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Introduction

- Energy Harvesting
- Thermoelectricity

*Three Effects*

1.) Seebeck Effect

Open circuit voltage (*Averaging Seebeck coefficient*)

\[
V_{OC} = \int_{T_c}^{T_h} \alpha(T) \, dT
\]

\[
\alpha = \alpha_p - \alpha_n
\]
Introduction

2.) Peltier Effect

\[ Q = \alpha_{total} \frac{T_H}{C} I \]

3.) Thomson Effect

Heat is proportional to electric current and temperature difference.

small enough to be neglected
Introduction

- Applications
Governing Equation

*Used in COMSOL*

\[ \rho C_P \frac{\partial T}{\partial t} + \nabla (-\kappa \nabla T + \alpha TJ) = Q \]

*Steady state (including Joule heating)*

\[ \frac{T_H - T_C}{R_T} - \frac{I^2 R_E}{2} + \alpha IT_H = Q_H \]

\[ \frac{T_H - T_C}{R_T} + \frac{I^2 R_E}{2} + \alpha IT_C = Q_C \]

Conduction heat flow

A *thermoelectric* effect node also adds:

\[ J = -\sigma \left( \nabla V + \alpha \nabla T \right) \]
Governing Equation

Terminal Voltage

\[ P = Q_H - Q_C = IV \]

\[ = V_{OC} I - I^2 R_E \]

Emf generated due to Seebeck effect
Loss due to Joule heating

\[ V_{\text{Terminal}} = V_{OC} - IR_E \]
Governing Equation

- Geometry factor

\[ R_E = R_{E,p} + R_{E,n} = \frac{l}{\sigma_p A} + \frac{l}{\sigma_n A} \]
\[ R_T = R_{T,p} \parallel R_{T,n} = \frac{l}{\kappa_p A + \kappa_n A} \]

When structure varying along the length

\[ R_E = \int dR = \frac{1}{\sigma} \int \frac{dx}{wy} \]
\[ K_T = \int dK = \kappa_{avg} \int \frac{wy}{dx} \]
Thermoelectric Unicouple

- Different shape & size of unicouple for simulation

Dimension: - 6.22 x 2.98 x 2.215 mm³
Tapering factor: - 0.25 & 0.5
Thermoelectric Unicouple

- Temperature dependent material properties for Mg2Si
  - High Seebeck coefficient measured in Ga-doped samples
  - Electrical conductivity slowly increases in the whole temperature range
  - Operate under large temperature ranges (250-850 K)
  - Electronic thermal conductivity is much lower than the lattice

- H. Ihou-Mouko et. al., Thermoelectric Properties and Electronic Structure of p-type Mg2Si and Mg2Si0.6Ge0.4 Compounds Doped with Ga, Journal of Alloys and Compounds, 509 (2011) 6503–6508.
Simulation in COMSOL

- **STEPS**

  - Electric Currents \( (ec) \)
  - Current Conservation 1
    
    Study 1, Stationary
    
    \[ \nabla \cdot \mathbf{j} = Q_i \]
    
    \[ \mathbf{j} = \sigma \mathbf{E} + \mathbf{J}_e \]
    
    \[ \mathbf{E} = -\nabla \mathbf{V} \]
  - Multiphysics
    
    Thermoelectric Effect 1 \( (tee1) \)
    
    Study 1, Stationary
    
    \[ \rho C_p \mathbf{u} \cdot \nabla T = \nabla \cdot (k \nabla T - \mathbf{P} \cdot \mathbf{j}) + Q \]

- **COMSOL Multiphysics 5.1**

- **HEAT TRANSFER IN SOLIDS \( (ht) \)**

- **HEAT TRANSFER IN SOLIDS 1**

- **Mesh 1**

- **Study 1**

  - Step 1: Stationary
    
    Sequence type:
    
    Physics-controlled mesh
    
    Element size:
    
    Finer

- **Results**

  - Data Sets
  - Derived Values
  - Tables

Marked Hidden
Results

Voltage vs. Current

- Rectangular
- Trapezoidal
- p(reduced)-n
- p(hourglass)-n
- p(pyramid)-n
- p(wrapped)-n

Terminal Voltage (V) vs. Input Current $I$ (A)
Results

Voltage vs. Temperature

ZT vs. Temperature
Results

- Heat flux vs. Current

![Graph showing heat flux vs. current for different shapes: Rectangular, Trapezoidal, p(reduced)-n, p(hourglass)-n, p(pyramid)-n, and p(wrapped)-n.](image)
References

- Christophe Goupil et al., Thermodynamics of Thermoelectric Phenomena and Applications, Entropy, (2011), 13, 1481-1517
- Yu Mu et al., Effect of Geometric Dimensions on Thermoelectric and Mechanical Performance for Mg2Si-based Thermoelectric Unicouple, Materials Science in Semiconductor Processing, 17 (2014) 21-26
- COMSOL Multiphysics 5.1, Heat Transfer Module User’s Guide
- H. Ihou-Mouko et. al., Thermoelectric Properties and Electronic Structure of p-type Mg2Si and Mg2Si0.6Ge0.4 Compounds Doped with Ga, Journal of Alloys and Compounds, 509 (2011) 6503–6508.
Thanking You!