Multiphysics Based Electrical Discharge Machining Simulation

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1.Introduction

Electric Discharge machining is a non-conventional process with good surface finish. It has found wide application in making intricate products like making moulds, aerospace parts and in surgical instruments [1]. The principle of working of wire EDM is based on material removal by applying series of repeated electric discharges between the work piece and the wire which serves as electrodes. The electric discharge takes place in the dielectric fluid which also acts as a flushing agent in wire EDM process. Commonly used dielectrics are deionized water, kerosene. The gap between the wire and the workpiece is maintained within (usually 20-40 microns) and voltage impressed causes ionization of the dielectric and hence the spark is generated. The material gets melted and removed due to the erosive effect of the electrical discharge. Its unique feature of using thermal energy to machine electrically conductive parts regardless of hardness has been its distinctive advantage in the manufacture of mold, die, and automotive aerospace and surgical components [6].

However, it suffers from few limitations such as low machining efficiency and poor surface finish. To overcome these limitations, a number of efforts have been made to develop such EDM systems that have capability of high material removal rate (MRR), high efficiency, high accuracy and precision without making any major alterations in its basic principle [7]. This method is defined as removing materials from a part by means of a series of repeated electrical discharges between tool called the electrode and the workpiece in the presence of a dielectric fluid [8]. This thermal erosion process is studied using hyperbolic heat conduction process on very short time the heat impulse works in the workpiece (ASTM 387Gr-91). Consequently a comparison is made between numerical and experimental results.
2. Mathematical Model

2.1 Problem specification:

Electric discharge machining process uses electric discharge for cutting the material. The maximum heat flux experienced by the workpiece due to interaction with the electric spark can be represented as [9].

\[ q_0 = \frac{4.56F_c VI}{\pi R_p^2} \]  

(1)

Where \( F_c \) is the fraction of power of EDM spark coming to the workpiece, \( V \) is the discharge voltage, \( I \) is the discharge current, and \( R_p \) is the spark radius.

The radius of the spark on the workpiece can be represented as

\[ R_p = 0.788 t_d^{3/4} \]  

(2)

Where \( t_d \) is the discharge time in \( \mu \) seconds.

The discharge time is 10-30 \( \mu \) seconds. It depends upon the conductivity of the material. Material removal rate (MRR) gets optimised by varying the discharge time.

2.2 Governing Equation and Boundary Conditions

Fraction of power of EDM spark that gets delivered to the cathode i.e. the workpiece is nearly 30%[2] rest gets utilised in expansion of the plasma channel and some energy is dissipated through the flushing action of the dielectric. The voltage applied is 200V across a gap of 40\( \mu \)m at the time of operation causing the dielectric to breakdown. The voltage then falls to 25v and the current rises to a constant value set by the operator [2].

The pulse on time for which the spark is on is usually less than 100\( \mu \)s. This high energy generates a plasma spark discharge surrounded by vapour bubble, which expands during the “on time” of the spark. Since the dielectric being a dense liquid restricts the expansion of the plasma channel, causing the input energy (V.I.t) confined in a very small volume. The local plasma temperatures reach as high as 20,000-40,000K due to energy densities of up to 3J/mm\(^3\). The heat flux entering the workpiece from the spark is of the order of 10\(^{11}\) W/m\(^2\). For such high flux interacting with the workpiece within very small interval of time Fourier heat conduction fails to predict the temperature distribution on the workpiece. Cattaneo and Vernotte [3-5] have independently, proposed a modification in Fourier’s law which is popularly known as Cattaneo-Vernotte’s constitutive equation

\[ q + \tau \frac{\partial T}{\partial t} = -k.\nabla T \]  

(3)

Where \( q \) is the heat flux vector, \( \tau \) is the thermal relaxation time; \( k \) is the thermal conductivity of workpiece. When energy equation is combined with equation 3 the governing equation gets modified to hyperbolic heat conduction equation very much similar to telegraph equation used.
in wave propagation problems similar to transmission line problem. It relates the voltage and current on an electrical transmission line in the form of set of coupled differential equations. The final equation takes can be stated as

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \nabla T$$  \hspace{1cm} (4)

Where $\alpha = \frac{k}{\rho c}$, $c$, $\nabla$ are thermal diffusivity, mass density, heat capacity and Laplace differential operator respectively.

