# Modeling Dielectric Heating: A First Principles Approach

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Abstract: Dielectric heating is an important, widely employed electromagnetic heating technology utilized by consumers, businesses and industry. Specific frequency bands are currently allocated, by international agreement [1], for use in the exploration, development, and application in operating devices. For example, consider that the readily available, commercial, consumer microwave ovens have been allocated an operating frequency of 2.45 GHz. In this paper, I present a First Principles Model of the applied dielectric heating process. This model is used to explore the physical differences manifested when different frequencies are utilized to execute the heat generation process on similar materials in similar geometries.

**Keywords:** dielectric, RF, microwave, complex relative permittivity.

#### 1. Introduction

There are basically two well-known electrical loss mechanisms in physical materials. In this case, Figure 1 shows an example of dielectric heating loss results that occur when energy at a frequency of 2.45 GHz is applied to whey gel [2]. The application of the 2.45 GHz energy generates volumetric heat and raises the temperature of the whey gel into the pasteurization temperature range (71.7 C for 15 seconds).

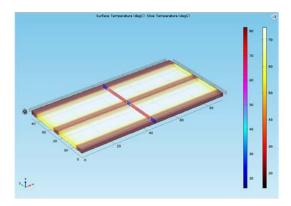


Figure 1. Dielectric Heating at 2.45 GHz.

Dielectric heating typically occurs in the absence of free carriers in nominally non-conductive materials [3]. Dielectric heating occurs as a result of the relative motion of bound electrons, ions, atoms, and molecules, in situ, resulting in vibrational energy coupling to the bulk of the material (heat).

The other electromagnetic loss mechanism is by Joule heating [4]. Joule heating involves charged free carrier flow through the particular material. The free carriers are usually electrons or electrons and ions. Joule heating occurs when a conductive material is subjected to an electromagnetic potential difference.

Different types of materials, due to their internal structure, are more or less sensitive to both the relative amplitude and the frequency of the applied electromagnetic field. As a result, for computational purposes, the characteristics of the material's electronic heating behavior is typically mathematically represented as a lumped-constant parameter named the relative permittivity (dielectric constant) [5]. The relative permittivity of a material is measured by how much it differs from the permittivity of Free Space. The permittivity of Free Space (a perfect vacuum) is a fundamental constant whose value is:

$$\varepsilon_0 = \frac{1}{c_0 \mu_0} \approx 8.85419 \, x \, 10^{-12} [F/m] \quad (1)$$

where:  $\varepsilon_{a} = permittivity of Free Space$ 

and:  $c_n = speed of light$ 

and:  $\mu_a = permeability of Free Space$ 

The permittivity of a material is:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{\mathbf{r}} \boldsymbol{\varepsilon}_{\mathbf{0}} \tag{2}$$

where:  $\varepsilon_r = relative permittivity$ 

and:  $\varepsilon_0 = permittivity of Free Space$ 

For very complicated materials that have intrinsic phase delays and energy losses, the relative permittivity is represented by a complex number, such as:

$$\hat{\varepsilon}_{\varepsilon} = \varepsilon'(\omega) + i\varepsilon''(\omega) \tag{3}$$

where:  $\varepsilon'(\omega)$  is the real part

and:  $\varepsilon''(\omega)$  is the imaginary part

and:  $\omega = 2 * \pi * f_0$ where:  $f_0 = frequency$ 

The loss equation is:

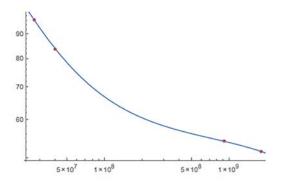
$$Q_{d} = 2 * \pi * f_{0} * \varepsilon'' * \varepsilon_{0} \left(\frac{V_{m}}{thickness}\right)^{2}$$
 (4)

where:  $f_0 = frequency$ 

## 2. Dielectric Heating Model Materials

As a first approximation, whey gel (~gelled milk) is essentially water, with added impurities (~10%), formed into a gelatinous mass. In the case of this model, it is assumed that the basic physical properties of liquid water, as available in the COMSOL Multiphysics Materials Library, can be modified sufficiently, as a First Principles Approximation, to ballpark the whey gel physical characteristics, by adding the measured complex permittivity values for whey gel [2] to the basic physical properties of water.

