Benchmarking tailored formulations of multiphase flow in porous media

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Multiphase flow situations

New uses of the underground

• Geological gas storage
  – Hydrogen storage
  – Carbon capture and storage

• Oil and gas industry
  – Enhance Oil Recovery

• Nuclear waste storage

Source: SKB

Source: CRC for GHG (CO2CRC)

Source: GSE 2013
Multiphase flow formulation

**ADE system**

Component mass conservation

$$\partial_t (\phi S_\alpha \rho_\alpha m_\alpha^k M^k) = -\nabla (q_\alpha \rho_\alpha m_\alpha^k M^k - \phi S_\alpha \rho_\alpha D_\alpha \nabla (m_\alpha^k M^k)) + Q_\alpha^k + T_\alpha^k$$

$$q_\alpha = -\frac{k_k r_\alpha}{\mu_\alpha} (\nabla p_\alpha - \rho_\alpha g)$$

$\alpha = \text{wetting \& non-wetting phase}$

$k = \text{components}$

Typically 4 equations

**Constitutive equations**

**Total saturation**

$$S_n + S_w = 1$$

**Capillarity pressure**

$$P_{cap} = P_n - P_w$$

$$P_{cap} = P_{cap}(S_w)$$

**Relative permeability**

$$k_{\text{r}_l}, k_{\text{r}_g} = f(S_w)$$

**Gas volume**

$$V_g = V_g(p_g, T, m_{l_{CO_2}}, m_{NaCl})$$

**Liquid density**

$$\rho_b = \rho_b(p_l, T, m_{l_{CO_2}}, m_{NaCl})$$

**Viscosity**

$$\mu_g = \mu_g(p_g, T, m_{l_{CO_2}}, m_{NaCl})$$
Multiphase flow formulation

Modeler alternatives

• How to represent the capillary pressure
  – Van Genuchten
  – Brooks & Corey
  – Specific interfacial area

• Which unknowns will be solved
  – Phase pressure & Phase saturation \( (S_\alpha, P_\alpha \} \)
  – “Global” variables (total pressure or capillary pressure)

• Which combination of governing equations will be solved
  – Global equation, phase equation, linear combinations,…

• Which state variables will be used
  – Thermodynamic relations
**Multiphase flow formulation**

**COMSOL Implementation**

Non-wetting + wetting eq.

\[ 0 = \nabla \left( k \lambda_n \left( P_w + \frac{dP_{cap}}{dS_n} \nabla S_n - \rho_n g \nabla z \right) \right) + \nabla \left( k \lambda_w \left( P_w - \rho_n g \nabla z \right) \right) + q_n + q_w \]

Non-wetting eq.

\[ \phi \partial_t (S_n) = \nabla \left( k \lambda_n \left( P_w + \frac{dP_{cap}}{dS_n} \nabla S_n - \rho_n g \nabla z \right) \right) + q_n \]

Matrix form:

\[
\begin{pmatrix}
0 & 0 \\
0 & \phi
\end{pmatrix}
\frac{\partial}{\partial t} \begin{pmatrix}
P_w \\
S_n
\end{pmatrix}
+ \nabla \cdot \left[
\begin{pmatrix}
\lambda & \lambda_n \frac{dP_{cap}}{dS_n} \\
\lambda_n & \lambda_n \frac{dP_{cap}}{dS_n}
\end{pmatrix}
\nabla \begin{pmatrix}
P_w \\
S_n
\end{pmatrix}
- kg \begin{pmatrix}
\lambda_w \rho_w + \lambda_n \rho_n \\
\lambda_n \rho_n
\end{pmatrix}
\right] = \begin{pmatrix}
q_n + q_w \\
q_n
\end{pmatrix}
\]

**COMSOL Coefficient Form PDE**

\[
e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \left( -c \nabla u - \alpha u + \gamma \right) + \beta \cdot \nabla u + au = f
\]
Multiphase flow

Benchmarking – Buckley-Leverett problem

- 1D displacement of oil by water
- Immiscible fluids
- No capillary pressure
- Various fluid viscosities
- Gravity effects neglected
- Homogeneous porous media

Buckley & Leverett, 1942
Multiphase flow

Benchmarking – Buckley-Leverett problem

Water Saturation after 300 days

\[ \frac{\mu_W}{\mu_O} = 1 \]

\[ \frac{\mu_W}{\mu_O} = 2 \]

\[ \frac{\mu_W}{\mu_O} = \frac{2}{3} \]
Multiphase flow

Benchmarking – McWorther problem

- 1D displacement of oil by water
- Immiscible fluids
- Capillary pressure
- Equal fluid viscosities
- Gravity effects neglected
- Homogeneous porous media

McWhorter & Sunada, 1990
Multiphase flow

Benchmarking – McWorther problem

Water Saturation after 2.78 hours
Multiphase flow

Benchmarking – Five spot problem

- 2D displacement of oil by water
- Immiscible fluids
- No capillary pressure
- Equal fluid viscosities
- Gravity effects neglected
- Homogeneous porous media

Chen, et al., 2006
Multiphase flow

Benchmarking – Five spot problem

Water Saturation

Time = 0 d

300 m

0.8

0.7

0.6

0.5

0.4

0.3

0.2

300 m
Multiphase flow

Benchmarking – CCS well leakage

- 2D model simplification of the 3D problem
- Immiscible fluids
- No capillary pressure
- $\rho$, $\mu$ are constant
- Isothermal

Ebigbo et al., 2007
Multiphase flow

Benchmarking – CCS well leakage

Time=0

CO₂ Saturation

30 m

100 m

30 m

500 m

500 m

30 m

100 m

30 m

500 m
Summary

• Formulations reproduced multiphase physical processes
• equations of state of oil, water, brine and $\text{CO}_2^{\text{sup}}$
  – easily extended to any other fluid

• The manner the equations are combined matters
• Each formulation has its own benefits and drawbacks
• The preferred may vary depending on the physics and numerical methods
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