Solid Rocket Motor Combustion Instability Modeling in COMSOL Multiphysics

Sean R. Fischbach
Mashall Space Flight Center / Qualis Corp. / Jacobs ESSSA Group
*MSFC Huntsville sean.r.fischbach@nasa.gov
Outline

• Introduction and problem statement
• Overview of Combustion Instability (CI) modeling
  – Industry standard approach and software
  – Acoustic wave equation model
  – Energy balance model
• Use of COMSOL
  – High Mach Number Flow (HMNF) module
  – Pressure Acoustics (PA) module
  – Coefficient Form Partial Differential Equation (PDE) module
• Results
Introduction

• CI in Solid Rocket Motors (SRM) is characterized by undesirable fluctuations of pressure, velocity, and temperature
  – Unsteady energy release from propellant surface
  – Internal fluid dynamics i.e. vortex shedding, turbulence, etc.
  – Chamber and grain geometry
• Modeling CI in SRMs requires accurate representation of the steady and unsteady flow parameters
• The present study investigates the feasibility and advantage of employing COMSOL in the prediction of CI in SRMs
Combustion Instability Modeling

• Solid Propellant Performance (SPP) ’04 program is the industry standard SRM ballistics prediction software.
  – One Dimensional fluid dynamics
  – Three dimensional grain geometry and regression
  – Includes various ballistics mechanisms (i.e. erosive burning, nozzle boundary layer loss…)

• Standard Stability Prediction (SSP) code uses outputs from SPP ‘04 to evaluate the Culick stability model.

• Culick/wave equation stability model
  – Flow parameters split into steady and unsteady terms
  – Inhomogenous wave equation including mean flow terms on the right hand side.
  – Unsteady terms modeled using 1-D homogenous wave equation

\[ \nabla^2 p' - \frac{1}{a^2} p''_t = -q \nabla \cdot (\bar{u} \cdot \nabla u' + u' \cdot \nabla \bar{u}) + \frac{1}{a^2} \bar{u} \cdot \nabla p'_t + \frac{\gamma}{a^2} p'_t \nabla \cdot \bar{u} \\
\]

\[ P = \bar{P} + p' e^{\alpha_{motor} t} \]

\[ \alpha_{motor} = \alpha_p c + \alpha_f t + \alpha_n d + \alpha_p d + \alpha_{blp} + \ldots \]

\[ \bar{P} = \text{mean chamber pressure} \]

\[ p' = \text{unsteady pressure} \]
Combustion Instability Modeling cont.

- Flandro/Jacob energy corollary model
  - Myers unsteady energy corollary used to model flow disturbances in the presence of mean flow
  - Flow parameters split into steady and unsteady parts
  - Model can account for acoustic, vortical, and thermal (entropy) oscillations

\[
\frac{\partial E_2}{\partial t} = D_2 - \nabla \cdot W_2
\]

\[
E_2 = \frac{p_1^2}{2\rho_0 a_0^2} + \rho_1 u_0 \cdot u_1 + \frac{1}{2} \rho_0 u_1^2 + \frac{\rho_0 \rho T_0 s_1^2}{2 C_p}
\]

\[
D_2 = -\rho_0 u_0 \cdot (u_1 \times \Omega_1) - \rho_1 u_1 \cdot (u_0 \times \Omega_0) - \rho_0 T_1 u_0 \cdot \nabla s_1 - \rho_0 s_1 u_1 \cdot \nabla T_0 - \rho_1 s_1 u_0 \cdot \nabla T_0 + m_1 \psi_1
\]

\[
W_2 = u_1 p_1 + \frac{u_0}{\rho_0} p_1 \rho_1 + \rho_0 u_1 (u_0 \cdot u_1) + \rho_1 u_0 (u_0 \cdot u_1)
\]