As shown in equation three (3), the complex permittivities for all materials comprise two components, the real component and the imaginary component. Figure 2 shows both the measured whey gel real permittivity component data at four individual frequencies and the power series polynomial curve created by me to fit that data.



**Figure 2**. Permittivity Real Part Data (red) and Polynomial Expansion Fit Curve (blue).

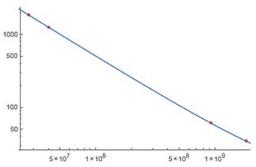
The formula for the real part Fit curve is:

$$\varepsilon' = 54.6919 - 2.05493 \times 10^{-9} * f_0$$

$$+1.27475 \times 10^{9} * f_0^{-1} - 4.04542 \times 10^{15} * f_0^{-2}$$
(5)

where:  $f_0 = frequency$ 

Figure 3 shows both the whey gel imaginary part permittivity component data measured at four individual frequencies and the power series polynomial curve created by me to fit that data.



**Figure 3**. Permittivity Imaginary Part Data (red) and Polynomial Expansion Fit Curve (blue).

The formula for the imaginary part Fit curve is:

$$\varepsilon'' = 6.88131 + 1.59761 \times 10^{-10} * f_0$$

$$+4.96322 \times 10^{10} * f_0^{-1} + 2.48106 \times 10^{16} * f_0^{-2}$$
(6)

where:  $f_0 = frequency$ 

### 3. The COMSOL Multiphysics Model

Figure 4 shows the COMSOL Multiphysics Model Builder tree, partially expanded. This model utilizes the Electric Currents (ec) and the Heat transfer in Solids (ht) Modules.

All of the model's Parameters are defined in the Global Parameters file. They include: input voltage (V\_in), frequency (f0), temperature (T), and dimensionless frequency (f01).

All of the model's variables are included in the Variables 1 file. They include: the dielectric heating energy loss equation (Qd), dimensionless temperature (T1), total complex relative

permittivity (rpw), the real part of the total complex relative permittivity (epr) and the imaginary part of the total complex relative permittivity (epi).

Figure 5 shows the Dielectric Heating geometry employed in this model. This geometry comprises two whey gel strips, surrounded by two air strips and separated by a single air strip. The geometries and volumes of the two whey gel strips are identical to each other. The geometries and volumes of the three air strips are identical to each other.

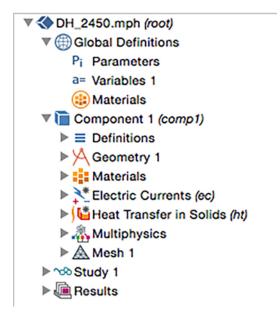


Figure 4. Dielectric Heating Model Builder Tree

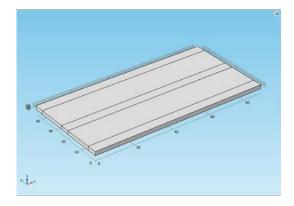


Figure 5. Geometry Configuration

Figure 6 shows the configuration of the Electric Current (ec) Module. The Electric Potential 1 is

applied to the top surface of the two whey gel strips. Ground 1 is applied to the entire lower surface of the model, including both whey strips (2) and all three air strips (3).

Figure 7 shows the configuration of the Heat Transfer in Solids (ht) module. Heat is generated volumetrically in the two whey gel strips. Heat is lost through all the external surfaces.