The hyperbolic differential equation sets the propagation speed of wave to a finite value, thus reaching a limit amounting to $\sqrt{\alpha/\tau}$, in $\tau > 0$. Several papers have been proposed for the solution of telegraph equation by various methods like finite difference method, differential quadrature method, interpolation scaling function, Lt inversion technique etc. This paper aims to Finite Element Method (FEM) solution to the telegraph equation using COMSOL multiphysics.

The present work studies both Fourier and Non Fourier heat conduction models. A complete comparative analysis is performed to understand the behaviour of EDM plasma channel.

2.2. (a) Fourier heat conduction model

In parabolic or Fourier heat conduction model heat signal travels with infinite speed, i.e. temperature gradient applied to one end is felt immediately to infinite distance on the other end. Energy balance for a rigid material without any heat generation can be written as

$$\rho c \frac{\partial T}{\partial t} + \nabla \cdot q = 0$$  \hspace{1cm} (5)

Where $\rho$ the density $c$ is the specific heat at a given temperature $T$ and $q$ is the flux vector. Submitting Fourier linear approximation of heat flux as

$$q = -k \nabla T$$  \hspace{1cm} (11)

Leads to well-known Fourier equation of heat conduction.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$  \hspace{1cm} (7)

Heat flux supplied to the workpiece is generated by applying voltage pulses; voltage pulse can be square or Gaussian distribution. The flux applied is given by the equations given below

$$q = q_0 \exp \left(-4.45 \frac{r^2}{R_p^2} \right)$$  \hspace{1cm} (8)

Governing equation

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_u \nabla T = \nabla \cdot (k \nabla T) + Q$$  \hspace{1cm} (9)

Boundary conditions

$$k \frac{\partial T}{\partial x}(x, 0, t) = q$$  \hspace{1cm} (10)

$$k \frac{\partial T}{\partial x} + h[T - T_\infty] = 0$$  \hspace{1cm} (11)

For all boundaries of workpiece in contact with water

Initial conditions

$$T(x, y, 0) = T_\infty$$  \hspace{1cm} (12)
Where \( \rho, C_p, q, T, t, Q, k, u, h \) are density, heat capacity, heat flux(pulsed), Temperature time, Internal energy, Thermal conductivity, velocity vector, heat transfer coefficient respectively. \( T_\infty = 273.16 \) K

### 2.2. (b) Non-Fourier heat conduction model

The hyperbolic heat conduction is very much similar to the relativistic heat conduction. It acknowledges the finite speed of heat propagation. It is based on microscopic evidence from the kinetic theory and statistical mechanics. The equation (7) show Fourier model assumes infinite speed of heat conduction. This means that a thermal disturbance can be detected instantaneously at an infinitely far distance from the source, which is physically unacceptable. One of the implications of the theory of relativity is that the principle of “no action at a distance” which requires finite speed of propagation of any signals, and necessitates a time lag between a cause and its effects. Thus the final modified equation for hyperbolic heat conduction can be written as

\[
\frac{1}{C^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\alpha} \frac{\partial T}{\partial t} = \nabla^2 T \tag{13}
\]

Where \( C^2 = \frac{\alpha}{\tau} \) C is called the second speed of sound in the medium, \( \alpha \) and \( \tau \) are thermal diffusivity and the thermal relaxation time of the heat conducting medium (free electrons in the case of metals) respectively.

### Governing equation

\[
\frac{\tau}{\alpha} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \tag{14}
\]

### Boundary Conditions

\[
k \frac{\partial T}{\partial y}(x, 0, t) = q \tag{15}
\]

\[
k \frac{\partial T}{\partial x} + h[T - T_\infty] = 0 \tag{16}
\]

At all boundaries of work piece in contact with water.

### Initial conditions

\[
T(x, y, 0) = T_\infty \tag{17}
\]

\[
\frac{\partial T}{\partial \tau}(x, y, 0) = 0 \tag{18}
\]

Where \( \tau, \alpha, q, h, T, t \) are Thermal relaxation time, thermal diffusivity flux, heat transfer coefficient, temperature, time respectively.

\( T_\infty = 273.16 \) K

### 2.3 Assumptions

EDM being a highly complex process some assumptions have been made for proper mathematical formulation of the problem.

1. Electric spark has been considered as concentrated heat source.
2. Material properties of the workpiece are temperature independent.
3. Heat transfer from the spark to the workpiece is only by means of conduction.
4. The size of workpiece is such that heat transfer is considered along one dimension only.
3. Use of COMSOL Multiphysics

The study of heat transfer in EDM includes hyperbolic heat transfer model resembling the telegrapher’s equation. As heat is also getting dissipated to dielectric heat transfer in fluids is also to be taken into consideration. COMSOL Multiphysics perfectly meets all the requirements of the problem.

