Numerical Analysis of Different Magnet Shapes on Heat Transfer Application using Ferrofluid

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Abstract: Ferro fluid cooling system is the next generation passive cooling systems for micro heat transfer applications. The Ferro fluid cooling system works under thermo magnetic convection using virtual pump by using Ferro fluid & external Magnet. The flow of the fluid is significantly dependent upon the position of the magnet & flux lines. The behavior changes accordingly & thus flow rate varies. In the proposed work, a uniform flow of fluid is achieved by using different shapes of the magnets irrespective of the change in position. The work discusses numerical & experimental analysis to study the heat transfer and flow characteristic of ferro-fluid by changing the shapes of magnets, which leads to different magnetic line paths. It was concluded that convex shape magnet gives better solution as compared to other shapes of magnet and a velocity of 1.592 mm/sec and fluid temperature of 331 K was obtained.

KEYWORDS: Ferro fluid, Magnetic flux, Magnet Shape, Temperature.

1. Introduction:

Magnetic fluid, which also known as a ferrofluid, is a colloidal suspension of single domain magnetic particles, with dimension of particle about 10 nm dispersed in a liquid carrier. The stability of the magnetic colloid depends on the thermal contribution and on the balance between attractive (Vander Waals and dipole-dipole) and repulsive (Steric and electrostatic) interactions. In order to avoid agglomeration, the magnetic particles have to be coated with a shell of an appropriate material like oleic acid.

A ferrofluid is a temperature sensitive magnetic fluid which means that its magnetization is function of temperature. An external magnetic field imposed on a ferrofluid with varying susceptibility (e.g. because of a temperature gradient) results in a non-uniform magnetic body force, which leads to a form of heat transfer called thermo-magnetic convection[1] and can be controlled by varying ferrofluid properties, the magnetic field strength and also the temperature distribution. This form of heat transfer can be useful when conventional convection heat transfer is inadequate; e.g., in miniature microscale devices or under reduced gravity conditions. It is a passive cooling technique and the heat from the system which otherwise is a waste can be used to induce the flow in presence of a magnetic field. The synergistic effect of magnetization and temperature gradient produces a high performance cooling. A prototype of a miniature automatic cooling device using ferrofluid has previously been been described by Love et al. [2] & Qiang Li at al. [3].

Strek and Jopek [4] simulated ferrofluid flow in a channel between two parallel plates. The flow state in a magnetic fluid [5] heat transport device is investigated numerically. A device with simple model geometry is considered, when it is placed vertically. The qualitative explanation is made from the results for the flow state of experimental device when the magnetic field is affected. Aminfar et al. [6] have investigated numerically the hydrothermal characteristics of a water based ferrofluid in vertical rectangular duct. A wire carrying the current along the length of the duct is placed to produce the magnetic field.

Wrobel et al. [7] studied thermo-magnetic convective flow of paramagnetic fluid in an annular enclosure with a round rod core and a cylindrical outer wall numerically and experimentally. Their results show, the magnetic convection of paramagnetic fluid can be controlled by strong magnetic field and the magnetizing force affects the heat transfer rate.

Kikura et al. [8] carried out experimental investigations in a cubical enclosure and concentric horizontal annuli under the influence of a varying magnetic field. The permanent magnet was placed at different sides of the enclosure and the effect of magnetic field gradient on the ferrofluid heat transfer was studied.

Lajvardi et al. [9] report an experimental work on the convective heat transfer of ferrofluid flowing through a heated copper tube in the laminar regime in the presence of magnetic field. A series of experiments were conducted to study the effect of external magnetic field and temperature distribution. Their results showed that fluid flow can be controlled by changing the

Excerpt from the Proceedings of the 2015 COMSOL Conference in Pune
position of the magnet. Significant enhancement on the heat transfer of ferrofluid by applying various orders of magnetic field is observed in this experiment. Also by increasing the ferrofluid concentration, a significant enhancement in heat transfer coefficient is observed. Although many theoretical and experimental investigations have been carried out for ferrofluid flow phenomena but the investigations on the dependence ferrofluid flow on the conductivity and orientation of magnetic field are sparse. These factors are discussed in this paper.

Further, it is assumed that conductivity can be varied without affecting the concentration of the fluid.