- Jacob recast the Myers energy model into the traditional alpha notation

**W:**

\[
\alpha_n = \frac{-\gamma}{2 E_n} \int n \cdot R_s \bar{u} p_n^2 dS - \frac{1}{2 E_n} \int \int \frac{1}{K_n^2} \left( \frac{d p_n}{dz} \right)^2 \bar{u}_b - r \frac{\rho_p}{\rho_g} (p')^2 dS_b
\]

**E:**

\[
\alpha'_n = \int \int \int -\nabla \left[ \rho_n u_n + \frac{u_0}{\rho_0} p_n \rho_n + \rho_0 u_n (u_0 \cdot u_n) + \rho_n u_0 (u_0 \cdot u_n) \right] - \rho_0 u_0 \cdot (u_n \times \Omega_n) - \rho_n u_n \cdot (u_0 \times \Omega_0) dV
\]
COMSOL Implementation of CI Theory

- A CI analysis of a simplified SRM was conducted using multiple modules of COMSOL multiphysics
- The HMNF module was used to model the SRM internal ballistics
  - Spalart-Allmaras turbulent flow model
  - Slip boundary condition on all chamber and nozzle walls
  - Gas injection modeled using St. Robert’s Law
- PA module was used to model the unsteady field variables
  - Geometry truncated at the Mach = 1 plane
  - Hard wall boundary used on all boundaries
- Acoustic Velocity Potential Equation (AVPE) modeled using the Coefficient Form PDE module.
  - AVPE is generated by combining the linearized conservation of mass and momentum equations
  - Retain mean flow effects on the acoustics as Mach numbers exceed 0.2.
- Results from the PA module and the AVPE are post processed in conjunction with the HMNF results to calculate alpha for both CI models
  - Alpha terms using the PA results are compared with SSP
  - Alpha terms using the AVPE are compared with the PA results to measure improvement
**HMNF Module**

- **Inlet/propellant boundary condition**
  - Regression rate of the solid propellant was modeled using, \( \dot{r} = ap^n \)
  - Conservation of mass at the propellant/flame surface provides the injection velocity, \( v_g = \frac{\dot{r}}{\rho_g} \frac{\rho_p}{\rho_g} \)
  - The assumption is made that the flame temperature is independent of burning pressure

- **The velocity is allowed to slip on the nozzle closure and cone walls**
  - Assists in extracting the M=1 plane
  - Acoustics are insensitive to near wall mean flow velocities

- **Mesh consists of 1,316,965 Tetrahedral, 61,233 Triangular, 855 Edge, and 68 Vertex elements with focus applied to the nozzle**

- **Stationary analysis with the wall distance initializer**

<table>
<thead>
<tr>
<th>Fluid Property</th>
<th>( k )</th>
<th>( M_n )</th>
<th>( \gamma )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.005315415 [lbf/(s*R)]</td>
<td>0.02775 [kg/mol]</td>
<td>1.1752</td>
<td>3.892E-6 [lbf*s/ft^2]</td>
</tr>
</tbody>
</table>
Pressure Acoustics and AVPE

- Sound Hard Wall / No Flux boundary conditions were applied to all boundaries
  - Assumes zero acoustic absorption or excitation at boundaries
- For the PA and AVPE models the required mean flow and material properties were supplied by the HMNF analysis
- AVPE allows for mean flow terms to affect the acoustics,

\[
\nabla^2 \psi - (\lambda/c)^2 \psi - M \cdot [M \cdot \nabla(\nabla \psi)] - 2(\lambda M/c + M \cdot \nabla M) \cdot \nabla \psi - 2\lambda \psi [M \cdot \nabla (1/c)] = 0
\]

- In the Coefficient Form PDE module the terms of the AVPE containing mean flow parameters were incorporated using domain source terms
- Mesh consists of 1,144,440 Tetrahedral, 67,286 Triangular, 818 Edge, and 60 Vertex elements with focus applied to the sonic line
- Eigenvalue studies were conducted for both modules
HMNF Results and SPP Comparison