3.1 Coefficient form of PDE

The coefficient form of PDE module has been used to simulate the heat transfer process from the plasma to the workpiece. It is applied to domain 1 and 2 i.e. the wire and the workpiece. The heat transfer being hyperbolic in nature, the coefficients have been defined in parameters with proper units.

To simulate the heat transfer the flux is provided in the form of step function, a step function is defined in the ‘Definitions’ and applied to one boundary of domain 1 which is treated as wire used for cutting the material.

3.2 Heat transfer in fluids

‘Heat transfer in fluids’ module is employed to domain 3 only. While applying the boundary conditions to the model outer boundary of domain 3 is kept insulated to avoid direct heat transfer from wire to dielectric. The heat energy coming to the cathode travels through domain 1 to domain 2 and finally to the water domain 3. To incorporate this type of heat transfer the two physics are coupled by using dirichlet’s and temperature boundary conditions.

3.3 Coupling of two physics

‘Coefficient form of PDE’ and ‘Heat transfer in fluids’ are coupled, so that smooth transition of heat can take place. At the interface of the domains the variables of the problem, here temperature is defined in interchanged manner. The boundary condition for domain 1 at the interface of domain 1 and 2 is taken to be the variable defining the temperature of domain 3b. In ‘heat transfer in fluids’ this coupling is done by employing ‘Temperature boundary conditions’. In coefficient form of PDE this is achieved by applying the dirichlet’s boundary condition.
3.4 Material properties

The Temperature and Material removal rate varies w.r.t materials. Thus material property is one of the most important features of the electric discharge machining EDM. The electrical and thermal conductivity of the material plays a major role in cutting process. The material properties for the workpiece are fed automatically by adding the material from the material library. The properties of water which is used as a dielectric is also fed from the material library. The additional materials properties are defined in ‘definitions’. Domain 2, 3 are ASTM A213/SA-387(9Cr Mo or G-91)[solid annealed] while domain 1 is water.

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>55[W/m*K]</td>
<td>55.000 kg·m·K/s</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>rho</td>
<td>7850[kg/(m³)]</td>
<td>7850.0 kg/m³</td>
<td>density</td>
</tr>
<tr>
<td>cp</td>
<td>470[J/(kg·K)]</td>
<td>470.00 J/(kg·K)</td>
<td>Specific heat capacity</td>
</tr>
<tr>
<td>tau</td>
<td>1e-6[s]</td>
<td>1.0000E-6 s</td>
<td>Thermal relaxation time</td>
</tr>
<tr>
<td>alpha</td>
<td>k/(rho*cp)</td>
<td>1.4907E-5 m²·K²/s</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>c1</td>
<td>1/alpha</td>
<td>67082 s/(m²·K²)</td>
<td>Coefficient 1</td>
</tr>
<tr>
<td>c2</td>
<td>tau/alpha</td>
<td>0.067082 s²/(m²·K²)</td>
<td>Coefficient2</td>
</tr>
<tr>
<td>q0</td>
<td>1e9[W/m²]</td>
<td>1.0000E9 W/m²</td>
<td>Maximum heat flux</td>
</tr>
</tbody>
</table>

Table 2 Cutting parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap current</td>
<td>1.50 A</td>
</tr>
<tr>
<td>Voltage</td>
<td>150 V</td>
</tr>
<tr>
<td>Arc efficiency</td>
<td>30%</td>
</tr>
<tr>
<td>Cutting time</td>
<td>1 s</td>
</tr>
<tr>
<td>Sample size</td>
<td>10x5x5 mm³</td>
</tr>
</tbody>
</table>

4. Results and Discussion

Parabolic heat conduction allows for the infinite speed of propagation of heat waves signals. This restricts the temperature of a spatial point to rise to higher value corresponding to the heat flux applied. This can be better understood in a way that as the heat signals arising from the source interacts with the point near the boundary of flux application; it gets immediately transferred to the next element without raising the temperature of the adjacent element to the expected limit. Thus the temperature gradient of a particular point of the sample does not show an agreement with the flux. The spatial and temporal variation of temperature at a particular time say 0.005 has been shown on the chapter. This show for a flux of the order of $10^{11}$ W/m² the temperature at the point of flux application is 1200 K, which is very near to the melting temperature of P-91.