2. Governing Equation:

Basic equations governing the ferrofluid flow are continuity equation, momentum equation, energy equation. The magnetostatic equation and Kelvin body force are explained below:

2.1. Continuity, Momentum, Energy and Magnetostatic equations

The mass conservation equation is given as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$  \hspace{1cm} (1)

Ferrofluids are generally incompressible, i.e.

$$\rho \nabla \cdot \mathbf{u} = 0$$  \hspace{1cm} (2)

Momentum equation is given by the following equation

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nabla \cdot (\mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} + \mathbf{F}$$  \hspace{1cm} (3)

Momentum equation for incompressible fluid

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nabla \cdot (\mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) + F$$  \hspace{1cm} (4)

$$F = (M. \nabla)B + \rho g$$  \hspace{1cm} (5)

Using force, the momentum equation for ferrofluid reduces to

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mu ((\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + (M. \nabla)B$$  \hspace{1cm} (6)

Permeability and relative permeability of the medium

$$\mu = \mu_0 (1 + \chi_m), \mu_r = \frac{\mu}{\mu_0}$$  \hspace{1cm} (7)

Magnetization of fluid

$$M = \chi_m H$$  \hspace{1cm} (8)

Magnetic flux density

$$B = \mu H$$  \hspace{1cm} (9)

Magnetic field

$$H = -\nabla \mathbf{v}_m$$  \hspace{1cm} (10)

$$\nabla \cdot B = 0$$  \hspace{1cm} (11)

Magnetic flux density due to Permanent Magnet

$$B = \mu H + B_r$$  \hspace{1cm} (12)

Where, $B_r$ is the remanent flux density of the permanent magnet (neodymium magnet NdFeB), which was placed in the position show in Figure 3, and little gap was maintained between the magnet and the heat source.

2.2. Kelvin body force

Nano particles in the magnetic fluid get magnetized in the presence of magnetic field. These magnetized ferrofluid particle experiences magnetic force in the vicinity of the magnetic field. By virtue of the stabilized suspension of ferrofluid particles in the liquid, the attractive magnetic force manifests itself as a body force on the liquid, analogous to the body force on a liquid due to gravity.

$$F = (M. \nabla)B$$  \hspace{1cm} (13)

Using value of $M$ and $B$ from the body force it can be written as

$$F = \mu_0 (\chi_m H \cdot \nabla) (1 + \chi_m) H$$  \hspace{1cm} (14)

Here $\chi_m$ is temperature dependent magnetic susceptibility.

$$\chi_m = \chi_m(T) = \frac{\chi_0}{1 + a(T - T_0)}$$  \hspace{1cm} (15)

The energy equation for incompressible ferrofluids follows modified Fourier’s law.

$$\rho c_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \nabla T \right) = k \nabla^2 T - \mu_0 \nabla \cdot \mathbf{M} \left( (\mathbf{v} \cdot \nabla) H \right)^2 + \mu \Phi$$  \hspace{1cm} (16)

Where, $\mu \Phi$ is the viscous dissipation is defined as

$$\Phi = 2 \left( \left( \frac{\partial u_x}{\partial x} \right)^2 + \left( \frac{\partial u_y}{\partial y} \right)^2 + \left( \frac{\partial u_z}{\partial z} \right)^2 \right) + \text{Excerpt from the Proceedings of the 2015 COMSOL Conference in Pune}$$
\[
\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial y}\right)^2 + \left(\frac{\partial u_y}{\partial x} + \frac{\partial u_z}{\partial y}\right)^2 + \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_z}{\partial x}\right)^2 - \frac{2}{3} (\nabla \cdot u)^2 \right) \tag{17}
\]

**Table 1: Analysis of Different Magnet Shapes**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (Kg/m(^3))</td>
<td>1050.0</td>
</tr>
<tr>
<td>Viscosity (Pa-s)</td>
<td>0.0030</td>
</tr>
<tr>
<td>Susceptibility</td>
<td>0.3860</td>
</tr>
<tr>
<td>Thermal conductivity (Wm(^{-1})K(^{-1}))</td>
<td>0.1500</td>
</tr>
<tr>
<td>Specific heat (J Kg(^{-1})K(^{-1}))</td>
<td>1715.0</td>
</tr>
<tr>
<td>Thermal expansion coefficient (K(^{-1}))</td>
<td>0.0009</td>
</tr>
<tr>
<td>Curie temperature (K)</td>
<td>353.00</td>
</tr>
</tbody>
</table>

