Numerical simulation of blood flow in a straight artery under the influence of magnetic field

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Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
Motivation

- Interaction of magnetic field with the flow of electrically conducting fluid (blood)
- Downstream Singularities in the cardiovascular system
- Application to Magnetic Resonance Imaging (MRI)
- Effect of magnetic field on the T wave in ECG

Tenforde, T.S., Progress in Biophysics & Molecular Biology, 87(2005) 279

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Representation of elementary volume
Governing Equations

The basic governing equations for magnetohydrodynamic (MHD) fluid flow are

\[ \nabla' \cdot \vec{V}' = 0 \]

\[
\rho \left( \frac{\partial \vec{V}'}{\partial t'} + (\vec{V}' \cdot \nabla') \vec{V}' \right) = -\vec{V}' p' + \eta \nabla'^2 \vec{V}' + \vec{J}' \times \vec{B}'
\]

The Maxwell's equations of electromagnetism

\[ \nabla' \cdot \vec{E}' = \frac{\rho_e}{\varepsilon} \]

\[ \nabla' \times \vec{E}' = -\frac{\partial \vec{B}'}{\partial t'} \]

\[ \nabla' \times \vec{B}' = \mu_e \vec{J}' \]

\[ \nabla' \cdot \vec{B}' = 0 \]

and the Ohm's law

\[ \vec{J}' = \sigma [\vec{E}' + \nabla' \times \vec{B}'] \]

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Normalized Equations & characteristics parameters

<table>
<thead>
<tr>
<th>Length</th>
<th>Velocity</th>
<th>Stress &amp; Pressure</th>
<th>Time</th>
<th>Magnetic field</th>
<th>Induced current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$V_0$</td>
<td>$\rho V_0^2$</td>
<td>$\frac{1}{\omega}$</td>
<td>$\frac{1}{a \sqrt{\eta}}$</td>
<td>$\frac{1}{\mu_e a^2 \sqrt{\eta}}$</td>
</tr>
</tbody>
</table>

$\nabla \cdot \vec{V} = 0$

$$\frac{\alpha^2}{Re} \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = -\nabla p + \frac{1}{Re} \nabla^2 \vec{V} + \frac{Ha}{ReR_m} (\nabla \times \vec{b}) \times \hat{i} + \frac{1}{ReR_m} (\nabla \times \vec{b}) \times \vec{b}$$

$$\frac{\alpha^2}{Re} \frac{\partial \vec{b}}{\partial t} + (\vec{V} \cdot \nabla) \vec{b} = (\vec{b} \cdot \nabla) \vec{V} + Ha (\hat{i} \cdot \nabla) \vec{V} + \frac{1}{R_m} \nabla^2 \vec{b}$$

<table>
<thead>
<tr>
<th>Reynolds number</th>
<th>Hartmann number</th>
<th>Magnetic Reynolds number</th>
<th>Womersley parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re = \frac{a \rho V_0}{\eta}$</td>
<td>$Ha = B_0 a \sqrt{\frac{\sigma}{\eta}}$</td>
<td>$R_m = a V_0 \mu_e \sigma$</td>
<td>$\alpha = a \sqrt{\frac{\omega}{\nu}}$</td>
</tr>
</tbody>
</table>

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## Boundary conditions

<table>
<thead>
<tr>
<th>Inlet</th>
<th>Wall</th>
<th>Plane of symmetry (XOZ-plane)</th>
<th>Plane of symmetry (YOZ-plane)</th>
<th>Outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w = \left[1 + \sin\left(t + \frac{3\pi}{2}\right)\right]$, $u = v = 0$; $w = 0$;</td>
<td>$u = v = 0$, $w = 0$;</td>
<td>$(\hat{n} \cdot \vec{V})\vec{V} = 0$;</td>
<td>$(\hat{n} \cdot \vec{V})\vec{V} = 0$;</td>
<td>$p = 0$;</td>
</tr>
<tr>
<td>$\hat{b} = \vec{0}$</td>
<td>$\hat{b} = \vec{0}$</td>
<td>$(\hat{n} \cdot \vec{V})\hat{b} = \vec{0}$</td>
<td>$\hat{b} = \vec{0}$</td>
<td>$(\hat{n} \cdot \vec{V})\hat{b} = \vec{0}$</td>
</tr>
</tbody>
</table>

Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
**Numerical methods and Mesh refinements**

- Direct numerical simulation were performed with Comsol Multiphysics software®
- Mesh statistics for the Quarter part of the tube

