness, Quality factor and sensitivity will improve. Also placing the anchor offset from the symmetry point will result in lower Quality factor and sensitivity.

Lorenz distribution can be used to plot the response curve of the resonator. Knowing the Quality factor, resonance frequency and assuming the maximum displacement to be equal to 5 nm, the response curve is plotted in Figure 5. As it is shown in Figure 5, additional mass loading on the resonator result in lowering the resonance frequency.

![Response curve for loaded and unloaded micro paddle](image)

**Figure 5.** Response curve for loaded and unloaded micro paddle.

### 8. Conclusions

In this study we investigate the effects of geometrical parameters on the behavior of the MEMS mass sensor. We showed that just by changing these parameters the Quality factor could be enhanced or inhibited which consequently could have significant results on the sensitivity of the sensor. It was shown that the sensitivity has a direct relation with the quality factor. In order to justify this simulation results, the fabrication of such a structure can be performed by nanofabrication method, such as Focused ion beam.

### 9. References