3. **Methodology and Numerical Analysis**

Analysing such problem is challenging because it involves fluid dynamics, thermal boundary conditions and temperature-dependent magnetic properties. COMSOL Multiphysics is an ideal tool that provides rich insight into the system’s behavior. The approach was to develop a model with different shapes of magnet and study their behaviour on ferrofluid while its takes heat from the heat source. The experiments involved studying the effect of different magnet shapes on ferrofluid on parameters like temperature and velocity induced in setup. These experiments tested 1) the effect of different shapes of magnet on ferrofluid; and 2) the magnetic field lines coming out of the magnet. In the model, temperature-dependent magnetic properties were incorporated into the force component of the momentum equation, which was coupled to the heat transfer module. Model calculations showed that the greatest force acted along the axis of the dipole field with the convex shape of the magnet and that the magnet is best placed midway between the heat source and sink.

![Figure 1. Different component of setup](image1.png)

The equations governing the ferrofluid flow under the effect of applied magnetic and gravitational field are magneto static equation, the mass conservation equation, momentum equation and the energy equation in the frame of Boussinesque approximation.

4. **Results & Discussions:**

Using Ferro-fluid in integration with heat and magnetic field can provide better solution to heat transfer problems.

**Table 2: Analysis of Different Magnet Shapes**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Shape of the magnet</th>
<th>Velocity (mm/sec)</th>
<th>Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Concave</td>
<td>1.532</td>
<td>319</td>
</tr>
<tr>
<td>2.</td>
<td>Convex</td>
<td>1.592</td>
<td>321</td>
</tr>
<tr>
<td>3.</td>
<td>Tapered</td>
<td>1.986</td>
<td>314</td>
</tr>
<tr>
<td>4.</td>
<td>Rev. Tapered</td>
<td>1.956</td>
<td>314</td>
</tr>
<tr>
<td>5.</td>
<td>Trapezoidal</td>
<td>1.137</td>
<td>328</td>
</tr>
<tr>
<td>6.</td>
<td>Rev. Trapezoidal</td>
<td>1.267</td>
<td>326</td>
</tr>
</tbody>
</table>

![Figure 2. Temperature plot for different shapes](image2.png)

![Figure 3. Velocity plot for different shapes](image3.png)

The synergize effect of magnetization and temperature gradient produces a flow & thus cooling takes place. In ferrofluid setup, the uniform flow of
the fluid is dependent upon the position & shape of the magnet. The results obtained after carrying out the analysis are in Table 2. The analysis of different shapes is shown and it was noticed that the convex shape has high performance in terms of optimized velocity and high fluid temperature as shown in Figure 4 and Figure 5 respectively. The magnetic flux lines are shown in Figure 6.

Figure 4. Velocity Contour in convex shape

Figure 5. Temperature in convex shape

Figure 6. Magnetic flux in convex shape

The magnetic flux lines for all the different shapes are shown below in all the different analysis carried out for different shapes from Figure 7 to Figure 11.

Figure 7. Concave shape of magnet

Figure 8. Tapered shape of magnet

Figure 9. Reverse tapered shape of magnet

Figure 10. Trapezoidal shape of magnet
5. Conclusion

- The positional optimization helped in achieving more flow rate within the same space and volume.
- Magnetic body force acting on the ferrofluid depends upon the shape and strength of the field line from the magnet.
- Magnetic body force induces the flow which reduces the temperature of the system for this study.
- The model was able to calculate reasonable magnitude of velocity and temperature. The higher velocity found with convex shape 1.532 mm/sec for 3mm inner diameter was obtained.

6. Acknowledgement

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7. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>Density (kg m⁻³)</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Temperature (K)</td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>Velocity (m s⁻¹)</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity (W m⁻¹ K⁻¹)</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>Magnetization vector (A m⁻¹)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Magnetic flux density (Wb m⁻²)</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Magnetic field (A m⁻¹)</td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>Magnetic permeability (Wb m⁻¹)</td>
<td></td>
</tr>
<tr>
<td>μ₀</td>
<td>Magnetic permeability of free space</td>
<td></td>
</tr>
<tr>
<td>χₐ₀</td>
<td>Total magnetic susceptibility</td>
<td></td>
</tr>
<tr>
<td>χ₀</td>
<td>Differential magnetic susceptibility of fluid</td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>Thermal expansion coefficient of fluid (°C⁻¹)</td>
<td></td>
</tr>
<tr>
<td>T₀</td>
<td>Reference temperature (K)</td>
<td></td>
</tr>
</tbody>
</table>

8. References