An experiment was carried out for obtaining the temperature of the plasma channel of the EDM. Optical emission spectroscopy with ‘Line pair method’ was implemented to obtain the temperature of the plasma channel. The experimental observation shows the temperature of the EDM plasma channel is of the order of 16000-18000K. Literature survey shows that only 30% of the heat generated in the plasma channel interacts with the specimen. Rest heat gets lost due to flushing action of the dielectric and radiation losses. A simple calculation shown in analytical solution section shows the temperature at the point of flux application reaches to 4000-6000K.

The Numerical modeling is done with COMSOL 5.0. The results with
comparative plots are presented below. A Comparative is also drawn to understand the behavior of system under both Fourier heat conduction model and Non Fourier heat conduction model.

Figure 3 Surface plot of Temp. at 1.2x10^{-4} sec (Non-Fourier heat conduction model)

Figure 4 Surface plot of Temp. at 5x10^{-4} sec (Non-Fourier heat conduction model)

Figure 5 Surface plot of Temp. at 0.002 sec (Non-Fourier heat conduction model)

The figure shows the comparison of Fourier and Non Fourier heat conduction. For hyperbolic heat conduction the temperature at the pint of flux application rises and then falls rapidly w.r.t time. The rate of change of temperature changes after some time and approaches towards the parabolic heat conduction behaviour. The time at which rate of change of temperature changes depends on the thermal relaxation time.

Figure 6 Temperature distribution on the workpiece (Fourier vs. Non Fourier comparison)

The behavior of the system at initial time of flux application is addressed in fig 7.8. It shows for Non-Fourier heat conduction the rise in temperature at a point is faster than that of Fourier heat conduction. As already explained the infinite speed of heat signals in parabolic heat conduction accounts for the low temperature gradient.

Figure 7 Comparison of Temperature rise at the point of Flux application

Spatial temperature variation with time governed by hyperbolic heat conduction is shown in fig 7.9. Temperature at different
points which are at a distance 0, 0.05mm, 0.2mm, 0.5mm, 0.6mm, 1mm, 2mm respectively from the cutting line.

Figure 8 Temperature at different from the cutting line (Fourier and Non-Fourier heat conduction comparative plot)

The plot shows that for far away points the behavior is similar to parabolic heat conduction. Temperature distribution is modeled up to 5 msec. Within such a small time interval the temperature falls from 3900K to 340K. This is validated through experiment using thermocouples at points 1mm, 2mm, 3mm, 3.5mm at time 5msec. The Numerical simulation results are found in good agreement with the experimental observations. The temperature at point of flux application is validated using optical emission spectroscopy results. The table 3 shows the results experiment.

Table 3 Experimental results

<table>
<thead>
<tr>
<th>Distance from point of flux application</th>
<th>Experimental observation of temperature (using K-type thermocouples) at Time (0.005sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm</td>
<td>303 K</td>
</tr>
<tr>
<td>2 mm</td>
<td>296 K</td>
</tr>
<tr>
<td>3 mm</td>
<td>295 K</td>
</tr>
<tr>
<td>3.5 mm</td>
<td>294 K</td>
</tr>
</tbody>
</table>

The results of parabolic heat conduction shows exaggerated temperature distribution in the work piece, though the hyperbolic heat conduction also gives similar temperature gradient profile after some time. But using parabolic heat conduction the high temperature of the plasma channel cannot be explained properly. This also fails to explain properly the cutting process. The Non Fourier heat conduction thus explains the true behaviour of the cutting process. For proper temperature gradient behavior corresponding to the heat flux applied the rise in temperature during period of flux application For Parabolic heat conduction the temperature rise is not fast and for low heat flux even the temperature does not reaches melting temperature properly. For points far away from the cutting point the

Excerpt from the Proceedings of the 2015 COMSOL Conference in Pune
temperature rise though similar to that of hyperbolic heat conduction.

5. Conclusion

Electric Discharge machining process involves very high Flux (order of $10^{11}$ W/m²) applied for very small interval of time. Model proposed here well describes the heat conduction occurring from the plasma channel to the workpiece. Moreover the temperature distribution obtained from numerical solution is found in good agreement with the experimental data. Heat conduction in such system occurs with finite speed of propagation of heat signals. The numerical simulation results confirm the wavy nature of the heat signals in Non–Fourier heat conduction. Future works includes improving the model so as to reach as close as possible to experimental results

6. References