<table>
<thead>
<tr>
<th>Cases</th>
<th>Maximum Element Size (ES)</th>
<th>Number of Elements (NE)</th>
<th>No. of Degrees of freedom (DOF)</th>
<th>Number of triangular element at the entry section</th>
<th>Number of triangular element at the surface of the vessel wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-I</td>
<td>0.30</td>
<td>46568</td>
<td>466982</td>
<td>38</td>
<td>6528</td>
</tr>
<tr>
<td>Case-II</td>
<td>0.20</td>
<td>163594</td>
<td>1538336</td>
<td>80</td>
<td>13486</td>
</tr>
<tr>
<td>Case-III</td>
<td>0.18</td>
<td>226680</td>
<td>2102756</td>
<td>89</td>
<td>17366</td>
</tr>
<tr>
<td>Case-IV</td>
<td>0.15</td>
<td>394192</td>
<td>3577693</td>
<td>125</td>
<td>25696</td>
</tr>
</tbody>
</table>
Absolute error in the axial velocity for four different cases when $Re=300$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$Ha=2$</th>
<th></th>
<th>$Ha=10$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case-I &amp; Case-II</td>
<td>Case-II &amp; Case-III</td>
<td>Case-III &amp; Case-IV</td>
<td>Case-I &amp; Case-II</td>
</tr>
<tr>
<td></td>
<td>$5 \times 10^{-5}$</td>
<td>$4 \times 10^{-5}$</td>
<td>$1 \times 10^{-5}$</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.0</td>
<td>$2 \times 10^{-4}$</td>
<td>$4 \times 10^{-5}$</td>
<td>$2 \times 10^{-5}$</td>
<td>$3 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.2</td>
<td>$3 \times 10^{-4}$</td>
<td>$6 \times 10^{-5}$</td>
<td>$1 \times 10^{-5}$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.4</td>
<td>$2 \times 10^{-4}$</td>
<td>$6 \times 10^{-5}$</td>
<td>$1 \times 10^{-5}$</td>
<td>$9 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.5</td>
<td>$8 \times 10^{-5}$</td>
<td>$1 \times 10^{-5}$</td>
<td>$3 \times 10^{-5}$</td>
<td>$1 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$6 \times 10^{-4}$</td>
<td>$8 \times 10^{-5}$</td>
<td>$3 \times 10^{-5}$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$B_0 = 9Ha$
Comparison of axial velocity profile in the developed flow region with the analytical solutions obtained by Fabri & Siestrunck (1960) for $Ha=1$ and 10.


Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
Steady flow development of axial velocity at different axial position along the tube, $Re=300, Ha=2$

Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
Axial velocity profile for different values of the Hartmann number $Ha$, when the flow is fully-developed with $Re=300$
Entrance length for steady case with different Reynolds number without applying magnetic field ($Ha=0$)

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Steady entrance length with Hartmann number $Ha$.

\[
Le \approx \frac{0.25 \text{Re}}{1 + 0.024\text{Ha} + 0.025\text{Ha}^2 - 0.0016\text{Ha}^3}
\]

Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
Unsteady entrance length with time for different Womersley parameter, when $Re=100$, $Ha=0.5$
Unsteady entrance length with time for different Hartmann number Ha, when Re=100, $\alpha = 5$

Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
Axial velocity profiles at the peak (solid) lines and at the minimal (dotted) lines for different values of the Womersley parameter (left) and for different Hartmann number $Ha$ (right).

Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
Plot with colour represents the axial induced magnetic field and arrow plot indicates the induced current density in steady case for Re=300, Ha=2

Variation of axial induced magnetic field for different Hartmann number $Ha$ in steady case for Re=300

Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore
Variation of different forces by unit volume on the fluid particle (pressure gradient force indicated by green colour, Viscous forces indicated by blue colour and Magnetic forces by red colour) with Hartmann Ha, in steady case for Re=300

Variation of flow resistance per unit length \( (\lambda = \frac{-\Delta p/\Delta z}{Q}) \) with Hartmann number Ha for Re=300
Plot shown in colour is the magnitude of the electric field and arrow plots indicate the direction of induced electric field,

\[ \vec{E} = \frac{1}{Rm} \nabla \times \vec{b} - \nabla \times (H_\alpha \hat{a} + \vec{b}) \]
Non-dimensional amplitude of the current density components $J_x$ and $J_y$ for $Ha=1$ and $Ha=3$ respectively.
Effect of steady and unsteady entrance length on induced voltage with $Ha$ and time respectively for $Ha = 2$, $Re = 100, \alpha = 5$
Reduction in mean velocity with Hartmann number $Ha$

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Conclusions

The steady entrance length is found to be increase with Reynolds number Re and decrease with the increase of Hartmann number Ha. The steady entrance length in terms of magnetic field strength can be approximated theoretically as shown earlier.

The sinusoidal variation with time in cycle is observed for unsteady entrance length. The unsteady entrance length is also decreases with Hartmann number Ha. The phase difference is observed in unsteady entrance length for the presence of both the parameters $\alpha$ and $Ha$

During pulsatile blood flow, the reverse flow can be strongly suppressed by applying strong magnetic field. The Womersley parameter has reducing effect on flow velocity in the core region and an enhancing effect in the boundary layer during its peak flow, while the trend is reversed in the case of minimal flow rate.
The interaction between induced currents and applied magnetic field causes reduction in flow velocity and thereby increases blood pressure in order to retain constant flow rate. The induced magnetic field forms two lobes on each side of the main current line and the induced currents re-circulates inside the vessel, due to the consideration of non-conducting vessel walls. Thus the effect of induced magnetic field should not be overcome.

The induced voltage also varying sinusoidally with time and proportional to the applied magnetic field only. It is worth mentioning here that the induced voltage does not depend on the development of flow field.

We may also conclude that finding analytical solutions with the interaction of electromagnetic field in unsteady case is not possible and thus the numerical simulation is the only way of its solution.


Thank you all for your kind attention