Iterative Learning Control for Spatio-Temporal Repetitive Processes

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Introduction

Process repeatability

- the same tracking error, oscillations and overshoot produced along each replicated trial,
- increasing tracking performance with knowledge of repetitive signals.
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Process repeatability
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- increasing tracking performance with knowledge of repetitive signals.

Challenges
- compensation of random disturbances,
- general control scheme to the repetitive spatio-temporal process.
Subject of the talk

use data from repetition of the same process controlled by PID several times to improve:

- quality of control,
- robustness with respect to model uncertainty.
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Motivations

- self–learning methodology,
- feedforward signals for subsequent trials through iterative update,
- high performance with low cost (transient tracking error),
- objects with a lot of measurement points,
- repetitive processes,
- and many more . . .
Consider $y_d(t)$ which denote a continuous reference trajectory defined over a finite time interval $T = [0, t_f]$, where $t_f < \infty$ denotes the trial length, then typical Iterative Learning Control law

$$v_{k+1}(t) = \mu v_k(t) + \eta \dot{e}_k(t)$$

where

- $k \geq 0$ — trial or cycle number,
- $v(t)$ — the system input along the trial,
- $\mu$ — momentum coefficients,
- $\eta$ — learning coefficients,
- $y_k(t)$ — system output,
- $e_k(t) = y_d(t) - y_k(t)$ — tracking error.
Learning controller

Could be splits into

- \( L \) — learning filter, inverse of process sensitivity,
- \( Q \) — low pass filter,
- \( P \) — object plant,

\[
\begin{align*}
L & \quad + \quad + \\
\quad & \quad + \\
\quad & \quad memory \\
\quad & \quad + \\
C \quad & \quad + \\
\quad & \quad - \\
P \quad & 
\end{align*}
\]
Illustrative example

Gas combustion chamber

- three dimensional model,
- inlet with constant concentration and velocity,
- inlet with constant concentration and controlled velocity,
- one outlet,
- mixing to achieve effective combustion (inside point).
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![Diagram of a gas combustion chamber with indicated inlet and outlet points.](image-url)
Illustrative example

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Mathematical model of the problem

- fluid flow: Navier-Stokes equations,
- mass balance: convection and diffusion application

\[
\rho \frac{\partial \mathbf{u}}{\partial t} - \nabla \cdot [\eta (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{u} p = \mathbf{F},
\]

\[\nabla \cdot \mathbf{u} = 0,\]

\[\delta_{ts} \frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) = R - \mathbf{u} \cdot \nabla c,\]

where
- \(\rho [\text{kg/m}^3]\) — density,
- \(\mathbf{u} [\text{m/s}]\) — velocity vector,
- \(\nabla\) — gradient operator
- \(\mathbf{F} [\text{N/m}^3]\) — volume force vector,
- \(c [\text{mol/m}^3]\) — concentration,
- \(\eta [\text{kg/m}^3]\) — dynamic viscosity,
- \(p [\text{Pa}]\) — pressure at output,
- \(R [\text{mol/(m}^3\text{s)}]\) — reaction rate,
- \(D [\text{m}^2\text{s}]\) — diffusion coefficient,
- \(\delta_{ts}\) — time scaling coefficient.
Boundary conditions

For mass balance:
- $c_t [\text{mol/m}^3]$ — concentration at upper input,
- $c_c [\text{mol/m}^3]$ — concentration at controlled input,
- $\mathbf{n} \cdot (-D \nabla c) = 0$ — output boundary condition,
- $\mathbf{N} \cdot \mathbf{n} = 0$ — for walls where molar flux $\mathbf{N} [\text{mol/m}^2 \cdot \text{s}]$

For fluid flow:
- $\mathbf{u} = (0, -u_t, 0)$ — constant inlet,
- $\mathbf{u} = (u_c, 0, 0)$ — controlled inlet,
- $p_0 = 0$ — pressure at output,
- $\mathbf{n} \cdot \mathbf{n} = 0$ — inlet sections,
- $\mathbf{u} = 0$ — walls.

transport of the reactants at the outlet and dispersal in the main direction of the convective flow was neglected.
Simulations results – concentration level

Time=1[s] Concentration, c [mol/m$^3$]
Simulations results – concentration level

Time=2[s] Concentration, c [mol/m³]
Simulations results – concentration level

Time=3[s] Concentration, c [mol/m³]
Simulations results – concentration level

Time=4[s] Concentration, c [mol/m^3]
Simulations results – concentration level

Time=5[s] Concentration, c [mol/m³]

[Graph showing concentration distribution over a time of 5 seconds]
Simulations results

Simple PID and constant reference tracking

Iterative Learning Control in 1\textsuperscript{st} and last trial
Simulations results

Error reached in 1\textsuperscript{st} and last trial of ILC of ILC

Error norm in each trial of ILC
Conclusion

- Summary of the contributions provided by this work to the state-of-the-art:
  - iterative learning control for distributed parameter system was presented as an promising approach for the improvement of control quality
  - control scheme was illustrated on the application to the fluid dynamics with the mass transport as an example of real chemical process.
- Further work:
  - more general methodology for combining sequential design and ILC techniques in order to increase the control quality for processes,
  - extensions to more wider range of systems,