<table>
<thead>
<tr>
<th></th>
<th>$P_h$ (psi)</th>
<th>$P_a$ (psi)</th>
<th>$\dot{m}$ (lb/s)</th>
<th>Thrust (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HMNF</td>
<td>1.02</td>
<td>1.03</td>
<td>1.04</td>
<td>1.02</td>
</tr>
<tr>
<td>% diff</td>
<td>1.95</td>
<td>2.64</td>
<td>3.88</td>
<td>1.65</td>
</tr>
</tbody>
</table>

- HMNF results normalized by the SSP value.
PA Results and SSP Comparison

Normalized acoustic pressure field

\[ \alpha_{PC} = \gamma \frac{1}{2E_n} \int \mathbf{n} \cdot R_s \overline{u} \overline{p}_n^2 \, dS \]

\[ \alpha_{FT} = -\frac{1}{2E_n} \int \frac{1}{K_n^2} \left( \frac{dp_n}{dx} \right)^2 \overline{u}_b \, dS_b \]

\[ \alpha_{RFC} = \frac{1}{2E_n} \int r \frac{\rho_p}{\rho_g} (p')^2 \, dS_b \quad \quad E_n = \iiint (p')^2 \, dV \]

<table>
<thead>
<tr>
<th>Freq. (Hz)</th>
<th>1L</th>
<th>2L</th>
<th>3L</th>
<th>4L</th>
<th>5L</th>
<th>6L</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA</td>
<td>115</td>
<td>231</td>
<td>346</td>
<td>462</td>
<td>578</td>
<td>695</td>
</tr>
<tr>
<td>SSP</td>
<td>116</td>
<td>233</td>
<td>350</td>
<td>467</td>
<td>584</td>
<td>701</td>
</tr>
<tr>
<td>% diff</td>
<td>0.86</td>
<td>0.86</td>
<td>1.14</td>
<td>1.07</td>
<td>1.03</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Alpha Comparison

- SSP-PC
- PA-PC
- SSP-FT
- PA-FT
- SSP-RFC
- PA-RFC

Jacobs ESSSA Group

10
AVPE Results and PA Comparison

\[
\alpha_{PC} = \frac{1}{2E_n^2} \int \int n \cdot \left( \rho_n u_n + \frac{u_0}{\rho_0} p_n \rho_n \right) S_b
\]

\[
\alpha_{ND} = \frac{1}{2E_n^2} \int \int n \cdot \left( \rho_n u_n + \frac{u_0}{\rho_0} p_n \rho_n \right) S_N
\]

\[
E_n^2 = \int \int \frac{p_n^2}{2\rho_0 a_0^2} + \rho_n u_0 \cdot u_n + \frac{1}{2} \rho_0 u_n^2 \, dV
\]

<table>
<thead>
<tr>
<th>Freq. (Hz)</th>
<th>1L</th>
<th>2L</th>
<th>3L</th>
<th>4L</th>
<th>5L</th>
<th>6L</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA</td>
<td>115</td>
<td>231</td>
<td>346</td>
<td>462</td>
<td>578</td>
<td>695</td>
</tr>
<tr>
<td>AVPE</td>
<td>115</td>
<td>230</td>
<td>345</td>
<td>460</td>
<td>576</td>
<td>692</td>
</tr>
<tr>
<td>% diff</td>
<td>0.0</td>
<td>0.43</td>
<td>0.29</td>
<td>0.43</td>
<td>0.35</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Alpha Comparison
Conclusions

• A simplified SRM was modeled using the COMSOL multiphysics finite element software
  – HMNF CFD was used to model mean flow parameters
  – PA and Coefficient PDE modules were used to model flow unsteadiness

• Pertinent ballistics parameters from the HMNF analysis compared well with the industry standard SPP

• Acoustic frequencies and CI alpha terms from the PA module compare well with the industry standard SSP

• Coefficient PDE results compare well with the PA results with the calculated CI terms showing the effect of a more accurate mode shape definition

• The present study demonstrates that COMSOL multiphysics can be used as a CI modeling tool and that the increased fidelity will result in improved results