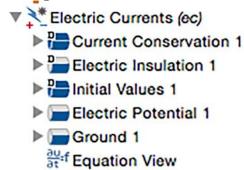


Figure 6. Electric Current (ec) Configuration

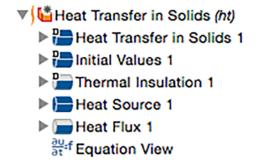


Figure 7. Heat Transfer in Solids (ht) Configuration

# 4. Solving the Dielectric Heating Model

The first step in solving the Dielectric Heating Model is to mesh the model. Figure 8 shows the models mesh.

The next step in the solution of the Dielectric Heating Model is to configure the solvers. Solution of this model requires the use of a two-step process. This Model is solved first in the Electric Currents Module using the Frequency Domain Solver. Next, it is solved in the Heat Transfer Module, using the Stationary Solver. Figure 9 shows the solver configuration.

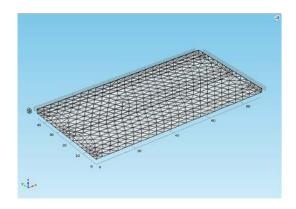


Figure 8. Dielectric Heating Model Mesh

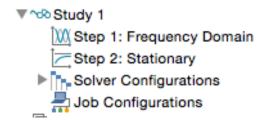


Figure 9. Solver Configuration

## **5. Dielectric Heating Solutions**

Figure 10, Figure 11, Figure 12, and Figure 13 show the calculated solutions for input frequencies of 2.45 GHz, 915 MHz, 40 MHz and 27 MHz, respectively.

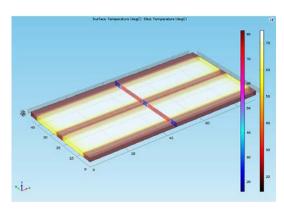


Figure 10. Model Calculation at 2.45 GHz

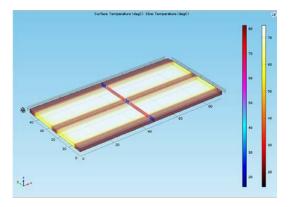


Figure 11. Model Calculation at 915 MHz

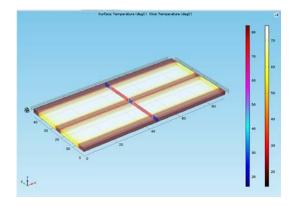


Figure 12. Model Calculation at 40 MHz

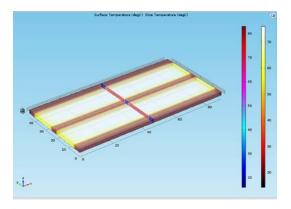
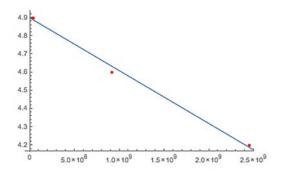


Figure 13. Model Calculation at 27 MHz

As can readily be observed, Figures 10, 11, 12, and 13 are almost identical. The input voltage was adjusted to achieve the same level of heating independent of frequency. Figure 14 shows the calculated data points for each test frequency and a Fit curve for the input voltage as a function of frequency.



**Figure 14**. Input Voltage vs. Frequency at a Constant Heating Level

The equation for the Input Voltage curve is:

$$V_{in} = 4.8991 - 2.90322 \times 10^{-10} * f_0$$
 (7)

where:  $f_0 = frequency$ 

#### 6. Conclusions

The dielectric heating process is a powerful tool. It is available for use over a broad spectrum of research and applications. The results of the modeling process are sensitive to the specific behavior of the material being modeled and needs to be explored thoroughly. In the case of materials with a large volumetric fraction of water, the required input voltage decreases as function of frequency, for the allocated industrial frequencies.

#### 7. References

- 1. Radio Regulations, 2012, ITU-R
- 2. Yifen Wang, et.al., J. Food Eng., 57 (2003) , pp. 257-268
- 3. https://en.wikipedia.org/wiki/ Dielectric\_heating
- 4. https://en.wikipedia.org/wiki/Joule heating
- 5. https://en.wikipedia.org/wiki/Permittivity